Error Performance of Serially Concatenated Space-Time Coding

Ibrahim Altunbas and Abbas Yongacoglu

Abstract: **In this paper, we investigate the error performance of a serially concatenated system using a nonrecursive convolutional code as the outer code and a recursive QPSK space-time trellis code as the inner code on quasi-static and rapid Rayleigh fading channels. At the receiver, we consider iterative decoding based on the maximum a posteriori (MAP) algorithm. The performance is evaluated by means of computer simulations and it is shown that better error performance can be obtained by using low complexity outer and/or inner codes and the Euclidean distance criterion based recursive space-time inner codes. We also obtain new systems with large number of trasmit and/or receive antennas providing good error performance.**

Index Terms: **Digital communication, information theory, serial concatenated coding, space-time coding, iterative decoding.**

I. INTRODUCTION

Telatar [1] and Foschini and Gans [2] have shown that the capacity of fading channels can be increased by employing multiple transmit and receive antennas. Since then, there has been an explosion of interest on multiple antenna systems due to the rapidly growing demand for reliable high data rate transmissions in wireless communications. Space-time coding is a way to increase the possible capacity [3] and [4]. It combines the benefits of forward error correction coding and diversity transmission to overcome the impairments of wireless channels. The design criteria for conventional space-time codes are maximizing the minimum rank and the minimum determinant of the distance matrices constructed from all possible pairs of distinct transmission sequences for quasi-static fading channel [3]. These design criteria are called the rank and determinant (RD) criteria. The rank criterion is used for achieving maximum diversity gain, while the determinant criterion is used for maximizing the coding gain. A number of trellis codes that provide maximum diversity and good coding advantage have also been presented in [3]. Later, some new codes (e.g., [5]) which have better error performance have been introduced. In [6], Chen *et al.* showed that, for quasi-static fading channel, RD design criteria are no longer valid for the large number of transmit and/or receive antenna case. When large number of transmit and/or receive antennas are used in the system, the multiple input multiple output (MIMO) channel converges to an additive white Gaussian noise (AWGN) channel and therefore the error performance is determined by the minimum Euclidean distance between all possible pairs of distinct transmission sequences. In this case, one should maximize the minimum Euclidean distance in the code. This criterion is called Euclidean distance (ED) criterion. For rapid fading channels, the design criteria for conventional space-time codes are maximizing the minimum symbol Hamming distance and minimum product distance [3]. These criteria are called Hamming distance and product distance (HDPD) criteria. In [7], it was shown that the ED criterion is also valid for rapid fading channels when the system has large number of transmit and/or receive antennas.

Turbo codes proposed by Berrou *et al.* [8] represent a recent breakthrough in coding theory. They were originally introduced as binary error-correcting codes built from the parallel concatenation of two recursive systematic convolutional codes exploiting iterative decoding algorithm. It was shown that turbo codes can perform close to the Shannon limit in AWGN channels. Beyond the form of parallel concatenation, different forms of concatenation such as serial concatenation [9] have been studied. In recent years, several schemes that combine space-time and parallel or serial concatenated codes (e.g., [10]–[13]) were proposed and it was shown that these schemes perform much better than conventional space-time codes of similar complexity. In [10], a serially concatenated coding scheme with a space-time code as the outer code and a rate-1 recursive code as the inner code on each transmit antenna was proposed. In [11], Tujkovic introduced some recursive forms of the Tarokh *et al.*'s codes [3] and proposed parallel and serial concatenated space-time coding schemes based on these recursive codes. The serial concatenation of convolutional outer codes or single parity check code based turbo product outer codes with space-time inner codes were proposed by Gulati and Narayanan [12]. In [13], Firmanto *et al.* reported some new parallel concatenated space-time coding schemes using recursive space-time trellis codes, as the constituent codes, which are designed according to the design criteria that depends on the diversity order [7].

In this paper, we investigate, by computer simulations, the performance of a serially concatenated system with a nonrecursive convolutional outer code and a recursive QPSK space-time trellis inner code on quasi-static and rapid Rayleigh fading channels. In such a serial concatenated scheme, since the inner code is a space-time code, full diversity is always obtained if the inner code has full diversity. Our aim is to investigate the effects of some system parameters such as complexity of the outer and/or inner code and type of the inner code and to obtain new serial concatenated systems with various number of transmit and receive antennas providing good error performance. We first con-

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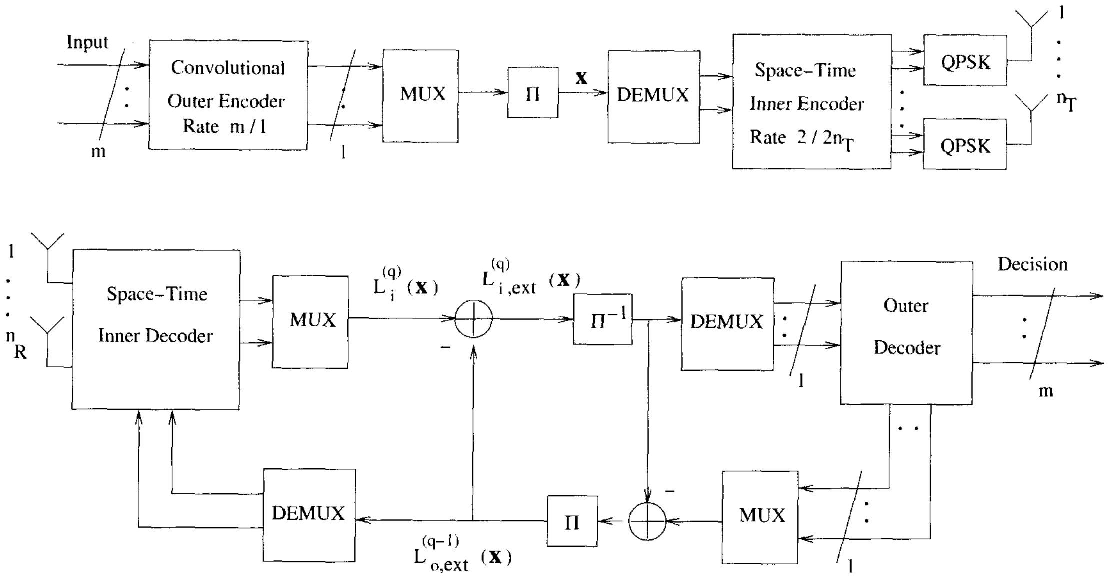

Fig. 1. Block diagram of the serial concatenated QPSK space-time system: Transmitter and Receiver.

sider the rate-1/2 convolutional outer code case which results in relatively low bandwidth efficiency (1 bit/sec/Hz), but improved error performance. Then we evaluate a high rate (4/5) convolutional outer code case which results in increased bandwidth efficiency (1.6 bits/sec/Hz) with respect to the rate-1/2 case and good error performance. Throughout the paper, we consider the case of coherent demodulation with ideal channel state information where the fading coefficients are perfectly known to the receiver (due to channel estimation) but unknown to the transmitter. We observe that when we use recursive forms of some of the previously known space-time trellis codes as the inner codes, the ones designed according to the ED criterion lead to better error performance for both types of channels. We show that the increasing number of states of the outer and inner codes actually results in worse performance when iterative decoding is used.

The rest of the paper is organized as follows. Section II describes the serially concatenated space-time system under consideration and presents the principles of the transmitter and receiver. The error performance of the system is evaluated by simulations in Section III. The conclusions are presented in Section IV.

II. SYSTEM MODEL

We consider a serially concatenated MIMO communication system that employs n_T antennas at the transmitter and n_R antennas at the receiver. Block diagram of the system is depicted in Fig. 1. In this system, a block of K independent data bits are encoded by a rate-m/l convolutional outer encoder whose output is a block of $N = K(l/m)$ coded bits. After the multiplexing, the binary sequence is interleaved by using a random interleaver (Π) with length N and the interleaved bit sequence $\mathrm{x} = (x_1, x_2, \cdots, x_N)$ is demultiplexed into two (even and odd) bit streams and then both streams are input to the inner encoder which is essentially a space-time trellis encoder. The inner encoder has a rate of $2/(2n_T)$ to encode incoming bit pairs to $2n_T$ output bits. The $2n_T$ bits of the output of the space-time encoder are mapped onto n_T QPSK symbols. In vector representation, the QPSK symbols can be shown as $0 \longleftrightarrow (\sqrt{E_s},0)$, $1 \longleftrightarrow (0,\sqrt{E_s})$, $2 \longleftrightarrow (-\sqrt{E_s},0)$, $3 \longleftrightarrow (0,-\sqrt{E_s})$ where E_s is the average energy per symbol. At time n, a QPSK symbol s_n^i is transmitted throught the ith transmit antenna, $i = 1, 2, \cdots, n_T$. All symbols are transmitted simultaneously, each from a different transmit antenna, and all symbols have the same transmission interval. This system achieves a bandwidth efficiency of $2m/l$ bits/sec/Hz.

Let the sequence $\mathrm{s}=[\mathrm{s}_1, \mathrm{s}_2, \cdots, \mathrm{s}_L]$ be transmitted where $\mathrm{s}_n = [s_n^1, s_n^2, \cdots, s_n^{n_T}]^T$, $n = 1, 2, \cdots, L$. Here, T denotes tranpose and L corresponds to the frame length of the transmitted symbol sequence for each antenna. After the demodulation (not shown in the figure), the corresponding $\mathrm{r}=[\mathrm{r}_1, \mathrm{r}_2, \cdots, \mathrm{r}_L]$ sequence is received at the receiver. Here $\mathrm{r}_n = [r_n^1, r_n^2, \cdots, r_n^{n_R}]^T$ and at time n, the received symbol at antenna j, $j = 1, 2, \cdots, n_R$ is given by

$$r_n^j = \sum_{i=1}^{n_T} \rho_n^{i,j} s_n^i + \eta_n^j, \qquad n = 1, 2, \cdots, L, \tag{1}$$

where $\rho_n^{i,j}$ is the fading coefficient for the path from transmit antenna i to receive antenna j and η_n^j is noise which is modeled as independent samples of a zero mean complex Gaussian random variable with variance $N_0/2$ per dimension. We define the signal to noise ratio (SNR) per receive antenna as E/N_0 where $E = n_T E_s$ is the total transmitted energy at each transmission interval.

We consider flat Rayleigh fading and spatially independent channels, i.e., fading is statistically independent from one transmitter-antenna pair to the other. Therefore, $\rho_n^{i,j}$ coefficients are modeled as samples of independent zero mean complex Gaussian random variables with variance 0.5 per dimension and satisfy $E\{|\rho_n^{i,j}|^2\}=1$. In this paper, we evaluate quasi-static and rapid Rayleigh fading channel cases. At the quasi-static fading case, the fading coefficients are constant over the entire duration of a frame but change independently from one frame to another. For the rapid fading case, the coefficients change independently from one symbol to another.

As shown in Fig. 1, the receiver uses a message passing decoder, which passes messages (extrinsic log likelihood ratios, LLRs) [8] between the soft output inner decoder (space-time decoder) and an outer decoder in an iterative fashion. Both inner and outer decoders use the nonbinary maximum a posteriori (MAP) [8] algorithm. Clearly, when $m = 1$, the outer decoder uses the binary MAP algorithm. A maximum of Q iterations between the inner and outer decoder are used. During the qth iteration ($q = 1, 2, \cdots, Q$), the inner decoder uses $\mathbf{R}^{(q)} = (\mathbf{r}, L_{o,ext}^{(q-1)}(\mathbf{x}))$ where $L_{o,ext}^{(q-1)}(\mathbf{x})$ is the interleaved extrinsic information (see Fig. 1) obtained from the outer decoder in the $(q-1)$th iteration. The space-time decoder produces LLRs for each bit in the sequence $\mathbf{x}$, given by

$$L_i^{(q)}(x_k) = \log \frac{P(x_k = 0|\mathbf{R}^{(q)})}{P(x_k = 1|\mathbf{R}^{(q)})}, \qquad (2)$$

where x_k is the kth element of $\mathbf{x}$, $k = 1, 2, \cdots, N$. The extrinsic information obtained from the inner decoder can be written as $L_{i,ext}^{(q)}(x_k) = L_i^{(q)}(x_k) - L_{o,ext}^{(q-1)}(x_k), \forall k$. This extrinsic information is deinterleaved by using a deinterleaver (Π^{-1}) and, after the demultiplexing, input to the outer decoder. The extrinsic information obtained from the outer decoder is interleaved and input to the space-time decoder.

III. SYSTEM PERFORMANCE

It is well known from [9] that in order to obtain "interleaving gain" and therefore good error performance in a serially concatenated system, the inner encoder must be a recursive encoder. There is no tight constraint on the outer encoder, however, it should be a nonrecursive encoder when the inner encoder is recursive. In [14], the authors proved that these design criteria are valid for serially concatenated space-time systems as well. Therefore, in this paper, we use recursive space-time encoders as the inner encoders.

We evaluate the error performance of the system considered on the quasi-static and rapid Rayleigh fading channels by computer simulations. In all simulations, the number of iterations is Q=8, each frame consists of L=130 symbols out of each transmit antenna and the interleaver between the outer and inner encoder is S-type random interleaver with length N=260 bits and S=5. We terminate both outer and inner encoders by using appropriate tail bits. We consider the rate-1/2 and 4/5 nonrecursive convolutional outer code cases and plot the frame error rate (FER) curves versus SNR per receive antenna.

A. Rate-1/2 Convolutional Outer Code

When the convolutional outer code rate is 1/2, the bandwidth efficiency of the system becomes 1 bit/sec/Hz with QPSK modulation, ignoring the tail bits used. In this subsection, the input block length is K=130 bits (in order to transmit L=130 QPSK symbols from each antenna) and we use a standart nonrecursive 4-state convolutional outer code with generator polynomial of [5 7], represented in octal, and having a free Hamming distance of 5.

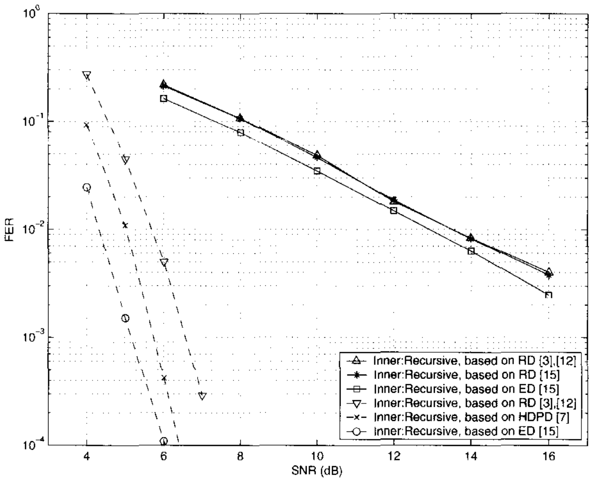

Fig. 2. Error performance of the serially concatenated system with different type of inner encoders. Outer rate-1/2 nonrecursive convolutional encoder, N=260, n_T=2, n_R=1. Solid: Quasi-static fading channel, Dash: Rapid fading channel.

Here, we first investigate the error performance of the serial concatenated systems employing recursive space-time inner encoders based on the ED criterion, the RD criteria and the HDPD criteria for n_T=2 and n_R=1. Toward this goal, we evaluate several systems and use recursive versions of the following 4-state space-time trellis codes as the inner codes in these systems: The first one is Chen *et al.*'s [6] and [15] space-time code based on the ED criterion, the second one is Tarokh *et al.*'s code [3] and [12] based on the RD criteria, the third one is again Chen *et al.*'s [15] code based on the RD criteria and the fourth one is Yuan *et al.*'s [7] code based on the HDPD criteria. In order to obtain recursive versions of the codes, we only redefined the input-output transitions of the original codes as done in [12]. Note that all of the inner codes achieve full diversity, therefore the systems using these codes as the inner code achieve full diversity.

Fig. 2 compares the frame error performance of the systems employing recursive space-time inner encoders based on the ED criterion and the RD criteria over quasi-static fading channel. Here, we evaluate three systems for quasi-static fading channel. As seen from the figure, even though the serially concatenated system has small number of antennas (n_T=2, n_R=1), the ED criterion for the inner code leads to better error performance. We observed the same property for the larger number of antennas as well. In Fig. 2, we also compare the performance of the systems using recursive space-time inner encoders based on the ED criterion, the RD criteria and the HDPD criteria over rapid fading channel. It appears from the figure, the system using inner code based on the ED criterion achieves better error performance over the rapid fading channel as well.

Fig. 3 illustrates the effect of number of the states of the outer and/or inner codes. Here, the system has n_T=3 transmit and n_R=1 receive antennas. We evaluate the systems using recursive versions of Chen *et al.*'s [15] 4 and 16-state ED criterion based space-time codes as the inner code and 4-state and 16-

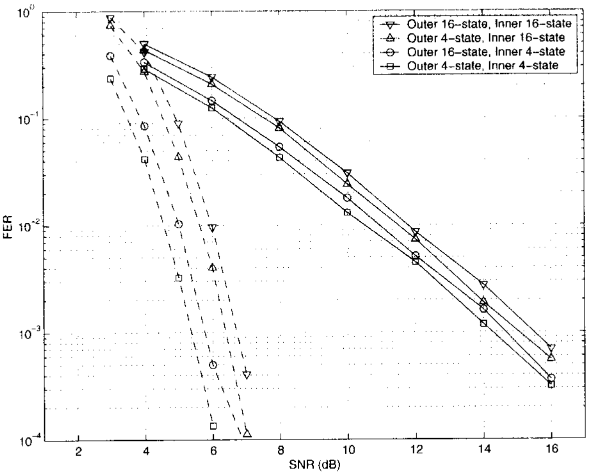

Fig. 3. Error performance of the serially concatenated system with different numbers of inner code state. Outer rate-1/2 nonrecursive convolutional encoder, N=260, n_T=3, n_R=1. Solid: Quasi-static fading channel, Dash: Rapid fading channel.

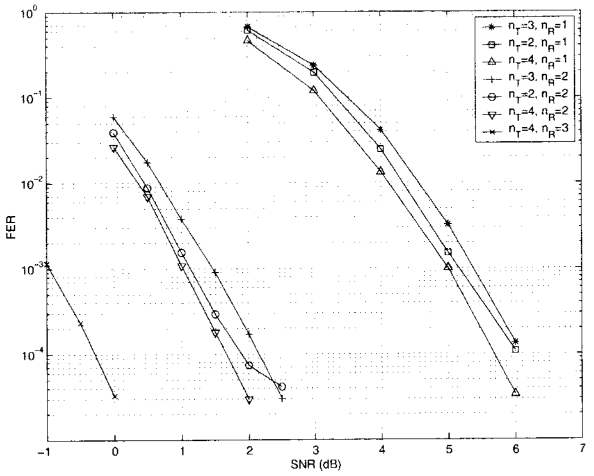

Fig. 5. Error performance of the serially concatenated systems with different number of antennas over rapid fading channels. Outer rate-1/2 nonrecursive convolutional encoder, N=260.

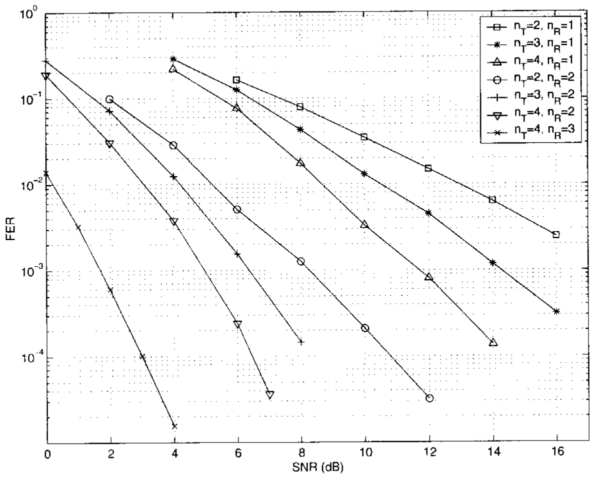

Fig. 4. Error performance of the serially concatenated systems with different number of antennas over quasi-static fading channels. Outer rate-1/2 nonrecursive convolutional encoder, N=260.

state nonrecursive convolutional codes as the outer codes. The 4-state outer code has a generator polynomial of [5 7] and free Hamming distance of 5 (the same code used above). The 16-state outer code has a generator polynomial of [23 35] and free Hamming distance of 7. As seen from the figure, the system with 4-state outer and 4-state inner code performs the best performance for both types of channels and increasing the number of states does not improve the performance, but rather makes it worse. This phenomenon is typical in the iterative decoding [12].

In Fig. 4, we show the performance when the system has various number of transmit and/or receive antennas over quasi-static fading channels. Here, the inner codes are recursive versions of the 4-state the ED criterion based space-time codes given in [15]. As seen, the SNR value required for a given FER decreases as the number of transmit and/or receive antennas increases. For example, at a FER of 10^{-2}, the system with n_T=2, n_R=2 provides 6.8 dB SNR improvement over the system with n_T=2, n_R=1. For n_R=2, at a FER of 10^{-3}, the performance improvement is 1.8 and 1.3 dB when n_T is increased from 2 to 3 and from 3 to 4, respectively. As expected, the system with n_T=4, n_R=3 achieves the best error performance with respect to the other systems evaluated.

In Fig. 5, the performance is demonstrated for several number of transmit and/or receive antennas over rapid fading channel. In general, the systems exhibit better performance as the number of transmit and/or receive antennas increases. For example, at a FER of 10^{-3}, for n_T=4, the system with n_R=3 yields 2 dB and 6 dB SNR gain over the systems with n_R=2 and n_R=1, respectively. However, for n_R=1 and also for n_R=2, when n_T is increased from 2 to 3, we observe worse error performance. Such kind of behaviour was also observed in [15] for conventional space-time codes over quasi-static fading channels. If we compare Fig. 4 and Fig. 5, we can see that increasing the number of transmit antennas provides larger SNR gain for quasi-static fading channels than that of rapid fading channels.

B. Rate-4/5 Convolutional Outer Code

When the convolutional outer code rate is 4/5, with QPSK modulation, the bandwidth efficiency of the system increases to 1.6 bits/sec/Hz. Here, we use input block length of K=208 bits and interleaver length of N=260 bits in order to transmit L=130 QPSK symbols from each antenna. We use a nonrecursive 4-state convolutional outer code with a generator polynomial of [67 15 26 52 57] and free Hamming distance of 2 [16]. We are aware of the case that such an outer code does not provide an interleaving gain [9] when a maximum likelihood decoder, which is almost impossible to implement in practice for concatenated systems, is employed at the receiver. This is due to the fact that its free Hamming distance is smaller than 3. In that case, to obtain a better error performance, a convolutional code with larger free Hamming distance can be used. However, the error performance behaviour of the iterative decoder for this case is quite different and increasing the free Hamming distance from 2 to 3 results in worse performance. Indeed, we have also simulated the system by using an 8-state rate-4/5 non-

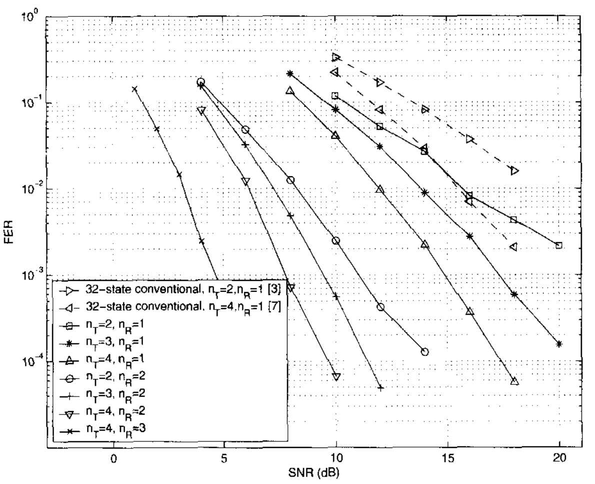

Fig. 6. Error performance of the serially concatenated systems with different number of antennas over quasi-static fading channels. Outer rate-4/5 nonrecursive convolutional encoder, N=260.

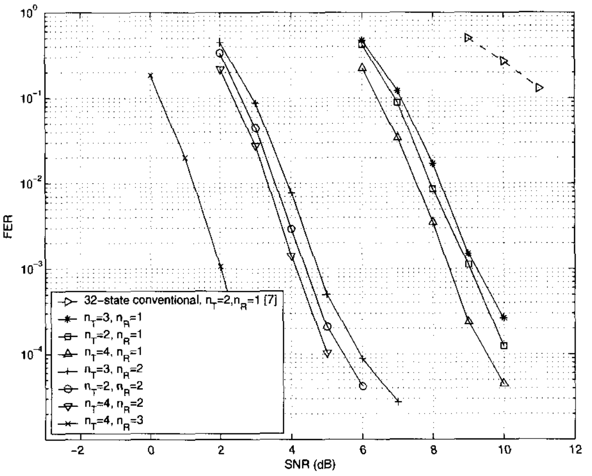

Fig. 7. Error performance of the serially concatenated systems with different number of antennas over rapid fading channels. Outer rate-4/5 nonrecursive convolutional encoder, N=260.

recursive convolutional outer code with generator polynomial of [013 023 056 132 174] and with free Hamming distance of 3 [16] and observed worse error performance with respect to that of 4-state rate-4/5 nonrecursive convolutional outer code. This phenomenon is similar to that in the previous section. Thus, we use a convolutional outer code with free Hamming distance of 2.

For n_T=2 and n_R=1, we have evaluated the performance of the systems with recursive space-time inner codes based on the ED, RD and HDPD criteria and observed that, as in the previous section, the ED criterion for inner code leads to better error performance for both quasi-static and rapid fading channels. The error performance of the system using the 4-state convolutional outer code and recursive space-time inner codes based on the ED criterion is illustrated in Fig. 6 and in Fig. 7 when the system has various number of transmit and/or receive antennas for quasi-static and rapid fading channels, respectively. The performance behavior of the systems is similar to those of the systems with rate-1/2 outer code. Increasing the number of transmit and/or receive antennas, generally, leads to better performance. We also compare our results with Tarokh *et al.*'s [3] and Yuan *et al.*'s [7] conventional 32-state QPSK space-time codes with 2 bits/sec/Hz bandwidth efficiency. Note that Tarokh *et al.*'s code is based on the RD criteria and one of Yuan *et al.*'s code (in Fig. 6) is based on the ED criterion, the other (in Fig. 7) is based on the HDPD criteria. As seen from the figures, the serial concatenated systems which actually have 0.4 bit/sec/Hz bandwidth efficiency penalty outperform the conventional codes.

IV. CONCLUSION

We studied a serially concatenated coding scheme with multiple transmit and/or receive antennas. In particular, we used a rate-1/2 and a rate-4/5 nonrecursive convolutional code as the outer code and a recursive QPSK space-time trellis code as the inner code. The channel was subject to quasi-static and rapid Rayleigh fading. We compared the performance of the systems which use recursive space- time inner codes based on the ED criterion, the RD criteria and the HDPD criteria and observed that the inner codes based on the ED criterion provide better error performance for both types of channels. We showed that the increasing number of states of the outer and/or inner codes actually results in worse performance when iterative decoding is used. We have obtained new systems with low complexity outer and inner codes and multiple transmit and/or receive antennas which give good error performance.

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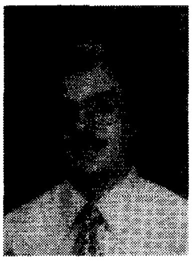

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