Noncoherent Detection of Orthogonal Modulation Combined with Alamouti Space-Time Coding

Marvin K. Simon and Jibing Wang

Abstract: **In this paper, we investigate the error probability performance of noncoherently detected orthogonal modulation combined with Alamouti space-time block coding. We find that there are two types of pair-wise error probabilities that characterize the performance. We employ methods that allow a direct evaluation of exact, closed-form expressions for these error probabilities. Theoretical as well as numerical results show that noncoherent orthogonal modulation combined with space-time block coding (STBC) achieves full spatial diversity. We derive an expression for approximate average bit error probability for M-ary orthogonal signaling that allows one to show the tradeoff between increased rate and performance degradation.**

Index Terms: **Digital communications, fading channels, noncoherent detection, space-time coding.**

I. INTRODUCTION

The next generation of broadband wireless communications systems is expected to provide users with wireless multimedia services such as high-speed Internet access, wireless television, and mobile computing [1]. The rapid growing demand for these services is driving the communication technology towards higher data rates and higher mobility transmissions over mobile radio channels. However, reliable communications are challenged by the physical limitation of the wireless channel. Multiple input multiple output (MIMO) wireless systems have become an active research area (see e.g., [2] and references therein). It is well known that MIMO systems promise very high data rates with low error probabilities, especially for the case that the channel is known at the receiver [3], [4]. Several design techniques for so-called space-time codes when the receiver has complete knowledge of the channel statistics have been developed (see e.g., [5]–[8]). This knowledge is often obtained from pilot signals periodically sent to the receiver. However, in certain scenarios, the assumption that channel state information is available at the receiver is questionable. In particular, in a rapidly changing mobile environment or when multiple transmit and receive antennas are employed, channel estimation is either costly or even impossible. As such, one is motivated to investigate space-time coding techniques that are capable of performing well without knowledge of channel estimates [9]–[21].

It is shown in [9]–[11] that high capacities with multiple antennas are achievable with no channel state information at either the transmitter or the receiver. For the noncoherent MIMO channel, Marzetta and Hochwald proposed a class of signals called unitary space-time signals and showed that by combining them with channel coding one could achieve a high fraction of the channel capacity [11], [13]. To help make noncoherent MIMO communication practical, differential modulation techniques have been proposed (see e.g., [15]–[21]). These techniques can be regarded as a generalization of the standard differential phase-shift-keying (DPSK) signals commonly used with a single antenna over an unknown channel.

Manuscript received January 31, 2003.

M. K. Simon is with the Jet Propulsion Laboratory. Currently a Visiting Scholar in the Department of Electrical Engineering, University of California, Los Angeles (UCLA), email: marvin.k.simon@jpl.nasa.gov.

J. Wang is with the Department of Electrical Engineering, University of California, Los Angeles (UCLA), email: wangjb@ucla.edu.

Traditionally, frequency-shift-keying (FSK) is a standard signaling scheme for noncoherent detection with a single transmit antenna. In this paper, we investigate the performance of noncoherent detection of an orthogonal FSK modulation combined with Alamouti's space-time block coding (STBC) scheme [7]. While, in principle, the performance of this scheme could be analyzed within the framework of unitary space- time signals in [11], here we employ methods that more directly allow an explicit evaluation of exact, compact closed-form expressions for the pair-wise error probabilities (PEP) in the presence of an arbitrary number of receive antennas. We find that there are two types of PEP that characterize the performance. For the type I PEP, we utilize the cumulative distribution function (CDF) of the decision statistic, expressed as an inverse Laplace transform involving the moment generating function (MGF) of this statistic, to obtain the exact PEP for Rayleigh fading channels. This technique has previously been applied with success to coherent space-time coding systems [22]. For the type II error, we employ the approach taken in [23] to evaluate the exact PEP. In the case of the latter, our closed-form PEP expression is given directly in terms of the MGF (and its derivatives) of the underlying fading model thereby allowing one to obtain this performance for the generalized fading channel, an example of which might be Nakagami fading. Our PEP results show that noncoherent FSK combined with STBC achieves full spatial diversity. We also derive an approximate average bit error probability (BEP) for M-ary orthogonal signaling to show the tradeoff between increased rate and performance degradation.

We begin by deriving the maximum-likelihood (ML) metric for noncoherent Alamouti space-time coding over a Rayleigh fading channel analogous to the approach used in [23, Chap. 7] for the single transmit antenna case. Following this, we use the ML metric to evaluate the various system performances described above, ending the paper with a discussion of numerical results illustrating the behavior of the expressions derived from the analyses. While, unlike the coherent and differential STC cases, the combination of an Alamouti code and noncoherent FSK modulation will not result in a metric that partitions so as to allow for separate decisions on the two component symbols of the space-time symbol, it nevertheless represents a simple con-

1229-2370/03/$10.00 © 2003 KICS

stant envelope STBC scheme that achieves full spatial diversity (whereas, for example, the designs in [11] and [13] do not). Furthermore, it is able to perform without the need for any channel state information whatsoever or assumptions on the continuity of the channel gain from space-time symbol to space-time symbol - our scheme only requires that the gain be constant across two FSK symbols. If, in the differential space-time modulation combined with an Alamouti code case the above assumption of continuity cannot be justified, i.e., the channel changes rapidly, then this system suffers from a severe performance degradation even for moderately fast fading [26].

II. THE MAXIMUM-LIKELIHOOD DECISION RULE FOR RAYLEIGH CHANNELS

To keep matters simple at first, we begin by assuming a single receive antenna. Let c_{1j} denote the complex channel gain from the jth transmit antenna ($j = 1, 2$) to the single receive antenna. Then for the Alamouti code [7], the two received complex baseband signals are given by

$$\begin{aligned} y(t) &= c_{11}x_1(t) + c_{12}x_2(t) + n(t),\ 0 \le t \le T_s \\ y(t) &= -c_{11}x_2^*(t - T_s) + c_{12}x_1^*(t) \\ &\quad + n(t - T_s),\ T_s \le t \le 2T_s, \end{aligned} \tag{1}$$

where $x_1(t), x_2(t)$ represents the complex data symbol pair transmitted over the channel and $n(t)$ is a complex Gaussian zero mean noise with $E\{n(t)n^*(\tau)\} = (N_0/E_s)\delta(t-\tau) \triangleq 2\sigma_N^2\delta(t-\tau)$. The conditional (on the channel) likelihood function is given by[1]

$$\begin{aligned} &p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{c}) \\ &= C_0\exp\Big\{-\tfrac{1}{2\sigma_N^2}\Big[\int_0^{T_s}|y(t)-(c_{11}x_1(t)+c_{12}x_2(t))|^2dt \\ &+ \int_{T_s}^{2T_s}|y(t)-(-c_{11}x_2^*(t-T_s)+c_{12}x_1^*(t-T_s))|^2dt\Big]\Big\}, \end{aligned} \tag{2}$$

which after some simplification can be expressed as

$$\begin{aligned} &p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{c}) \\ &= C_0\exp\Big\{-\tfrac{1}{2\sigma_N^2}\Big[\int_0^{T_s}|y(t)|^2dt+\int_{T_s}^{2T_s}|y(t)|^2dt \\ &+(|c_{11}|^2+|c_{12}|^2)\big(\int_0^{T_s}|x_1(t)|^2dt+\int_0^{T_s}|x_2(t)|^2dt\big) \\ &-2|c_{11}|\Big|\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big| \\ &\quad\times\cos(\theta_{11}+\phi_1) \\ &-2|c_{12}|\Big|\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big| \\ &\quad\times\cos(\theta_{12}+\phi_2)\Big]\Big\}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \theta_{11} &= \arg c_{11}, \\ \phi_1 &= \arg\Big(\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big), \\ \theta_{12} &= \arg c_{12}, \end{aligned}$$

$$\phi_1 = \arg\Big(\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big). \tag{4}$$

Assuming uniformly distributed channel phases θ_{11} and θ_{12} (as would be the case for Rayleigh fading), and averaging over these phases, we obtain

$$\begin{aligned} &p(\boldsymbol{y}|\boldsymbol{x},|c_{11}|,|c_{12}|) \\ &= C_0\exp\Big\{-\tfrac{1}{2\sigma_N^2}(|c_{11}|^2+|c_{12}|^2) \\ &\qquad\times\big(\int_0^{T_s}|x_1(t)|^2dt+\int_0^{T_s}|x_2(t)|^2dt\big)\Big\} \\ &\times I_0\Big(\tfrac{1}{\sigma_N^2}|c_{11}|\Big|\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big|\Big) \\ &\times I_0\Big(\tfrac{1}{\sigma_N^2}|c_{12}|\Big|\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big|\Big), \end{aligned} \tag{5}$$

where we have also absorbed the data independent terms involving into the constant C_0. Next, we assume a modulation such that $x_1(t)$ and $x_2(t)$ are normalized to unit energy. Then (5) simplifies to

$$\begin{aligned} &p(\boldsymbol{y}|\boldsymbol{x},|c_{11}|,|c_{12}|) \\ &= C_0\exp\Big(-\tfrac{|c_{11}|^2}{\sigma_N^2}\Big) \\ &\times I_0\Big(\tfrac{1}{\sigma_N^2}|c_{11}|\Big|\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big|\Big) \\ &\times\exp\Big(-\tfrac{|c_{12}|^2}{\sigma_N^2}\Big) \\ &\times I_0\Big(\tfrac{1}{\sigma_N^2}|c_{12}|\Big|\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big|\Big). \end{aligned} \tag{6}$$

Finally, assuming that $|c_{11}|$ and $|c_{12}|$ are independent (but not necessarily identically) Rayleigh distributed amplitudes, then averaging over these distributions we obtain after some simplification the unconditional likelihood function

$$\begin{aligned} &p(\boldsymbol{y}|\boldsymbol{x}) \\ &= C_0\Big(\tfrac{1}{1+2\bar\gamma_{11}}\Big)\Big(\tfrac{1}{1+2\bar\gamma_{12}}\Big) \\ &\times\exp\Big\{\tfrac{E_s}{N_0}\Big[\tfrac{\bar\gamma_{11}}{1+2\bar\gamma_{11}}\Big|\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big|^2 \\ &+\tfrac{\bar\gamma_{12}}{1+2\bar\gamma_{12}}\Big|\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big|^2\Big]\Big\}, \end{aligned} \tag{7}$$

where

$$\gamma_{11} = |c_{11}|^2E_s/N_0, \gamma_{12} = |c_{12}|^2E_s/N_0 \tag{8}$$

are the instantaneous fading SNRs per transmit antenna for the two channels and $\bar\gamma_{11}, \bar\gamma_{12}$ are their corresponding statistical averages[2]. Taking the natural logarithm of (7) and ignoring data-independent terms and normalization constants we obtain

$$\begin{aligned} &\ln p(\boldsymbol{y}|\boldsymbol{x}) \\ &= \tfrac{\bar\gamma_{11}}{1+2\bar\gamma_{11}}\Big|\int_0^{T_s}y(t)x_1^*(t)dt-\int_{T_s}^{2T_s}y(t)x_2(t-T_s)dt\Big|^2 \\ &+\tfrac{\bar\gamma_{12}}{1+2\bar\gamma_{12}}\Big|\int_0^{T_s}y(t)x_2^*(t)dt+\int_{T_s}^{2T_s}y(t)x_1(t-T_s)dt\Big|^2 \end{aligned} \tag{9}$$

[1] The vector notation $\boldsymbol{y}$ and $\boldsymbol{x}$ is used merely as a shorthand means of representing the corresponding waveforms $y(t)$ and $x(t)$ as might be the case in a Karhunen-Loeve or sampling expansion of these processes. Furthermore, C_0 is a normalization constant that is irrelevant in so far as the detection process is concerned and $\boldsymbol{c}$ is the vector of channel gains c_{11} and c_{12}.

[2] Where convenient, we shall use an overbar to denote statistical expectation.

i.e., a weighted square-law metric. Thus, for the Rayleigh channel, the maximum-likelihood (ML) decision is to choose $x_1(t) = \hat{x}_1(t)$and $x_2(t) = \hat{x}_2(t)$ such that

$$\begin{aligned}&\hat{x}_1(t), \hat{x}_2(t)\\ &= \max_{x_1,x_2}{}^{-1} \frac{\bar{\gamma}_{11}}{1+2\bar{\gamma}_{11}} \left| \int_0^{T_s} y(t)x_1^*(t)dt - \int_{T_s}^{2T_s} y(t)x_2(t-T_s)dt \right|^2\\ &+ \frac{\bar{\gamma}_{12}}{1+2\bar{\gamma}_{12}} \left| \int_0^{T_s} y(t)x_2^*(t)dt + \int_{T_s}^{2T_s} y(t)x_1(t-T_s)dt \right|^2, \quad (10)\end{aligned}$$

which for identically distributed channels, i.e., $\bar{\gamma}_{11} = \bar{\gamma}_{12} = \bar{\gamma}$, simplifies to

$$\begin{aligned}&\hat{x}_1(t), \hat{x}_2(t)\\ &= \max_{x_1,x_2}{}^{-1} \left| \int_0^{T_s} y(t)x_1^*(t)dt - \int_{T_s}^{2T_s} y(t)x_2(t-T_s)dt \right|^2\\ &\quad + \left| \int_0^{T_s} y(t)x_2^*(t)dt + \int_{T_s}^{2T_s} y(t)x_1(t-T_s)dt \right|^2\\ &\triangleq \max_{x_1,x_2}{}^{-1} m(y, x_1, x_2). \quad (11)\end{aligned}$$

Note that, unlike the coherent detection case, the metric of (9) does not partition to allow for separate decisions on $x_1(t)$ and $x_2(t)$.

It is interesting to note that if one were to try to use say M-PSK as the modulation whereupon $x_1(t)$ and $x_2(t)$ would take on values $e^{j\theta_1}$ and $e^{j\theta_2}$ with θ_i ranging over the set $2\pi m/M, m = 0, 1, \ldots, M-1$, then the metric to be maximized in (11) becomes

$$m(y, x_1, x_2) = 2\left(\left| \int_0^{T_s} y(t)dt \right|^2 + \left| \int_{T_s}^{2T_s} y(t)dt \right|^2 \right) \quad (12)$$

which is independent of the data. Thus, as one might expect, M-PSK cannot be used for noncoherent detection on the space-time channel.

Using (1), the following components are needed to form the ML metric:

$$\begin{aligned}Z_1 &\triangleq \int_0^{T_s} y(t)\hat{x}_1^*(t)dt - \int_{T_s}^{2T_s} y(t)\hat{x}_2(t-T_s)dt\\ &= c_{11} \int_0^{T_s} \left[x_1(t)\hat{x}_1^*(t) + x_2^*(t)\hat{x}_2(t) \right] dt\\ &+ c_{12} \int_0^{T_s} \left[x_2(t)\hat{x}_1^*(t) - x_1^*(t)\hat{x}_2(t) \right] dt + N_1,\\ Z_2 &\triangleq \int_0^{T_s} y(t)\hat{x}_2^*(t)dt + \int_{T_s}^{2T_s} y(t)\hat{x}_1(t-T_s)dt\\ &= c_{11} \int_0^{T_s} \left[x_1(t)\hat{x}_2^*(t) - x_2^*(t)\hat{x}_1(t) \right] dt\\ &+ c_{12} \int_0^{T_s} \left[x_2(t)\hat{x}_2^*(t) + x_1^*(t)\hat{x}_1(t) \right] dt + N_2, \quad (13)\end{aligned}$$

where

$$\begin{aligned}N_1 &\triangleq \int_0^{T_s} n(t)\hat{x}_1^*(t)dt - \int_{T_s}^{2T_s} n(t)\hat{x}_2(t-T_s)dt,\\ N_2 &\triangleq \int_0^{T_s} n(t)\hat{x}_2^*(t)dt + \int_{T_s}^{2T_s} n(t)\hat{x}_1(t-T_s)dt. \quad (14)\end{aligned}$$

Furthermore, it is straightforward to show that

$$\begin{aligned}\overline{N_1 N_2^*} &= 0,\\ \overline{|N_1|^2} = \overline{|N_2|^2} &= 2N_0/E_s. \quad (15)\end{aligned}$$

III. ERROR PROBABILITY PERFORMANCE FOR BINARY ORTHOGONAL FSK MODULATION

Assume now that $x_1(t)$ and $x_2(t)$ take on values from a binary orthogonal FSK modulation with signals $s_1(t)$ and $s_2(t)$ such that

$$\begin{aligned}&\int_0^{T_s} s_1(t)s_2^*(t)dt = 0,\\ &\int_0^{T_s} |s_1(t)|^2 dt = \int_0^{T_s} |s_2(t)|^2 dt = 1. \quad (16)\end{aligned}$$

Furthermore, let $s_1(t)$ correspond to a binary "0" and $s_2(t)$ correspond to a binary "1". Then, if the actual signals transmitted are say $x_1(t) = s_1(t), x_2(t) = s_1(t)$ (corresponding to the binary data sequence 0,0), there are three possible error sequences that can occur. We now compute the PEP for these three error sequences.

From (13) and the orthogonality property in (16), the following ML metrics result:

A. $\hat{x}_1(t) = s_1(t), \hat{x}_2(t) = s_2(t)$

$$\begin{aligned}&|Z_1^A|^2 + |Z_2^A|^2\\ &= |c_{11} + c_{12} + N_1^A|^2 + |-c_{11} + c_{12} + N_2^A|^2. \quad (17)\end{aligned}$$

B. $\hat{x}_1(t) = s_2(t), \hat{x}_2(t) = s_1(t)$

$$\begin{aligned}&|Z_1^B|^2 + |Z_2^B|^2\\ &= |c_{11} - c_{12} + N_1^B|^2 + |c_{11} + c_{12} + N_2^B|^2. \quad (18)\end{aligned}$$

C. $\hat{x}_1(t) = s_2(t), \hat{x}_2(t) = s_2(t)$

$$|Z_1^C|^2 + |Z_2^C|^2 = |N_1^C|^2 + |N_2^C|^2. \quad (19)$$

For the correctly detected sequence, we have

D. $\hat{x}_1(t) = s_1(t), \hat{x}_2(t) = s_1(t)$

$$|Z_1^D|^2 + |Z_2^D|^2 = |2c_{11} + N_1^D|^2 + |2c_{12} + N_2^D|^2. \quad (20)$$

In (17) – (20), the noises are defined as follows:

$$\begin{aligned}N_1^A &\triangleq \int_0^{T_s} n(t)s_1^*(t)dt - \int_{T_s}^{2T_s} n(t)s_2(t-T_s)dt,\\ N_2^A &\triangleq \int_0^{T_s} n(t)s_2^*(t)dt + \int_{T_s}^{2T_s} n(t)s_1(t-T_s)dt,\\ N_1^B &\triangleq \int_0^{T_s} n(t)s_2^*(t)dt - \int_{T_s}^{2T_s} n(t)s_1(t-T_s)dt,\\ N_2^B &\triangleq \int_0^{T_s} n(t)s_1^*(t)dt + \int_{T_s}^{2T_s} n(t)s_2(t-T_s)dt,\\ N_1^C &\triangleq \int_0^{T_s} n(t)s_2^*(t)dt - \int_{T_s}^{2T_s} n(t)s_2(t-T_s)dt,\\ N_2^C &\triangleq \int_0^{T_s} n(t)s_2^*(t)dt + \int_{T_s}^{2T_s} n(t)s_2(t-T_s)dt,\end{aligned}$$

$$N_1^D \triangleq \int_0^{T_s} n(t)s_1^*(t)dt - \int_{T_s}^{2T_s} n(t)s_1(t-T_s)dt,$$
$$N_2^D \triangleq \int_0^{T_s} n(t)s_1^*(t)dt + \int_{T_s}^{2T_s} n(t)s_1(t-T_s)dt. \quad (21)$$

From a comparison of (17) and (18), it is simple to see that the PEP for the error events of case A and B will be identical. We define this PEP, which results in 1 bit error, as being of type I and is computed as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = \Pr\Big\{\sum_{i=1}^{2}|Z_i^A|^2 > \sum_{i=1}^{2}|Z_i^D|^2\Big\} = \Pr\Big\{\sum_{i=1}^{2}|Z_i^D|^2 - \sum_{i=1}^{2}|Z_i^A|^2 < 0\Big\}. \quad (22)$$

Defining $\xi = \sum_{i=1}^{2}|Z_i^D|^2 - \sum_{i=1}^{2}|Z_i^A|^2$ then the probability required in (22) can be evaluated in terms of the MGF of ξ, $M_\xi(s) = E\{e^{s\xi}\} = \int_{-\infty}^{\infty} e^{s\xi}p_\xi(\xi)d\xi$, as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = \Pr\{\xi < 0\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{M_\xi(-s)}{s}ds. \quad (23)$$

To compute the MGF $M_\xi(s)$ for the Rayleigh channel, we make use of a result of Turin [24] for quadratic forms of zero mean complex Gaussian random variables, namely,

$$E\{-s(\boldsymbol{v}^T)^*\boldsymbol{A}\boldsymbol{v}\} = \Big(\det[\boldsymbol{I} + s\Omega\boldsymbol{A}]\Big)^{-1}, \quad (24)$$

where $\boldsymbol{v}$ is a column vector of such random variables, $\boldsymbol{A}$ is an arbitrary Hermetian matrix, and $\Omega = E\{\boldsymbol{v}(\boldsymbol{v}^T)^*\}$ is the covariance matrix of $\boldsymbol{v}$. In our application, $\boldsymbol{v}^T = [Z_1^D Z_2^D Z_1^A Z_2^A]$ and

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad (25)$$

in which case

$$(\boldsymbol{v}^T)^*\boldsymbol{A}\boldsymbol{v} = \sum_{i=1}^{2}|Z_i^D|^2 - \sum_{i=1}^{2}|Z_i^A|^2 = \xi. \quad (26)$$

Using the definitions of Z_i^A and Z_i^D in (17) and (20) and the noise correlation properties in (21), it is straightforward to show that the covariance matrix Ω is obtained as (27) shown at the bottom of this page. After multiplication by the matrix $\boldsymbol{A}$ and adding the identity matrix, we get (28) as shown at the bottom of this page, where $s_0 \triangleq 2sN_0/E_s$ and $\bar{\gamma}_{11}, \bar{\gamma}_{12}$, are defined in (8).

Next, assuming i.i.d. fading, i.e., $\bar{\gamma}_{11} = \bar{\gamma}_{12} = \bar{\gamma}$, (28) simplifies to (29) as shown at the bottom of this page. Finally, taking the determinant of the matrix in (29), we obtain

$$\det[\boldsymbol{I} + s\Omega\boldsymbol{A}] = \Big(\frac{1}{2}(1+2\bar{\gamma})s_0^2 - \bar{\gamma}s_0 - 1\Big)^2. \quad (30)$$

Using (30) in (24) and then substituting the result in (23), we obtain the PEP

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{1}{s_0\Big(\frac{1}{2}(1+2\bar{\gamma})s_0^2 - \bar{\gamma}s_0 - 1\Big)^2}ds_0$$
$$= \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{\Big(\frac{1}{2}(1+2\bar{\gamma})\Big)^{-2}}{s_0\Big(s_0 - \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}+\bar{\gamma}}{1+2\bar{\gamma}}\Big)^2} \times \frac{1}{\Big(s_0 + \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}-\bar{\gamma}}{1+2\bar{\gamma}}\Big)^2}ds_0, \quad (31)$$

which can be evaluated by the residue method as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = -\sum_{\text{R.H.P Poles}}\text{Residue}\left[\frac{\Big(\frac{1}{2}(1+2\bar{\gamma})\Big)^{-2}}{s_0\Big(s_0 - \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}+\bar{\gamma}}{1+2\bar{\gamma}}\Big)^2}\right.$$

$$\Omega = \begin{bmatrix} 4\overline{|c_{11}|^2} + \frac{2N_0}{E_s} & 0 & 2\overline{|c_{11}|^2} + \frac{N_0}{E_s} & -2\overline{|c_{11}|^2} - \frac{N_0}{E_s} \\ 0 & 4\overline{|c_{12}|^2} + \frac{2N_0}{E_s} & 2\overline{|c_{12}|^2} + \frac{N_0}{E_s} & 2\overline{|c_{12}|^2} + \frac{N_0}{E_s} \\ 2\overline{|c_{11}|^2} + \frac{N_0}{E_s} & 2\overline{|c_{12}|^2} + \frac{N_0}{E_s} & \overline{|c_{11}|^2} + \overline{|c_{12}|^2} + \frac{2N_0}{E_s} & \overline{|c_{12}|^2} - \overline{|c_{11}|^2} \\ -2\overline{|c_{11}|^2} - \frac{N_0}{E_s} & 2\overline{|c_{12}|^2} + \frac{N_0}{E_s} & \overline{|c_{12}|^2} - \overline{|c_{11}|^2} & \overline{|c_{11}|^2} + \overline{|c_{12}|^2} + \frac{2N_0}{E_s} \end{bmatrix} \quad (27)$$

$$\boldsymbol{I} + s\Omega\boldsymbol{A} = \begin{bmatrix} 1+(1+2\bar{\gamma}_{11})s_0 & 0 & -\frac{1}{2}(1+2\bar{\gamma}_{11})s_0 & \frac{1}{2}(1+2\bar{\gamma}_{11})s_0 \\ 0 & 1+(1+2\bar{\gamma}_{12})s_0 & -\frac{1}{2}(1+2\bar{\gamma}_{12})s_0 & -\frac{1}{2}(1+2\bar{\gamma}_{12})s_0 \\ \frac{1}{2}(1+2\bar{\gamma}_{11})s_0 & \frac{1}{2}(1+2\bar{\gamma}_{12})s_0 & 1-\Big(1+\frac{1}{2}(\bar{\gamma}_{11}+\bar{\gamma}_{12})\Big)s_0 & \frac{1}{2}(\bar{\gamma}_{11}-\bar{\gamma}_{12})s_0 \\ -\frac{1}{2}(1+2\bar{\gamma}_{11})s_0 & \frac{1}{2}(1+2\bar{\gamma}_{12})s_0 & \frac{1}{2}(\bar{\gamma}_{11}-\bar{\gamma}_{12})s_0 & 1-\Big(1+\frac{1}{2}(\bar{\gamma}_{11}+\bar{\gamma}_{12})\Big)s_0 \end{bmatrix} \quad (28)$$

$$\boldsymbol{I} + s\Omega\boldsymbol{A} = \begin{bmatrix} 1+(1+2\bar{\gamma})s_0 & 0 & -\frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 \\ 0 & 1+(1+2\bar{\gamma})s_0 & -\frac{1}{2}(1+2\bar{\gamma})s_0 & -\frac{1}{2}(1+2\bar{\gamma})s_0 \\ \frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 & 1-(1+\bar{\gamma})s_0 & 0 \\ -\frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 & 0 & 1-(1+\bar{\gamma})s_0 \end{bmatrix} \quad (29)$$

$$\times \frac{1}{\left(s_0 + \frac{\sqrt{2(1+2\bar\gamma)+\bar\gamma^2}-\bar\gamma}{1+2\bar\gamma}\right)^2}\Bigg]$$

$$= -\lim_{s_0 = \frac{\sqrt{2(1+2\bar\gamma)+\bar\gamma^2}+\bar\gamma}{1+2\bar\gamma}} \frac{d}{ds_0} \frac{\left(\frac{1}{2}(1+2\bar\gamma)\right)^{-2}}{s_0\left(s_0 + \frac{\sqrt{2(1+2\bar\gamma)+\bar\gamma^2}-\bar\gamma}{1+2\bar\gamma}\right)^2}$$

$$= \frac{(2\sqrt{2(1+2\bar\gamma)+\bar\gamma^2}+\bar\gamma)(1+2\bar\gamma)^2}{(\sqrt{2(1+2\bar\gamma)+\bar\gamma^2}+\bar\gamma)^2(\sqrt{2(1+2\bar\gamma)+\bar\gamma^2})^3}, \quad (32)$$

which for large $\bar\gamma$ varies as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I \cong \frac{3}{\bar\gamma^3}. \quad (33)$$

For the error event of case C, we define the conditional (on the channel gains) PEP, which results in 2 bit errors, as being of type II and is computed as

$$P\left(\boldsymbol{X} \to \hat{\boldsymbol{X}} \middle| |c_{11}|, |c_{12}|\right)_{II} = \Pr\left\{\sum_{i=1}^{2} |Z_i^C|^2 > \sum_{i=1}^{2} |Z_i^D|^2\right\} = \Pr\left\{\sum_{i=1}^{2} |Z_i^D|^2 - \sum_{i=1}^{2} |Z_i^C|^2 < 0\right\}. \quad (34)$$

To evaluate the conditional PEP in (34), we make use of the results in Appendix 9A of the Simon/Alouini book [23].[3] In particular, letting $A = 1, B = -1, C = 0$ and $L = 2$ in (9A.2), then from (9A.1), we have

$$\Pr\{D < 0\} = \Pr\left\{\sum_{i=1}^{2} |X_i|^2 - \sum_{i=1}^{2} |Y_i|^2 < 0\right\}, \quad (35)$$

where the pairs $\{X_i, Y_i\}$ are mutually independent complex Gaussian random variables. (Note that X_i and Y_i can be correlated as will be the case here.) Also, the X_is must have identical variance and likewise for the Y_is. Thus, associating X_i with Z_i^D and Y_i with either Z_i^A, the type II conditional PEP can be computed in closed form from $\Pr\{D < 0\}$ of (35).

The parameters needed in (9A.3) – (9A.5) of [23] are as follows:

$$\mu_{xx} = \frac{1}{2}E\{|N_1^D|^2\} = \frac{1}{2}E\{|N_2^D|^2\} = \frac{N_0}{E_s},$$
$$\mu_{yy} = \frac{1}{2}E\{|N_1^C|^2\} = \frac{1}{2}E\{|N_2^C|^2\} = \frac{N_0}{E_s},$$
$$\mu_{xy} = \frac{1}{2}E\{N_1^D(N_1^C)^*\} = \frac{1}{2}E\{N_2^D(N_2C)^*\} = 0,$$

$$\omega = 0,$$
$$\nu_1 = \nu_2 = \sqrt{\frac{1}{4(\mu_{xx}\mu_{yy} - |\mu_{xy}|^2)}} = \frac{1}{2}\frac{E_s}{N_0}. \quad (36)$$

Furthermore,

$$\xi_{11} = 2\left(|\bar Z_1^D|^2\mu_{yy} + |\bar Z_1^C|^2\mu_{xx} - (\bar Z_1^D)^*\bar Z_1^C\mu_{xy} - \bar Z_1^D(\bar Z_1^C)^*\mu_{xy}^*\right) = 8\frac{N_0}{E_s}|c_{11}|^2,$$
$$\xi_{12} = 2\left(|\bar Z_2^D|^2\mu_{yy} + |\bar Z_2^C|^2\mu_{xx} - (\bar Z_2^D)^*\bar Z_2^C\mu_{xy} - \bar Z_2^D(\bar Z_2^C)^*\mu_{xy}^*\right) = 8\frac{N_0}{E_s}|c_{12}|^2,$$
$$\xi_1 = \xi_{11} + \xi_{12} = 8\frac{N_0}{E_s}\left(|c_{11}|^2 + |c_{12}|^2\right), \quad (37)$$

and

$$\xi_{21} = |\bar Z_1^D|^2 - |\bar Z_1^C|^2 = 4|c_{11}|^2,$$
$$\xi_{22} = |\bar Z_2^D|^2 - |\bar Z_2^C|^2 = 4|c_{12}|^2,$$
$$\xi_2 = \xi_{21} + \xi_{22} = 4\left(|c_{11}|^2 + |c_{12}|^2\right). \quad (38)$$

Finally,

$$a = \left[\frac{2\nu_1^2\nu_2(\xi_1\nu_2 - \xi_2)}{(\nu_1+\nu_2)^2}\right]^{1/2} = 0,$$
$$b = \left[\frac{2\nu_2^2\nu_1(\xi_1\nu_1 + \xi_2)}{(\nu_1+\nu_2)^2}\right]^{1/2} = [2(\gamma_{11} + \gamma_{12})]^{1/2}. \quad (39)$$

Since from (4.46) and (4.23) of [23]

$$Q_m(0,\beta) = \sum_{n=0}^{m-1} \exp\left(-\frac{\beta^2}{2}\right)\frac{(\beta^2/2)^n}{n!},$$
$$Q_m(\beta, 0) = 1, \quad (40)$$

then the conditional PEP is computed from (9A.15) as

$$P\left(\boldsymbol{X} \to \hat{\boldsymbol{X}} \middle| \gamma_{11}, \gamma_{12}\right)_{II} = \frac{1}{2}\left(1 + \frac{b^2}{8}\right)\exp\left(-\frac{b^2}{2}\right). \quad (41)$$

To evaluate the average PEP $P\left(\boldsymbol{X} \to \hat{\boldsymbol{X}}\right)_{II}$ one now averages over the distributions of γ_{11} and γ_{12}. Recognizing that $b^2/2 = \gamma_{11} + \gamma_{12}$, then averaging over the i.i.d. channel statistics, the unconditional PEP of the second type becomes[4]

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II} = \frac{1}{2}M_\gamma(-1)\left(M_\gamma(-1) + \frac{1}{2}M_\gamma^{(1)}(-1)\right), \quad (42)$$

where $M_\gamma^{(n)}(s) = d^n M_\gamma(s)/ds^n$.

[3]While the PEP for the type II errors could also be evaluated using the inverse Laplace transform method of (23), the advantage of the approach taken here is somewhat more general in that the expression for the PEP so obtained is expressed directly in terms of the MGF (and its derivatives) of the fading process and thus allows its evaluation for arbitrary channel fading statistics. The reason for this is that the latter approach first evaluates the *conditional* (on the fading) PEP and then averages over the (arbitrary) fading statistics whereas the former approach directly evaluates the unconditional PEP *including* the fading statistics and as such is only valid for complex Gaussian (Rayleigh or Rican envelope) fading amplitudes. It should be pointed out, however, that while offering the above advantage, unfortunately, because of the nonzero correlation between certain noise components, e.g., N_1^A and N_2^D, the approach used to obtain the type II PEP cannot be used to evaluate the type I PEP.

[4]The reader is reminded that although this PEP can be computed for arbitrary fading channel statistics, the decision metric that results in this performance is optimum only for the Rayleigh channel as per the derivation in Section I. Thus, the performance obtained from this computation will at best be suboptimum.

For the case of Rayleigh fading,

$$M_\gamma(s) = \frac{1}{1-s\bar{\gamma}},$$
$$M_\gamma^{(1)}(s) = \frac{\bar{\gamma}}{(1-s\bar{\gamma})^2}, \quad (43)$$

and thus the PEP of the type II error event is obtained from (42) in closed form as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II} = \frac{1}{2(1+\bar{\gamma})}\left(\frac{1}{1+\bar{\gamma}} + \frac{1}{2}\frac{\bar{\gamma}}{(1+\bar{\gamma})^2}\right) = \frac{1+3\bar{\gamma}/2}{2(1+\bar{\gamma})^3} \quad (44)$$

which for large $\bar{\gamma}$ varies as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II} \cong \frac{3}{4\bar{\gamma}^2}. \quad (45)$$

Note that both PEPs evaluate to 1/2 as they should at $\bar{\gamma} = 0$.

Finally, the average bit error probability (BEP) can be approximately evaluated from the two types of PEP as follows. For each input of 2 bits, the Alamouti STBC generates a pair of binary signals, the code has rate $r = 1$ bps/Hz. Each of the two type I error events results in a single bit error whereas the type II error event results in 2 bit errors. Thus, the average BEP is approximately given by

$$P_b(E) \cong \frac{1}{2}\left[2\left(1 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I\right) + 2 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}\right] = P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I + P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}. \quad (46)$$

IV. ERROR PROBABILITY PERFORMANCE FOR M-ARY ORTHOGONAL FSK MODULATION

Consider now a 4-ary orthogonal modulation where the signals $s_1(t), s_2(t), s_3(t), s_4(t)$ are arbitrarily assigned to the bit sequences $00, 01, 10, 11$, respectively and satisfy the orthogonality conditions analogous to (16). Since now 4 input bits generate a pair of Alamouti symbols, the code has rate $r = 2$ bps/Hz. There are still two types of PEP. For example, if as before the actual transmitted sequence is $x_1(t) = s_1(t), x_2(t) = s_1(t)$ (corresponding to the binary data sequence $00, 00$), then there are a total of 15 possible error sequences that can occur - 6 of type I and 9 of type II. For the 6 type I error events, 4 of them result in a single bit error while 2 of them result in 2 bit errors each. For the 9 type II error events, 4 of them result in 2 bit errors, 4 of them result in 3 bit errors and 1 of them results in 4 bit errors. Hence, the average BEP is approximated by

$$\begin{aligned} P_b(E) &\cong \frac{1}{4}\Big[4\left(1 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I\right) + 2\left(2 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I\right) \\ &\quad +4\left(2 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}\right) + 4\left(3 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}\right) \\ &\quad +1\left(4 \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}\right)\Big] \\ &= 2P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I + 6P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}. \end{aligned} \quad (47)$$

Generalizing to $M = 2^m$-ary orthogonal modulation (code rate $r = \log_2 M$ bps/Hz), there are $2(M-1)$ type I errors resulting in a total of $M \log_2 M$ bit errors and $M^2 - 2(M-1) - 1$ type II errors resulting in a total of $M(M-1)\log_2 M$ bit errors. Hence, the average BEP is approximated by

$$\begin{aligned} P_b(E) &\cong \frac{1}{2\log_2 M}\Big[M \log_2 M \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I \\ &\quad +M(M-1)\log_2 M \times P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}\Big] \\ &= \frac{M}{2}P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I + \frac{M(M-1)}{2}P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II}. \end{aligned} \quad (48)$$

V. EXTENSION TO THE CASE OF MORE THAN ONE RECEIVE ANTENNA

Assume now that the receiver contains L_r antennas. Then, analogous to (1), the $2L_r$ received symbols are given by

$$\begin{aligned} y_l(t) &= c_{l1}x_1(t) + c_{l2}x_2(t) + n_l(t), 0 \le t \le T_s \\ y_l(t) &= -c_{l1}x_2^*(t-T_s) + c_{l2}x_1^*(t-T_s) \\ &\quad + n_l(t),\ T_s \le t \le 2T_s,\ l = 1, 2, \ldots, L_r. \end{aligned} \quad (49)$$

Following steps analogous to those in Section II, it can be shown that the log-likelihood ratio of (9) is now given by

$$\begin{aligned} \ln p(\boldsymbol{y}|\boldsymbol{x}) &= \sum_{l=1}^{L_r}\Bigg[\frac{\bar{\gamma}_{l1}}{1+2\bar{\gamma}_{l1}}\left|\int_0^{T_s} y_l(t)x_1^*(t)dt - \int_{T_s}^{2T_s} y_l(t)x_2(t-T_s)dt\right|^2 \\ &\quad + \frac{\bar{\gamma}_{l2}}{1+2\bar{\gamma}_{l2}}\left|\int_0^{T_s} y_l(t)x_2^*(t)dt + \int_{T_s}^{2T_s} y_l(t)x_1(t-T_s)dt\right|^2\Bigg]. \end{aligned} \quad (50)$$

Thus, the ML decision rule is to choose $x_1(t) = \hat{x}_1(t)$and $x_2(t) = \hat{x}_2(t)$ such that

$$\begin{aligned} \hat{x}_1(t), \hat{x}_2(t) &= \max_{x_1, x_2}{}^{-1}\sum_{l=1}^{L_r}\Bigg[\frac{\bar{\gamma}_{l1}}{1+2\bar{\gamma}_{l1}}\left|\int_0^{T_s} y_l(t)x_1^*(t)dt \right. \\ &\quad \left. - \int_{T_s}^{2T_s} y_l(t)x_2(t-T_s)dt\right|^2 \\ &\quad + \frac{\bar{\gamma}_{l2}}{1+2\bar{\gamma}_{l2}}\left|\int_0^{T_s} y_l(t)x_2^*(t)dt + \int_{T_s}^{2T_s} y_l(t)x_1(t-T_s)dt\right|^2\Bigg], \end{aligned} \quad (51)$$

or for identically distributed channels, i.e., $\bar{\gamma}_{l1} = \bar{\gamma}_{l2} = \bar{\gamma}$,

$$\begin{aligned} \hat{x}_1(t), \hat{x}_2(t) &= \max_{x_1, x_2}{}^{-1}\sum_{l=1}^{L_r}\Bigg[\left|\int_0^{T_s} y_l(t)x_1^*(t)dt - \int_{T_s}^{2T_s} y_l(t)x_2(t-T_s)dt\right|^2 \\ &\quad + \left|\int_0^{T_s} y_l(t)x_2^*(t)dt + \int_{T_s}^{2T_s} y_l(t)x_1(t-T_s)dt\right|^2\Bigg]. \end{aligned} \quad (52)$$

Analogous to (13), we define

$$\begin{aligned} Z_l &\triangleq \int_0^{T_s} y_l(t)\hat{x}_1^*(t)dt - \int_{T_s}^{2T_s} y_l(t)\hat{x}_2(t-T_s)dt \\ &= c_{l1}\int_0^{T_s}\left[x_1(t)\hat{x}_1^*(t) + x_2^*(t)\hat{x}_2(t)\right]dt \\ &\quad + c_{l2}\int_0^{T_s}\left[x_2(t)\hat{x}_1^*(t) - x_1^*(t)\hat{x}_2(t)\right]dt + N_l, \end{aligned}$$

$$Z_{l+L_r} \triangleq \int_0^{T_s} y_l(t)\hat{x}_2^*(t)dt + \int_{T_s}^{2T_s} y_l(t)\hat{x}_1(t-T_s)dt$$
$$= c_{l1}\int_0^{T_s}\Big[x_1(t)\hat{x}_2^*(t) - x_2^*(t)\hat{x}_1(t)\Big]dt$$
$$+ c_{l2}\int_0^{T_s}\Big[x_2(t)\hat{x}_2^*(t) + x_1^*(t)\hat{x}_1(t)\Big]dt + N_{l+N_r}, \quad (53)$$

where

$$N_l \triangleq \int_0^{T_s} n_l(t)\hat{x}_1^*(t)dt - \int_{T_s}^{2T_s} n_l(t)\hat{x}_2(t-T_s)dt,$$
$$N_{l+N_r} \triangleq \int_0^{T_s} n_l(t)\hat{x}_2^*(t)d + \int_{T_s}^{2T_s} n_l(t)\hat{x}_1(t-T_s)dt,$$
$$l = 1, 2, \ldots, L_r, \quad (54)$$

and have the properties

$$\overline{N_i N_j^*} = 0,$$
$$\overline{|N_l|^2} = 2N_0/E_s,\ l = 1, 2, \ldots, 2L_r. \quad (55)$$

To compute the error probability performance for binary orthogonal FSK modulation, we proceed as in Section III. Analogous to (17) – (20) we have

A. $\hat{x}_1(t) = s_1(t), \hat{x}_2(t) = s_2(t)$

$$\sum_{l=1}^{2L_r}|Z_l^A|^2 = \sum_{l=1}^{L_r}|c_{l1} + c_{l2} + N_l^A|^2 + \sum_{l=1}^{L_r}|-c_{l1} + c_{l2} + N_{l+L_r}^A|^2. \quad (56)$$

B. $\hat{x}_1(t) = s_2(t), \hat{x}_2(t) = s_1(t)$

$$\sum_{l=1}^{2L_r}|Z_l^B|^2 = \sum_{l=1}^{L_r}|c_{l1} - c_{l2} + N_l^B|^2 + \sum_{l=1}^{L_r}|c_{l1} + c_{l2} + N_{l+L_r}^B|^2. \quad (57)$$

C. $\hat{x}_1(t) = s_2(t), \hat{x}_2(t) = s_2(t)$

$$\sum_{l=1}^{2L_r}|Z_l^C|^2 = \sum_{l=1}^{2L_r}|N_l^C|^2. \quad (58)$$

For the correctly detected sequence, we have

D. $\hat{x}_1(t) = s_1(t), \hat{x}_2(t) = s_1(t)$

$$\sum_{l=1}^{2L_r}|Z_l^D|^2 = \sum_{l=1}^{L_r}|2c_{l1} + N_l^D|^2 + \sum_{l=1}^{L_r}|2c_{l1} + N_{l+L_r}^D|^2. \quad (59)$$

In (56) -(59), the noises are defined as follows:

$$N_l^A \triangleq \int_0^{T_s} n_l(t)s_1^*(t)dt - \int_{T_s}^{2T_s} n_l(t)s_2(t-T_s)dt,$$
$$N_{l+L_r}^A \triangleq \int_0^{T_s} n_l(t)s_2^*(t)dt + \int_{T_s}^{2T_s} n_l(t)s_1(t-T_s)dt,$$
$$N_l^B \triangleq \int_0^{T_s} n_l(t)s_2^*(t)dt - \int_{T_s}^{2T_s} n_l(t)s_1(t-T_s)dt,$$
$$N_{l+L_r}^B \triangleq \int_0^{T_s} n_l(t)s_1^*(t)dt + \int_{T_s}^{2T_s} n_l(t)s_2(t-T_s)dt,$$
$$N_l^C \triangleq \int_0^{T_s} n_l(t)s_2^*(t)dt - \int_{T_s}^{2T_s} n_l(t)s_2(t-T_s)dt,$$
$$N_{l+L_r}^C \triangleq \int_0^{T_s} n_l(t)s_2^*(t)dt + \int_{T_s}^{2T_s} n_l(t)s_2(t-T_s)dt,$$
$$N_l^D \triangleq \int_0^{T_s} n_l(t)s_1^*(t)dt - \int_{T_s}^{2T_s} n_l(t)s_1(t-T_s)dt,$$
$$N_{l+L_r}^D \triangleq \int_0^{T_s} n_l(t)s_1^*(t)dt + \int_{T_s}^{2T_s} n_l(t)s_1(t-T_s)dt. \quad (60)$$

Consider first the type I error. Again we can use the result of Turin [24] to evaluate the quadratic form necessary to compute the PEP. Here, we define the vector v needed in (24) by $\boldsymbol{v}^T = \Big[Z_1^D Z_{1+L_r}^D Z_2^D Z_{2+L_r}^D \cdots Z_{L_r}^D Z_{2L_r}^D Z_1^A Z_{1+L_r}^A Z_2^A Z_{2+L_r}^A \cdots Z_{L_r}^A Z_{2L_r}^A\Big]$. When this is done, then analogous to (29), the matrix $\boldsymbol{I} + s\Omega\boldsymbol{A}$ can be partitioned into four block diagonal $2L_r \times 2L_r$ matrices as follows:

$$\boldsymbol{I} + s\Omega\boldsymbol{A} = \begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{M}_2 \\ \boldsymbol{M}_3 & \boldsymbol{M}_4 \end{bmatrix}, \quad (61)$$

where

$$\boldsymbol{M}_1 = [1 + (1+2\bar{\gamma})]\boldsymbol{I}_{L_r},$$
$$\boldsymbol{M}_2 = \boldsymbol{I}_{L_r} \otimes \begin{bmatrix} -\frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 \\ -\frac{1}{2}(1+2\bar{\gamma})s_0 & -\frac{1}{2}(1+2\bar{\gamma})s_0 \end{bmatrix},$$
$$\boldsymbol{M}_3 = \boldsymbol{I}_{L_r} \otimes \begin{bmatrix} \frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 \\ -\frac{1}{2}(1+2\bar{\gamma})s_0 & \frac{1}{2}(1+2\bar{\gamma})s_0 \end{bmatrix},$$
$$\boldsymbol{M}_4 = [1 - (1+\bar{\gamma})]\boldsymbol{I}_{L_r}, \quad (62)$$

and $\otimes$ denotes Kronecker product. Since

$$\det[\boldsymbol{I} + s\Omega\boldsymbol{A}] = \det(\boldsymbol{M}_4)\det(\boldsymbol{M}_1 - \boldsymbol{M}_2\boldsymbol{M}_4^{-1}\boldsymbol{M}_3), \quad (63)$$

then it is straightforward to show that (30) generalizes to

$$\det[\boldsymbol{I} + s\Omega\boldsymbol{A}] = \Big(\frac{1}{2}(1+2\bar{\gamma})s_0^2 - \bar{\gamma}s_0 - 1\Big)^{2L_r}, \quad (64)$$

resulting in the type I PEP

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{1}{s_0\Big(\frac{1}{2}(1+2\bar{\gamma})s_0^2 - \bar{\gamma}s_0 - 1\Big)^{2L_r}}ds_0$$
$$= \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\Big(\frac{1}{2}(1+2\bar{\gamma})\Big)^{-2L_r}}{s_0\Big(s_0 - \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^{2L_r}}+\bar{\gamma}}{1+2\bar{\gamma}}\Big)^{2L_r}} \times \frac{1}{\Big(s_0 + \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}-\bar{\gamma}}{1+2\bar{\gamma}}\Big)^2}ds_0. \quad (65)$$

Once again using the residue method we obtain

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = -\sum_{\text{R.H.P Poles}} \text{Residue}\left[\frac{\left(\frac{1}{2}(1+2\bar{\gamma})\right)^{-2L_r}}{s_0\left(s_0 - \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}+\bar{\gamma}}{1+2\bar{\gamma}}\right)^{2L_r}} \times \frac{1}{\left(s_0 + \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}-\bar{\gamma}}{1+2\bar{\gamma}}\right)^{2L_r}}\right]$$
$$= -\lim_{s_0=\frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}+\bar{\gamma}}{1+2\bar{\gamma}}} \frac{1}{(2L_r-1)!}\frac{d^{2L_r-1}}{ds_0^{2L_r-1}} \times \frac{\left(\frac{1}{2}(1+2\bar{\gamma})\right)^{-2L_r}}{s_0\left(s_0 + \frac{\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}-\bar{\gamma}}{1+2\bar{\gamma}}\right)^{2L_r}}. \quad (66)$$

It can be shown that

$$\frac{1}{(n-1)!}\frac{d^{n-1}}{ds_0^{n-1}}\frac{1}{s_0(s_0+a)^n} = (-1)^{n-1}\frac{\sum_{l=1}^{n}\binom{2n-1}{n-l}s_0^{n-l}a^{l-1}}{s_0^n(s_0+a)^{2n-1}}. \quad (67)$$

Thus, letting $n = 2L_r$, we obtain after some simplification the desired closed-form expression for the type I error as

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_I = (1+2\bar{\gamma})^{2L_r}\sum_{l=1}^{2L_r}\binom{4L_r-1}{2L_r-l} \times \frac{(2\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}+\bar{\gamma})^{-l}(2\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2}-\bar{\gamma})^{l-1}}{2^{2L_r-1}(\sqrt{2(1+2\bar{\gamma})+\bar{\gamma}^2})^{4L_r-1}}, \quad (68)$$

which reduces to (32) for $L_r = 1$.

Consider next the type II error. The parameters needed in (9A.3) - (9A.5) of [23] are as follows:

$$\mu_{xx} = \frac{1}{2}E\{|N_l^D|^2\} = \frac{N_0}{E_s},$$
$$\mu_{yy} = \frac{1}{2}E\{|N_l^C|^2\} = \frac{N_0}{E_s},$$
$$\mu_{xy} = \frac{1}{2}E\{N_l^D(N_l^C)^*\} = 0,$$
$$\omega = 0,$$
$$\nu_1 = \nu_2 = \sqrt{\frac{1}{4(\mu_{xx}\mu_{yy} - |\mu_{xy}|^2)}} = \frac{1}{2}\frac{E_s}{N_0}. \quad (69)$$

Furthermore,

$$\xi_1 = 8\frac{N_0}{E_s}\sum_{l-1}^{L_r}\left(|c_{l1}|^2 + |c_{l2}|^2\right),$$
$$\xi_2 = 4\sum_{l=1}^{L_r}\left(|c_{l1}|^2 + |c_{l2}|^2\right), \quad (70)$$

and

$$a = 0, \quad b = \left[2\sum_{l=1}^{L_r}(\gamma_{11}+\gamma_{12})\right]^{1/2}. \quad (71)$$

Once again using (40), then the conditional PEP as computed from (9A.15) of [23] is given by

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}}|\gamma_{11},\gamma_{12}; l = 1,2,\ldots,L_r)_{II} = \frac{1}{2} + \frac{1}{2^{4L_r-1}} \times\sum_{l=1}^{2L_r}\binom{4L_r-1}{2L_r-l}\left[\sum_{n=0}^{l-1}\frac{(b^2/2)^n}{n!}\exp\left(-\frac{b^2}{2}\right) - 1\right]. \quad (72)$$

Recognizing that $\sum_{l=1}^{2L_r}\binom{4L_r-1}{2L_r-l} = 2^{4L_r-2}$, then (72) simplifies to

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}}|\gamma_{11},\gamma_{12}; l = 1,2,\ldots,L_r)_{II} = \frac{1}{2^{4L_r-1}}\sum_{l=1}^{2L_r}\binom{4L_r-1}{2L_r-l}\sum_{n=0}^{l-1}\frac{(b^2/2)^n}{n!}\exp\left(-\frac{b^2}{2}\right). \quad (73)$$

To compute the average PEP, one now averages over the distributions of γ_{11} and γ_{12}. Noting that $b^2/2 = \sum_{l=1}^{L_r}(\gamma_{11}+\gamma_{12})$, then making use of the multinomial generating function, we obtain the following statistical average for the i.i.d. case:

$$\overline{\left(\frac{b^2}{2}\right)^n \exp\left(-\frac{b^2}{2}\right)} = \sum_{n_1+n_2+\cdots+n_{2L_r}=n}\frac{n!}{n_1!n_2!\cdots n_{2L_r}!}\prod_{l=1}^{2L_r}M_\gamma^{(n_l)}(-1). \quad (74)$$

Thus, substituting (74) into (73), the average type II PEP becomes

$$P(\boldsymbol{X} \to \hat{\boldsymbol{X}})_{II} = \frac{1}{2^{4L_r-1}}\sum_{l=1}^{2L_r}\binom{4L_r-1}{2L_r-l}\sum_{n=0}^{l-1} \times \sum_{n_1+n_2+\cdots+n_{2L_r}=n}\frac{n!}{n_1!n_2!\cdots n_{2L_r}!}\prod_{l=1}^{2L_r}M_\gamma^{(n_l)}(-1). \quad (75)$$

For the Rayleigh fading channel, it is straightforward to show that

$$M_\gamma^{(n)}(-1) = \frac{n!\bar{\gamma}^n}{(1+\bar{\gamma})^{n+1}}, \ n = 0,1,2,\ldots \quad (76)$$

Thus, using (76) in (75), one obtains a closed-form expression for the average type II PEP over the Rayleigh channel. Asymptotically, in the limit of large SNR, (76) behaves as

$$M_\gamma^{(n)}(-1) \cong \frac{n!}{\bar{\gamma}}, \ n = 0,1,2,\ldots \quad (77)$$

Since each term in (75) is of order $2L_r$ in the MGF and its derivatives, then asymptotically for large SNR, the average PEP varies as $\bar{\gamma}^{-2L_r}$ which immediately identifies the full diversity (order $2L_r$) of the type II PEP performance.

As an example, consider the case $L_r = 2$. Then, from (74) we obtain

$$\begin{aligned}\overline{\exp\left(-\frac{b^2}{2}\right)} &= \left(M_\gamma(-1)\right)^4,\\ \overline{\frac{b^2}{2}\exp\left(-\frac{b^2}{2}\right)} &= 4\left(M_\gamma(-1)\right)^3\left(M_\gamma^{(1)}(-1)\right),\\ \overline{\left(\frac{b^2}{2}\right)^2\exp\left(-\frac{b^2}{2}\right)} &= 4\left(M_\gamma(-1)\right)^3 M_\gamma^{(2)}(-1)\\ &\quad +12\left(M_\gamma(-1)\right)^2\left(M_\gamma^{(1)}(-1)\right),\\ \overline{\left(\frac{b^2}{2}\right)^3\exp\left(-\frac{b^2}{2}\right)} &= 4\left(M_\gamma(-1)\right)^3 M_\gamma^{(3)}(-1)\\ &\quad +36\left(M_\gamma(-1)\right)^2\left(M_\gamma^{(1)}(-1)\right)^2\left(M_\gamma^{(2)}(-1)\right)\\ &\quad +24M_\gamma(-1)\left(M_\gamma^{(1)}(-1)\right)^3, \end{aligned} \tag{78}$$

which when used in (75) together with (76) gives for the Rayleigh channel

$$\begin{aligned} P(\boldsymbol{X}\to\hat{\boldsymbol{X}})_{II} &= \frac{1}{128}\left[64\frac{1}{(1+\bar{\gamma})^4}+116\frac{\bar{\gamma}}{(1+\bar{\gamma})^5}\right.\\ &\quad \left.+80\frac{\bar{\gamma}^2}{(1+\bar{\gamma})^6}+20\frac{\bar{\gamma}^3}{(1+\bar{\gamma})^7}\right]\\ &= \frac{1}{2(1+\bar{\gamma})^4}\left[1+\frac{29}{16}\left(\frac{\bar{\gamma}}{(1+\bar{\gamma})}\right)\right.\\ &\quad \left.+\frac{5}{4}\left(\frac{\bar{\gamma}}{(1+\bar{\gamma})}\right)^2+\frac{5}{16}\left(\frac{\bar{\gamma}}{(1+\bar{\gamma})}\right)^3\right]. \end{aligned} \tag{79}$$

Finally, one can again use (68) and (75) in (48) to approximately evaluate the average BEP for M-ary orthogonal modulation.

VI. NUMERICAL RESULTS

In this section, we provide numerical results to illustrate the behavior of the expressions derived from the theoretical analyses. In the numerical calculations, the signal-to-noise ratio (SNR) is normalized per receive antenna, i.e., SNR= $2\bar{\gamma}$. Fig. 1 shows the PEPs and the approximate average BEP versus SNR for binary orthogonal FSK with a single receive antenna. For comparison purposes, this figure also contains a BEP performance curve for noncoherent binary orthogonal FSK with a single transmit antenna. We can clearly see the advantage of transmit diversity obtained by combining noncoherent orthogonal modulation with STBC. Fig. 2 illustrates the approximate average BEP versus SNR for M-ary orthogonal FSK with a single receive antenna. The tradeoff between increased rate and performance degradation can be observed. Similar curves are shown in Fig. 3 and Fig. 4 for systems with more than one receive antenna. From the slope of the curves, we conclude that noncoherent detection of orthogonal modulation combined with Alamouti STBC achieves full spatial diversity.

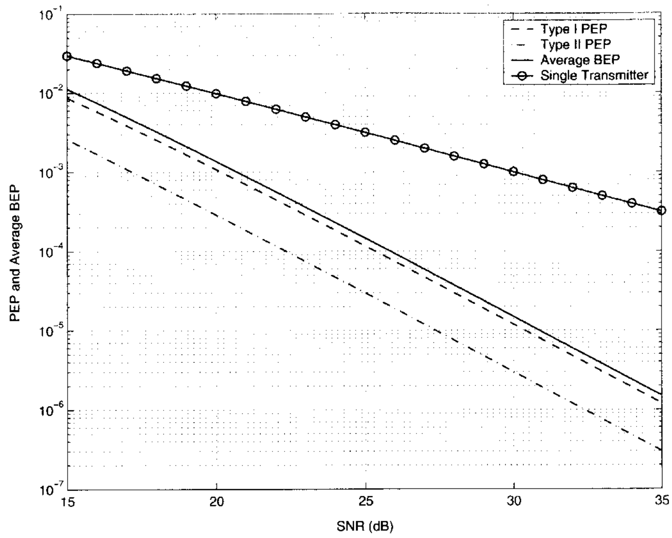

Fig. 1. PEPs and the approximate average BEP vs. SNR for binary orthogonal FSK with a single receive antenna.

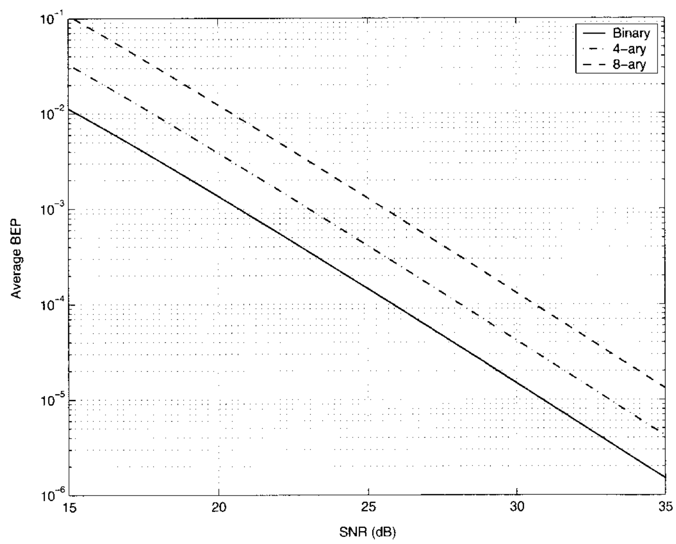

Fig. 2. The approximate average BEP vs. SNR for M-ary orthogonal FSK with a single receive antenna.

Before concluding, it is worthy of note that the key results of this paper for the PEP performance as given by (68) and (75) agree exactly with those derived from (B.10) in [11] and (B1) in [12] which are given in another (integral) form (see also [25]).

VII. CONCLUSIONS

In this paper, we have investigated the performance of noncoherent detection of the orthogonal modulation combined with STBC. We derived exact, compact closed-form expressions for the PEP. We employed two methods, namely, the inverse Laplace transform and the MGF-based approach to evaluate the PEP. The MGF-based approach allows us to obtain the performance for the generalized fading channel. Theoretical and numerical results show that noncoherent detection of orthogonal modulation combined with Alamouti STBC achieves full spatial diversity. We also derived an approximate average bit error

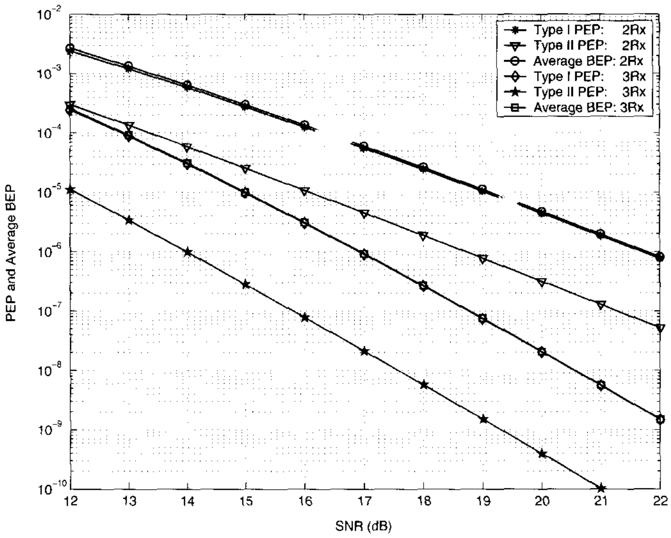

Fig. 3. PEPs and the approximate average BEP vs. SNR for binary orthogonal FSK with multiple receive antennas.

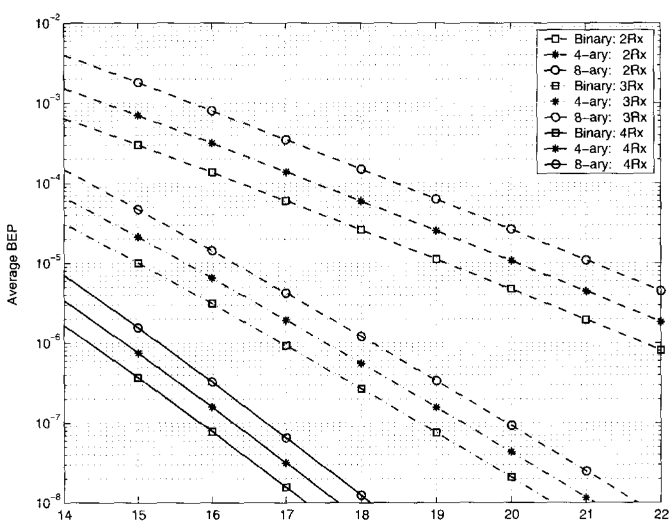

Fig. 4. The approximate average BEP vs. SNR for M-ary orthogonal FSK with multiple receive antennas.

probability (BEP) for M-ary orthogonal signaling to characterize the tradeoff between increased rate and performance degradation.

REFERENCES

[1] "Special issue on the European path towards UMTS," *IEEE Pers. Commun. Mag.*, vol. 2, Feb. 1995.

[2] E. Biglieri, G. Caire, and G. Taricco, "Recent results on coding for the multiple-antenna transmission systems," in *Proc IEEE ISSSTA'2000*, Sept. 2000, pp. 117–121.

[3] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, pp. 585–595, Nov.-Dec. 1999.

[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, pp. 311–335, Mar. 1998.

[5] J.-C. Guey *et al.*, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," in *Proc. IEEE VTC'96*, pp. 136–140.

[6] V. Tarokh, N. Seshadri, and A.R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[7] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[8] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[9] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 139–157, Jan. 1999.

[10] L. Zheng and D. Tse, "Communicating on the Grassmann manifold: A geometric approach to the non-coherent multiple antenna channel," *IEEE Trans. Inform. Theory*, vol. 48, pp. 359–383, Feb. 2002.

[11] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 46, pp. 543–564, Mar. 2000.

[12] B. M. Hochwald, T. L. Marzetta, and B. Hassibi, "Space-time autocoding," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2761–2781, Nov. 2001.

[13] B. M. Hochwald *et al.*, "Systematic design of unitary space-time constellation," *IEEE Trans. Inform. Theory*, vol. 46, pp. 1962–1973, Sept. 2000.

[14] D. Warrier and U. Madhow, "Spectrally efficient noncoherent communication," *IEEE Trans. Inform. Theory*, vol. 48, pp. 651–668, Mar. 2002.

[15] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 1169–1174 July 2000.

[16] B.L. Hughes, "Differential space-time modulation," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2567–2578, Nov. 2000.

[17] B. M. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, vol. 48, pp. 2041–2052, Dec. 2000.

[18] H. Jafarkhani and V. Tarokh, "Multiple transmit antenna differential detection from generalized orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2626–2631, Sept. 2001.

[19] A. Shokrollahi *et al.*, "Representation theory for high-rate multiple-antenna code design," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2335–2367, Sept. 2001.

[20] X. B. Liang and X. G. Xia, "Unitary signal constellations for differential space-time modulation with two transmit antennas: Parametric codes, optimal designs and bounds," *IEEE Trans. Inform. Theory*, vol. 48, pp. 2291–2232, Aug. 2002.

[21] B. Hassibi and B. M Hochwald, "Cayley differential space-time codes," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1485–1503, Aug. 2002.

[22] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inform. Theory*, vol. 48, no. 2, pp. 510–513, Feb. 2002.

[23] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels: A Uniform Approach to Performance Evaluation*, New York, NY: John Wiley & Sons, Inc., Aug. 2000.

[24] G. L. Turin, "The characteristic function of Hermetian quadratic forms in complex normal random variables," *Biometrika*, pp. 199–201, June 1960.

[25] H. F. Lu *et al.*, "On the performance of space-time codes," submitted to publication in the *IEEE Trans. Inform. Theory*.

[26] R. Schober and L. H.-J. Lampre, "Noncoherent receivers for differential space-time modulation," *IEEE Trans. Commun.*, vol. 50, no. 5, pp. 768–777, May 2002.

Marvin K. Simon is currently a Principal Scientist at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California where for the last 35 years he has performed research as applied to the design of NASA's deep-space and near-earth missions resulting in the issuance of 9 patents and 23 NASA Tech Briefs. Dr. Simon is known as an internationally acclaimed authority on the subject of digital communications with particular emphasis in the disciplines of modulation and demodulation, synchronization techniques for space, satellite and radio communications, trellis-coded modulation, spread spectrum and multiple access communications, and communication over fading channels. In the past, Dr. Simon also held a joint appointment with the Electrical Engineering Department at Caltech where for 6 years he was responsible for teaching the first year graduate level three-quarter sequence of courses on random processes and digital communications.

He has published over 180 papers on the above subjects and is co-author of 10 textbooks including, *Telecommunication Systems Engineering* (Prentice-Hall, 1973 and Dover Press, 1991), *Phase-Locked Loops and Their Application* (IEEE Press, 1978), *Spread Spectrum Communications, Vols. I, II, and III*

(Computer Science Press, 1984 and McGraw-Hill, 1994), *An Introduction to Trellis Coded Modulation with Applications* (MacMillan, 1991), *Digital Communication Techniques: Vol. I* (Prentice-Hall, 1994) and *Digital Communication Over Fading Channels: A Unified Approach to Performance Analysis* (John Wiley & Sons, 2000), *Probability Distributions Involving Gaussian Random Variables - A Handbook for Engineers and Scientists* (Kluwer, 2002) and *Bandwidth-Efficient Digital Modulation with Application to Deep-Space Communication* (John Wiley & Sons, 2003). His work has also appeared in the textbook *Deep Space Telecommunication Systems Engineering* (Plenum Press, 1984) and he is co-author of a chapter entitled *Spread Spectrum Communications* in the *Mobile Communications Handbook* (CRC Press, 1995), Communications Handbook (CRC Press, 1997) and the Electrical Engineering Handbook (CRC Press, 1997). He is the co-recipient of the 1988 Prize Paper Award in Communications of the IEEE Transactions on Vehicular Technology for his work on trellis coded differential detection systems and also the 1999 Prize Paper of the IEEE Vehicular Technology Conference for his work on switched diversity. He is a Fellow of the IEEE and a Fellow of the IAE. Among his awards are the NASA Exceptional Service Medal, NASA Exceptional Engineering Achievement Medal, IEEE Edwin H. Armstrong Achievement Award and most recently the IEEE Millennium Medal all in recognition of outstanding contributions to the field of digital communications and leadership in advancing this discipline.

Jibing Wang is a PhD candidate at Electrical Engineering Department at University of California, Los Angeles. His research interest lies in the area of wireless communications, including space-time coding, MIMO OFDM, multiuser communications, etc. He is a recipient of a Microsoft Fellowship and a University of California Fellowship.