# A Decomposition Approach for Fixed Channel Assignment Problems in Large-Scale Cellular Networks

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*Abstract:* **Due to insufficient available bandwidth resources and the continuously growing demand for cellular communication services, the channel assignment problem has become increasingly important. To trace the optimal assignment, several heuristic strategies have been proposed. So far, most of them focus on the small-scale systems containing no more than 25 cells and they use an anachronistic cost model, which does not satisfy the requirements of most existing cellular operators, to measure the solution quality. Solving the small-scale channel assignment problems could not be applied into existing large scale cellular networks' practice. This article proposes a decomposition approach to solve the fixed channel assignment problem (FCAP) for large-scale cellular networks through partitioning the whole cellular network into several smaller sub-networks and then designing a sequential branch-and-bound algorithm that is made to solve the FCAP for them sequentially. The key issue of partition is to minimize the dependences of the sub-networks so that the proposed heuristics for solving smaller problems will suffer fewer constraints in searching for better assignments. The proposed algorithms perform well based on experimental results and they were applied to the Taiwan Cellular Cooperation (TCC) in ChungLi city to find better assignments for its network.**

***Index Terms:***

## I. INTRODUCTION

Pursuing an optimal frequency assignment for a cellular network is a great challenge, especially when the scale of cellular network is large and the constraints are stringent. To obtain an optimal solution, previous studies in the literatures proposed several approaches. The proposed approaches could be generally classified into two categories: 1) fixed channel assignment (FCA), where channels are permanently allocated to each cell and 2) dynamic channel assignment (DCA), where all channels that are available to every cell are allocated dynamically upon request. The FCA schemes are simple and adopted by many of the current service providers; however, they do not adapt to changing traffic and user call distributions [1], [2]. On the other hand, at the cost of higher complexity, DCA does provide flexibility and traffic adaptability. Some hybrid schemes [3], [4] combine the FCA and the DCA approaches through introducing their proposed channel borrowing algorithms. In these schemes, the system first applies an FCA algorithm to sufficiently pre-assign channel resources to all the cells and then applies their channel borrowing algorithms to adjust the assignments according to the real-time traffic status.

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In this article, the main efforts are focused on FCA approach, which pre-assigns frequencies to all the cells with the consideration of channel interferences and expected traffic load and is adopted by the Taiwan Cellular Cooperate (TCC) for managing their radio resources. The key issue for FCA is the way to appropriately pre-assign all available frequencies to individual cells to reach optimal solutions.

Obtaining the optimal solution generally implies minimizing a cost function while satisfying a set of constraints. In the literatures, the cost model of the fixed channel assignment problem (FCAP) could be stated as follows [2],[5]–[12]:

$$\text{Minimize } Z. \tag{1}$$

Subject to

$$\forall\ 1 \le i \le N \quad \sum_{p=1}^{Z} f_{i,p} \ge d_i, \tag{2}$$

$$|p-q| \ge C_{i,j} \ \text{ if } f_{i,p} = f_{j,q} = 1 \\ \text{where } 1 \le p,q \le Z \text{ and } 1 \le i,j \le N, \tag{3}$$

$$\forall\ 1 \le i \le N \text{ and } 1 \le p \le Z \ \ f_{i,p} \in \{0,1\}. \tag{4}$$

Where

- $Z$ : The number of available frequencies.
- $N$ : $L = \{l_1, l_2, \cdots, l_N\}$ is the set of all cells in the cellular network.
- $D$ : The least demand vector with $N$ components in which each component $d_i$ of vector $D$ corresponds to the fewest number of frequencies required in the cell $l_i$.
- $C$ : The compatibility $N \times N$ matrix in which each element $C_{i,j}$ describes the minimum frequency separation between the frequencies used simultaneously in the cells $l_i$ and $l_j$. This matrix is used to assure that the computed frequency assignment does not lead to any interference between different calls.

$$\forall\ 1 \le i \le N \text{ and } 1 \le p \le Z$$

$$f_{i,p} = \begin{cases} 1, \text{ if the } p^{\text{th}} \text{ frequency is assigned to the } i^{\text{th}} \text{ cell} \\ 0, \text{ otherwise} \end{cases} \tag{5}$$

The optimization problems above are attempting to minimize the total number of available frequencies. However, for many cellular network operators, their available bandwidths (the numbers of frequencies) are usually given and fixed in advance. It is clear that the above cost model is not adequate for such practices. To solve the FCAP with the limited bandwidth constraint, we have proposed a new cost model as (6)–(13) in [1] where the available bandwidth become a hard constraint, and other constraints and the demand constraint (2) were relaxed and converted to be the cost that we try to minimize. Previously mentioned other constraints include constraints such as the electronic compatibility (EMC) constraints (3), which include the adjacent channel constraint (ACC) and the cosite constraint (CSC).

Minimize

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{p=1}^{Z}\sum_{q=1}^{Z} f(i,j,p,q) + \alpha\sum_{i=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_i}\int_{T\times f_i}^{\infty}(y-T\times f_i)e^{-\frac{1}{2}\left(\frac{y-\mu_i}{\sigma_i}\right)^2}dy. \quad (6)$$

Subject to

$$\forall\ 1\le i\le N \text{ and } 1\le p\le Z\ \ f_{i,p}\in\{0,1\}, \quad (7)$$

where

$$\forall\ 1\le i\le N\ \ f_i=\sum_{p=1}^{Z} f_{i,p}, \quad (8)$$

$\forall\ 1\le i\le N$ and $1\le p,q\le Z$,

$$f(i,j,p,q) = \begin{cases} 0, & \text{if } |p-q|\ge C_{i,j} \\ f_{i,p}f_{j,q}\Psi_C(C_{i,j}-|p-q|), & \text{if } |p-q|<C_{i,j} \text{ and } i=j \\ f_{i,p}f_{j,q}\Psi_A(C_{i,j}-|p-q|), & \text{otherwise} \end{cases} \quad (9)$$

$\mu_i =$ The expected number of requested channels in the cell $l_i$ (10)

$\sigma_i =$ The standard deviation of the number of requests channel in the cell $l_i$ (11)

$T =$ The number of channels supportedby a frequency (TDMA) (12)

$\Psi_A$ and $\Psi_C$ are the cost functions of violating the ACC and CSC (13)

$C_{i,j}, Z, L$ and $N$ are the same as the cost model (1) – (5).

The study in [1] also proposed a genetic algorithm for solving several 21-cell FCAPs under a limited bandwidth constraint. For complete appropriate channel assignment perspectives, cellular operators, such as Taiwan Cellular Cooperation (TCC), desire an algorithm for solving the whole cellular network containing thousands of cells. Thus, in this paper, we continue the study in solving the FCAP under a limited bandwidth constraint for large-scale cellular networks.

The main difficulty of assigning frequencies for large-scale cellular networks is the time complexity. Time complexity increases dramatically with the number of cells [1]. To solve this problem, [13] proposed a decomposition approach and a sequential branch-and-bound algorithm. However, the proposed decomposition algorithm does not satisfy the requirements for several existing operators (such as TCC) with the limited bandwidth constraint since their algorithm adopts the cost model (1) – (5) to measure the solution quality. Because the EMC and demand constraint were relaxed in (6) – (13), the weight between each two cells should be redefined. Besides this, the assigning algorithm, which assigns frequencies to all the cells of each cluster, needs to consider the boundary restrictions carefully because several frequencies are restricted to be assigned to the marginal cells. The proposed algorithms in [13] do not consider the boundary restriction problem because the cost model (1) - (5) does not generate any feasible solution that violates any EMC and the demand constraints.

This paper is organized as follows. Section II roughly describes the algorithm for the large-scale FCAP with the limited bandwidth constraint. The algorithm consists of clustering, coupling and assigning algorithms. To provide the weights for clustering and coupling algorithms, we propose several measures in Section III. Section IV presents the details of the clustering, coupling and assigning algorithms. Experimental results that show the performance of our designed algorithms is provided in Section V. The conclusion and future works are drawn in Section VI.

## II. PROBLEM DESCRIPTION

Since the optimization problem (6)–(13) is NP-complete, solving the problem through the direct help of existing algorithms seems impracticable for large-scale cellular networks. Due to this difficulty, we adopt the decomposition approach to solve the large-scale FCAP under a limited bandwidth constraint in this paper. Decomposing the problem with large-scale cellular networks into several sub-problems containing smaller scale networks introduces the boundary cells problem. The boundary cells, which will be explained in Section II-B, decrease the freedom of solving the latter-solved sub-problems because they may conceal some hidden interference cost, which will be explained in Section II-D. To reduce the $un-freedom^{+}$ in solving the latter-solved sub-problems, decomposing and coupling algorithms should be carefully designed to reduce the $un-freedom$ from boundary cells. Because it is impossible to eliminate all boundary cells, the assigning algorithm must minimize not only the solution costs for each sub-problem, but also the hidden interference costs. We will overview our decomposition approach in this section and then propose several measures for our solutions to complete the whole algorithm in the next two sections.

### A. Brief Overview to Our Decomposition Algorithm

Because the time complexity of solving the FCAPs rises dramatically with the number of cells, most previous studies in the literatures have only considered 21 cells in their proposed so-

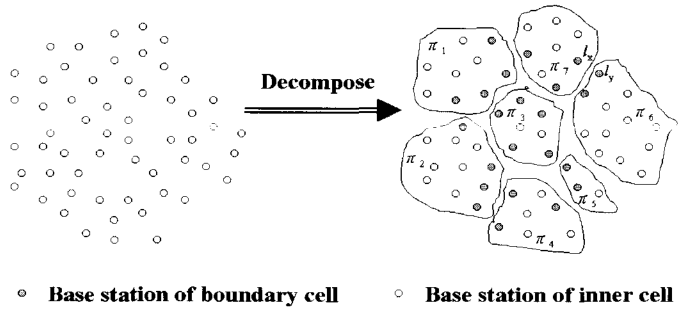


Fig. 1. Decomposition after step 1.

lutions. Therefore, we design the results from each cluster of our decomposition to contain no more than a certain number of cells. Let $\aleph$ be the maximal number of cells of each cluster, our algorithm for solving large-scale FCAP under a limited bandwidth constraint could be roughly stated as follows.

Step 1: Partition all cells into several clusters, each one of which contains no more than $\aleph$ cells and we denote $\Im$ to be the set of all clusters. (Decomposition)

Step 2: If $\Im = \phi$, terminate this algorithm.

Step 3: Find an appropriate cluster $\pi \in \Im$ and set $\Im = \Im - \{\pi\}$. (Coupling)

Step 4: Assign appropriate frequencies to all the cells of $\pi$ through minimizing the cost function (6). And then go to step 2. (Frequencies Assigning).

### B. Assigning Difficulty and Decomposition

The first task for us is to propose a decomposition algorithm for step 1 to find the best partition of the cellular network. Fig. 1 shows an example of decomposition result of step 1. In Fig. 1, a whole cellular network is decomposed into 7 clusters $\{\pi_1, \cdots, \pi_7\}$. After this step, several cells become boundary cells. A cell is called a boundary cell if it may be interfered by some cells in other clusters. The boundary cells increase the $un-freedom$ of assigning frequencies in their clusters because some frequencies are restricted to be assigned there. For example in Fig. 1, if we have solved the frequency assignment problem in $\pi_6$ where the $5^{\text{th}}$ frequency was assigned to cell $l_y$, and $C_{x,y} = 2$ is given. When we try to solve the frequency assignment problem in $\pi_7$, either all the $4^{\text{th}}$, the $5^{\text{th}}$, and the $6^{\text{th}}$ frequencies could not to be assigned to the cell $l_x$ or the corresponding interference cost will be increased. In this situation, we say that the $4^{\text{th}}$, the $5^{\text{th}}$ and the $6^{\text{th}}$ frequencies are restricted in cell $l_x$. This indicates that, for a particular cluster $\pi$, the difficulty of solving the frequency assignment in $\pi$ increases with the number of total restricted frequencies within. In Section III, we will propose several measures for measuring the un-freedom of assigning frequencies to all the cells in each cluster. Under the help of the measures, a decomposition algorithm is proposed in Section IV-A to minimize the un-freedom of each cluster it generates.

### C. Coupling and Frequency Assignment

**FrequencyAssignment**

Although we have proposed an assigning algorithm in [1] for solving the FCAPs with no more than 21 cells, however, assigning frequencies to all the cells in each individual cluster is more difficult. The difficulties arise from the boundary cells of each cluster. Assigning frequencies to the boundary cells must be carefully done because they conceal some hidden interference cost. For example in Fig. 1 with coupling sequence $\{\pi_7, \pi_1, \pi_2, \pi_3, \pi_6, \pi_5, \pi_4\}$ and $C_{x,y} = 2$, when we consider assigning the $5^{\text{th}}$ frequency to the cell $l_x$, we must take into account the possibility that we may assign the $4^{\text{th}}$, the $5^{\text{th}}$ or the $6^{\text{th}}$ frequency to the cell $l_y$ when we try to solve the channel assignment problem of $\pi_6$. On the other hand, if the assigning algorithm is forced to assign several restricted frequencies to the boundary cell $l_y$, the algorithm needs a mean to choose the most appropriate restricted frequencies to minimize not only the solution cost for $\pi_6$, but also the hidden interference costs which are increased in corresponding clusters.

Instead of preventing the hidden interference, we adopt remedial approach to minimize the hidden interference cost because predicting the future assignment is difficult. To facilitate minimizing the hidden interference cost, we modify the solution format proposed in [1] to (14) in Section III-A. The new solution format provides the current total hidden interference cost, which is concealed in each restricted frequency, for the assigning algorithm to find the best assignment in the corresponding clusters. In Section IV-C, we propose a frequency assignment algorithm, which takes into account the hidden interference cost.

**CouplingSequence**

The second task is to propose a rule to determine the most appropriate cluster of $\Im$ in step three. A coupling sequence for the partition $\Im$ is a permutation of $\Im$ which indicates the order of assigning frequencies. For example in Fig. 1, the coupling sequence $\{\pi_3, \pi_1, \pi_4, \pi_5, \pi_2, \pi_7, \pi_6\}$ indicates that our algorithm first solves the frequency assignment problem of $\pi_3$, and then $\pi_1$ until the problem of $\pi_6$ is solved. The coupling sequence may affect the quality of solutions generated by our algorithm.

For each boundary cell $l$ in the cluster of $\pi$, it is clear that the number of restricted frequencies of $l$ increases with the order where $\pi$ is in the coupling sequence. Consider the scenario where the sub-problem in cluster $\pi$ is solved after all other subproblems were solved. If the traffic load of most cells in $\pi$ is huge, then finding a solution, which assigns no or fewer restricted frequencies to the boundary cells of $\pi$, is difficult. In this scenario, the assigning algorithm in step 4 has a higher probability to assign more restricted frequencies to the boundary cells. This not only increases the solution cost in $\pi$, but also brings the hidden interference costs to the pre-solved clusters. However, if the traffic load of most cells in $\pi$ is light, then the assigning algorithm has higher probability of finding better solutions, which could assign no or fewer restricted frequencies to the boundary cells while minimizing their blocking cost. This shows that different coupling sequence brings different solution costs and indicates that reducing the solution cost through finding an appropriate coupling sequence is promising. We propose a coupling algorithm in Section IV-B.

## III. THE NOTATIONS AND MEASURES FOR OUR ALGORITHMS

For the goal of minimizing the un-freedom for the latter-solved sub-problems, we need measures for the decomposition and coupling algorithms. Whenever a cluster $X$ is determined, that is, the frequencies of each cell of $X$ have been determined, then, for any cell $l$ whose frequencies are not determined, we could easily calculate the number of restricted frequencies of $l$ from $X$. We proposed several deterministic measures in Section III-A. Although the deterministic measures are useful for the coupling algorithm, however, they become useless for the decomposition algorithm because no determined cells exist. All cells in this step are undetermined and hence we need to develop a set of non-deterministic measures to estimate the expected number of restricted frequencies and expected un-freedom based on only the traffic load of each cell and the compatibility matrix.

### A. Notations, Solution Formats and Definitions

For each $1 \leq i \leq N$, we denote the array $F_i = \{f_{i,1}, \cdots, f_{i,Z}\}$ to be the assignment of frequencies to cell $l_i$. To provide information about hidden interference cost for the assigning algorithm in step 4, we extend the domain of $f_{i,p}$ defined in (4) and (7) to (14) below.

$\forall\ 1 \leq i \leq N$ and $1 \leq p \leq Z$,

$$f_{i,p} = \begin{cases} 1, \text{ if the system has assigned the } p^{\text{th}} \text{ frequency to the cell } l_i \\ 0, \text{ if the system dose not assign the } p^{\text{th}} \text{ frequency to the cell } l_i \\ r < 0, \text{ if the } p\text{th frequency is restricted in the cell } l_i \text{ with cost } -r \\ *, \text{ if the } p\text{th frequency is not restricted in the cell } l_i, \end{cases} \quad (14)$$

Based on the definition (14), we say that the cell $l_i$ is determined if $f_{i,p} \in \{0, 1\}$ for all $1 \leq p \leq Z$ and a cluster $\pi$ is determined if all its cells are determined. To facilitate the explanation of the measures for decomposition and coupling algorithms, we define the following operators for determined cells. Fig. 2 shows the examples for the operators.

$\forall a = [a_1, \cdots, a_z], b = [b_1, \cdots, b_z]$, if both $a$ and $b$ are elements of $G = \{[x_1, \cdots, x_z] | x_i \in \{0, 1\}\}$ and $c$ is an non-negative integer, then the addition, scalar multiplication and norm operators are defined as follows.

Addition:

$$a + b = [max(a_1, b_1), \cdots, max(a_Z, b_Z)], \quad (15)$$

$ca = [d_1, \cdots, d_z]$, where

Scalar Multiplication:

$$d_p = \begin{cases} 1, \text{if } \exists q \text{ satisfies } 1 \leq q \leq Z,\ a_q = 1 \\ \quad \text{and } |p - q| < c \qquad \forall\, 1 \leq p \leq Z \\ 0, \text{o.w.} \end{cases} \quad (16)$$

Norm:

$$||a|| = \sum_{p=1}^{Z} a_p. \quad (17)$$

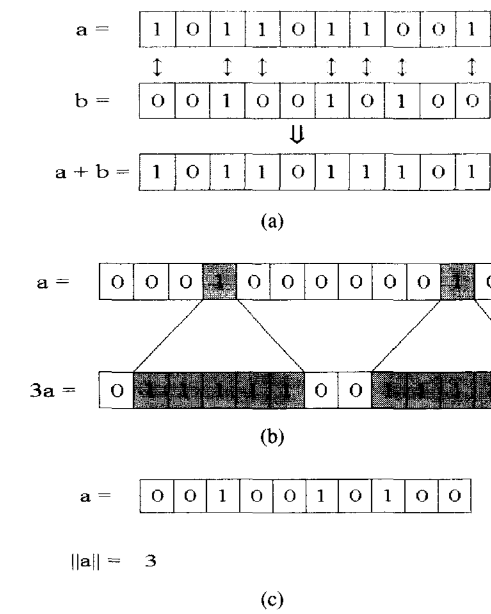

Fig. 2. The operators on F: (a) Addition, (b) scale-multiplication, (c) norm

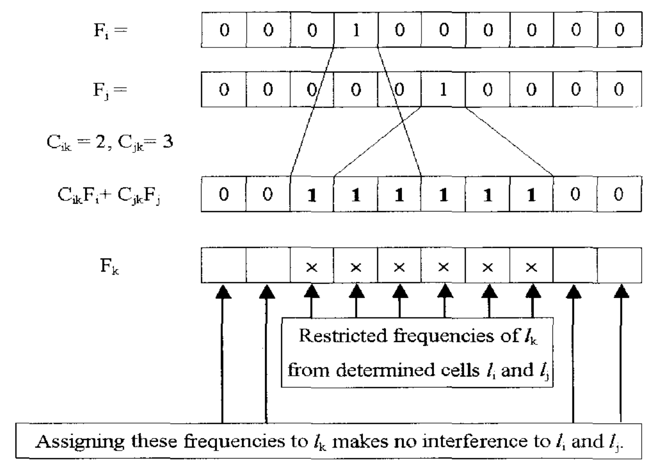


Fig. 3. Restricted frequencies calculation.

### B. Determined Measures: Straits and Freedoms

Fig. 3 shows an example of calculating the restricted frequencies for each undetermined cell. In Fig. 3, $F_i$ and $F_j$ are the assignments of the two determined cells $l_i$ and $l_j$, and $F_k$ indicates the restricted frequencies in the undetermined cell $l_k$. The content of $F_k$ could be easily derived from the result of $C_{i,k}F_i + C_{j,k}F_j$. The $p^{th}$ frequency is restricted by $l_i$ and $l_j$ in $l_k$ if and only if the $p^{th}$ element of $C_{i,k}F_i + C_{j,k}F_j$ is 1. Let X be a set of determined cells and $l_j$ be an undetermined cell, according to above discussions, the result of $\sum_{l_i \in X} C_{i,j}F_i$ indicates the frequencies which are restricted by $X$ in the cell $l_j$.

If $X$ is the set of all determined cells, then it is clear that the larger value of $|| \sum_{l_i \in X} C_{i,j}F_i||$ implies the more punitive limitation for assigning frequencies to $l_j$ since there are at most Z -$|| \sum_{l_i \in X} C_{i,j}F_i||$ frequencies, which are not restricted by $X$, in $l_j$. Thus, $|| \sum_{l_i \in X} C_{i,j}F_i||$ could be a measure to determine the strait of assigning frequencies in the cell $l_j$ under the restriction from $X$, and we hence denote it as $Strait(l_j|X)$.

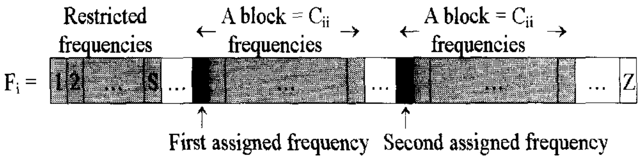


Fig. 4. Freedom of assigning frequencies in cell $l_i$.

The difficulty of assigning frequencies to an undetermined cell arises not only from its restricted frequencies, but also from its traffic load. Although the traffic load of the cell $l_i$ is represented by its mean $\mu_i$ and standard deviation $\sigma_i$ in the cost model (6) – (13), however, we prefer to use another measure, which is denoted by $D_i$, to represent the traffic load for the cell li because it facilitates the presentation of traffic load. In this paper, we use the same new call blocking rate $P_{BR} \in I(0,1)$ to derive $D_i$ from $\mu_i$ and $\sigma_i$ for each cell $l_i$ by solving equation (18) below.

$$D_i = \min\{d \text{ is an non-negative integer} \mid \frac{1}{\sqrt{2\pi}\sigma_i}\int_{T\times d}^{\infty} e^{-\frac{1}{2}(\frac{y-\mu_i}{\sigma_i})^2} dy \le P_{BR}\} \tag{18}$$

According to the Theorem 1 in [1], $\frac{1}{\sqrt{2\pi}\sigma_i}\int_{T\times d}^{\infty} e^{-\frac{1}{2}(\frac{y-\mu_i}{\sigma_i})^2} dy \le P_{BR}$ implies that if we assign d frequencies to the cell $l_i$, then the new call blocking rate in the cell $l_i$ is expected to be less than or equal to $P_{BR}$.

The freedom of assigning frequencies to each cell should take into account not only the number of restricted frequencies, but also the traffic load. Fig. 4 shows an example of assigning frequencies to cell $l_i$, which contains S restricted frequencies. If we hope that the blocking rate in $l_i$ is lower than or equal to $P_{BR}$ and no EMC constraint will be violated, then the assigning algorithm has to choose $D_i$ frequencies from the Z - S unrestricted frequencies of $l_i$. To avoid the interferences between the $D_i$ frequencies, the separations between the frequencies chosen by the assigning algorithm should be larger than or equal to $C_{i,i}$. Due to this reason, we can envision that each choice forms a block containing $C_{i,i}$ frequencies and the $D_i$ blocks could share only the $Z - S$ unrestricted frequencies in cell $l_i$. Thus, the freedom of the $D_i$ blocks in the $Z - S$ unrestricted frequencies increases with the value $(Z - S)/D_iC_{i,i}$. According to this deduction, we propose $Free(l_i|X) = (Z - Strait(l_i|X))/C_{i,i}D_i$ to be another measure for determining the freedom of assigning frequencies to in the cell $l_i$ under the restriction from the determined set of cells $X$. According to the above discussions, we formally define the measures for measuring the difficulty of frequency assignment in equations (19) - (24), where $l_i$ is a determined cell, $X$ is a set of determined cells, $l_j$ is an undetermined cell and $Y$ is a set of undetermined cells.

$$Strait(l_j|l_i) = ||C_{i,j}F_i||, \tag{19}$$

$$Free(l_j|l_i) = \frac{Z - Strait(l_j|l_i)}{D_jC_{j,j}}, \tag{20}$$

$$Strait(l_j|X) = ||\sum_{l_i\in X} C_{i,j}F_i||, \tag{21}$$

$$Free(l_j|X) = \frac{Z - Strait(l_j|X)}{D_jC_{j,j}}, \tag{22}$$

$$Strait(Y|X) = \sum_{l_j\in Y} Strait(l_j|X), \tag{23}$$

$$Free(Y|X) = \frac{\sum_{l_j\in Y, Strait(l_j|X)\neq 0} Free(l_j|X)}{\sum_{l_j\in Y, Strait(l_j|X)\neq 0} 1}. \tag{24}$$

### C. Undetermined Measures: UStrait and UFree

In the previous subsection, we have proposed two types of determined measures $Strait(.|X)$ and $Free(.—X)$ for a determined set of cells $X$. However, when we tried to decompose the whole network into several clusters, no cell was determined. In this situation, we have to propose corresponding measures for the decomposition algorithm in step 1. When $X$ is an undetermined cell or set of cells, we propose two measures $UStrait(Y|X)$ and $UFree(Y|X)$ to measure the expected straits and freedoms of $Y$ restricted by $X$. Before the measures are proposed, we assume that $C_{i,i} > 2C_{i,j}$ for all $i$ and $j \neq i$. According to our observation from the compatibility matrices of several benchmarks [1],[2],[5]–[12], this assumption is true. Under this assumption, the following are clearly true if all the EMC constraint should not be violated and the new call blocking rate in $l_j$ is required to be lower than or equal to $P_{BR}$.

$$Strait(l_j|l_i) \ge C_{i,j}D_i \text{ and } Free(l_j|l_i) \le \frac{Z - Strait(l_j|l_i)}{D_jC_{j,j}}. \tag{25}$$

And hence we define the two measures $UStrait$ and $UFree$ for each two undetermined cells $l_iadl_j$ as follows

$$UStrait(l_j|l_i) = C_{i,j}D_i \text{ and } UFree(l_j|l_i) = \frac{Z - UStrait(l_j|l_i)}{D_jC_{j,j}}. \tag{26}$$

When $X$ and $Y$ are two disjoint sets of undetermined cells, the calculation of $UStrait(Y|X)$ becomes really complex because, in each boundary cell of $Y$, the frequencies restricted by different cells of $X$ may or may not overlap, and the interference among the cells of $X$ increases the difficulty of estimation. To find an approximation for the expected value of $UStrait(l_j|X)$ where $X$ is an undetermined set of cells, we assume that $C_{i,j}$ = 0 for all $l_i$ and $l_j$ of $X$ to simplify the analysis. Although this assumption is not always true, however, the experimental results show that our approximation based on this assumption is close to the expected value of $Strait(l_j|X)^+$. We estimate the value of $Ustrait(l_j|X)$ below.

Let $P_{i,j,k}$ be the probability that a random assignment of $l_i$ will not restrict the $j^{th}$ frequency assignment of $l_k$. Based on this definition, we have,

$$P_{i,j,k} = Pr\{\forall h \in [max\{j - C_{i,k} + 1, 0\}, min\{j + C_{i,k} - 1, Z\}], f_{i,h} = 0\}$$

If $C_{i,k}$ = 0, it is clear that $P_{i,j,k} = 1$ for all $j$. Fig. 5 shows an assignment of $l_i$ in which the $j^{th}$ frequency of $l_k$ will not be

| 1~x-1 | x | j-x≥C_ik | j | y-j≥C_ik | y | w-y≥C_ii | w | w+1~Z |
|---|---|---|---|---|---|---|---|---|
| ... | 1 | 0...0 | 0 | 0...0 | 1 | 0...0 | 1 | ... |

Fig. 5. The constraint of frequencies distribution of a cell.

restricted by $l_i$ under the condition that $C_{i,k} \neq 0$. In Fig. 5, in order to satisfy the CCC in the cell $l_i$, the condition of $|x-y| \geq C_{i,i}$ is necessary if $f_{i,x} = f_{i,y} = 1$. Based on this constraint, an estimation of $P_{i,j,k}$ is derived below.

$$\begin{aligned} P_{i,j,k} &= Pr\{\forall h \in [max\{j - C_{i,k}+1, 0\}, \\ &\qquad min\{j + C_{i,k} - 1, Z\}], f_{i,h} = 0\} \\ &\approx \frac{C_{D_i}^{Z-2C_{i,k}+1-(D_i-2)(C_{i,i}-2)-(C_{i,i}-2C_{i,k})}}{C_{D_i}^{Z-(D_i-1)(C_{i,i}-1)}}, \end{aligned}$$

where $C_r^n = \frac{n!}{r!(n-r)!}$

Thus, $\prod_{l_i \in X, C_{i,k} \neq 0} \frac{C_{D_i}^{Z-2C_{i,k}+1-(D_i-2)(C_{i,i}-2)-(C_{i,i}-2C_{i,k})}}{C_{D_i}^{Z-(D_i-1)(C_{i,i}-1)}}$ is an estimation of the probability that assigning frequencies to all the cells of $X$ randomly will not restrict the $j^{th}$ frequency in $l_k$ and hence the expected number of frequencies, which could be assigned to cell $l_k$ without violating the EMC constraints to all the cells of $X$, is close to

$$\sum_{j=1}^{Z} \prod_{l_i \in X, C_{i,k} \neq 0} \frac{C_{D_i}^{Z-2C_{i,k}+1-(D_i-2)(C_{i,i}-2)-(C_{i,i}-2C_{i,k})}}{C_{D_i}^{Z-(D_i-1)(C_{i,i}-1)}}. \quad (27)$$

Although the exceptions may occur if, for all $x < j$, $f_{i,x} = 0$ or if, for all $y > j$, $f_{i,y} = 0$, the probabilities of the exceptions are expected to decrease when the size of the cell $X$, the value of $Z$ and the value of $D_i$ increase. Based on this reason, we adopt $Z$-Eq.(27) to calculate the value of $UStrait(l_k|X)$, although $Z$-Eq.(27) gives only an approximation of $UStrait(l_k|X)$.

Similar with the functions of $Strait(.|X)$ and $Free(.|X)$, we propose the following equations below to measure the corresponding expected values. With the help of Eq.(27), we have ability to estimate them.

$$UFree(l_k|X) = \frac{Z - UStrit(l_k|X)}{D_k C_{k,k}}, \quad (28)$$

$$UStrait(Y|X) = \sum_{l_k} UStrait(l_k|X), \quad (29)$$

$$UFree(Y|X) = \frac{\sum_{l_k, Strait(l_k|X) \neq 0} UFree(l_k|X)}{\sum_{l_k \in Y, Strait(l_k|X) \neq 0} 1}, \quad (30)$$

## IV. THE PROPOSED ALGORITHMS

### A. Decomposition Algorithms

The goal of the decomposition algorithms in this section is to find a partition with minimal expected interferences between

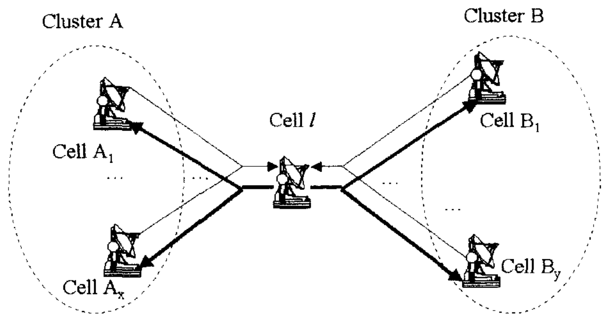


Fig. 6. The weights of a cell and its adjacent clusters.

any two different clusters. For each un-classified cell $l$, the concept behind the decomposition algorithm is either constructing a cluster for $l$, or classifying $l$ to the cluster $X$, which provides more restrictions between $X$ and $l$. Thus, a cell $l$ should belong to cluster $X$ if both $UStrait(l|X)$ and $UStrait(X|\{l\})$ are large, or both $UFree(l|X)$ and $UFree(X|\{l\})$ are small. Fig. 6 below explains the proposed decision rule for our decomposition algorithms.

In Fig. 6, cell $l$ has two adjacent clusters $A$ and $B^+$. According to step 1 in Section II-A, l *should not be classified to cluster* $A$ if $A$ has contained more than or equal to $\aleph$ cells. If all of its adjacent clusters contain more than or equal to $\aleph$ cells, or $l$ has no adjacent clusters, then a new cluster should be constructed for $l$.

Let's focus on the case that both $A$ and $B$ contain no more than $\aleph$ cells. If we classify $l$ to $A$, then the expected straits of $A$ to $B$ and the expected strait of $B$ to $A$ are $UStrait(A \cup \{l\}|B)$ *and* $UStrait(B|A \cup \{l\})$, respectively. If we adopt the $UStrait$ to be the measure of our clustering algorithm, then cluster $A$ is more appropriate to contain cell $l$ than cluster $B$ if $UStrait(A \cup \{l\}|B) + UStrait(B|A \cup \{l\}) < UStrait(B \cup \{l\}|A) + UStrait(A|B \cup \{l\})$.

Although the possibility is small, we still need to consider the case that $UStrait(A \cup \{l\}|B) + UStrait(B|A \cup \{l\}) = UStrait(B \cup \{l\}|A) + UStrait(A|B \cup \{l\})$. When this situation occurs, $l$ should be classified to $A$ if $UStrait(l|A) < UStrait(l|B)$. This could reduce the difficulty of assigning frequencies to all the cells in both $A$ and $B$. If, both relations $UStrait(A \cup \{l\}|B) + UStrait(B|A \cup \{l\}) = UStrait(B \cup \{l\}|A) + UStrait(A|B \cup \{l\})$ and $UStrait(l|A) = UStrait(l|B)$ are true, to balance the difficulties of assigning frequencies, we assign $l$ to $A$ if $A$ contains less cells than $B$. Based on the above reason, let $S$ be the set of all cells that does not belongs to any cluster and $P$ be the set of all generated clusters, we propose our decomposition algorithm based on the measure $UStrait$ as follows.

*Strait − based algorithm*

Step 1 : Let $i = 0, j = 0, \Pi = \phi$

Step 2 : if $S = \phi$,terminate this algorithm and $\Pi$ is the results of this algorithm.

Step 3 : $i = i + 1, S = S - \{l_i\}$

Step 4 : Let $\Lambda_0 = \{\pi \in \Pi$— there exists $l_k \in \pi$ such that $C_{i,k} \neq 0$ *and* $||\pi|| < \aleph\}$. If $\Lambda_0 \neq \phi$, go to step 6

Step 5 : Let $j = j + 1, \pi_j = \{l_i\}$ and $\Pi = \Pi \cup \{\pi_j\}$ and go to step 2

Step 6 :

Step 6.1 : Find $\Lambda_1 \subseteq \Lambda_0$ satisfying

$$UStrait(\pi_x \bigcup\{l_i\}| \bigcup_{\pi\in\Lambda_0,\pi\neq\pi_x} \pi) + \sum_{\pi\Lambda_0,\pi\neq\pi_x} UStrait(\pi|\pi_x \bigcup\{l_i\}) \leq UStrait(\pi_y \bigcup\{l_i\}| \bigcup_{\pi\in\Lambda_0,\pi\neq\pi_y} \pi) + \sum_{\pi\Lambda_0,\pi\neq\pi_y} UStrait(\pi|\pi_y \bigcup\{l_i\}$$

for all $\pi_y \in \Lambda_0$ and $\pi_x \in \Lambda_0$.
And for $all \pi_x \in \Lambda_1, ||\pi_x|| \leq \aleph$.

Step 6.2 : Find $\Lambda_2 \subseteq \Lambda_1$ satisfying $UStrait(l_i|\pi_x) \leq UStrait(l_i|\pi_y)$ for all $\pi_y \in \Lambda_1$and $\pi_x \in \Lambda_2$

Step 6.3 : Find an element $\pi_x \in \Lambda_1$ satisfying Go to step 2
Similarly, we could propose another decomposition algorithm called Freedom-based algorithm, which adopts the measure $UFree$, through replacing the steps 6.1 and 6.2 of the Strait-based algorithm with the procedures below.

Step 6.1 : Find $\Lambda_1 \subseteq \Lambda_0$ satisfying

$$UFree(\pi_x \bigcup\{l_i\}| \bigcup_{\pi\in\Lambda_0,\pi\neq\pi_x} \pi) + \sum_{\pi\Lambda_0,\pi\neq\pi_x} UFree(\pi|\pi_x \bigcup\{l_i\}) \geq UFree(\pi_y \bigcup\{l_i\}| \bigcup_{\pi\in\Lambda_0,\pi\neq\pi_y} \pi) + \sum_{\pi\Lambda_0,\pi\neq\pi_y} UFree(\pi|\pi_y \bigcup\{l_i\})$$

for all $\pi_y \in \Lambda_0$ *and* $\pi_x \in \Lambda_0$.
And for all $\pi_x \in \Lambda_1, ||\pi_x|| \leq \aleph$.

Step 6.2 : Find $\Lambda_2 \subseteq \Lambda_1$ satisfying $UFree(l_i|\pi_x) \geq UFree(l_i|\pi_y)$ for all $\pi_y \in \Lambda_1$ and $\pi_x \in \Lambda_2$

### *B. The Coupling Rules*

According to our algorithm proposed in Section II-A, the coupling rules are used to choose the next undetermined cluster for the assigning algorithm proposed in Section IV-C. According to the discussion in Section II-C, we conclude that solving a subproblem of a cluster will become more difficult if most of its adjacent clusters were determined. This suggests that cluster $A$ should be determined earlier than cluster $B$ if cluster $A$ is harder to be determined than cluster $B$.

The difficulty of determining a cluster may arise from its determined adjacent clusters or from the interferences among its cells. For each $\pi \in \Pi$, if we define

$$UStrait(\pi) = \sum_{l\in\pi} UStrait(l|\pi - \{l\}). \qquad (31)$$

According to the definition (31), $UStrait(\pi)$ generally increases with the difficulty of determining $\pi$ because for each $l \in \pi$, $UStrait(l|\pi - \{l\})$ is the expected number of frequencies of $l$, which are restricted by other cells of $\pi$.Thus, we use $UStrait(\pi)$ to measure the difficulty arising from the interferences among the cells of $\pi$.

The principle behind the coupling rule is determining which cluster is more difficult to be determined early on. When we apply the coupling rule in step 3 in Section II-A, some clusters may have been determined. Let $\Pi_d$ be the set of all determined clusters and $\Pi_u$ be the set of all undetermined clusters. Then the difficulties of determining $p$, which arise from $\Pi_d$ and $\Pi_u$, are $Strait(\pi| \bigcup_{X\in\Pi_d} X)$ and $UStrait(\pi| \bigcup_{Y\in\Pi_u-\{\pi\}} Y)$. Because we still have a chance to reduce through designing better assigning algorithms in Step 4 in Section II-A, the weight of $UStrait(\pi| \bigcup_{Y\in\Pi_u-\{\pi\}} Y)$ should be lower than the weight of $\Pi_u$, are $Strait(\pi| \bigcup_{X\in\Pi_d} X)$. Based on the above discussion, we propose our coupling rule based on the measures UStrait and Strait as follows.

**Strait – basedcouplingrule**

Select $\pi \in \Pi_u$ satisfying

$$Strait(\pi) + \beta_1 \times Strait(\pi| \bigcup_{X\in\Pi_d} X) + \beta_2 \times UStrait(\pi| \bigcup_{Y\in\Pi_u-\{\pi\}} Y) \geq Strait(\zeta) + \beta_1 \times Strait(\zeta| \bigcup_{X\in\Pi_d} X) + \beta_2 \times UStrait(\zeta| \bigcup_{Y\in\Pi_u-\{\zeta\}} Y) \quad \text{for all } \zeta \in \Pi_u.$$

Determining the weights $\beta_1$ and $\beta_2$ is difficult. Because the assignments in all the cells of $\Pi_u$ are undetermined, we still have a chance to reduce the value $UStrait(\pi| \bigcup_{Y\in\Pi_u-\{\pi\}} Y)$ through appropriately assigning frequencies to all the cells of $\Pi_u$. According to this, it is reasonable to think $\beta_1 > \beta_2$. In all the experiments in this paper, we set $\beta_1 = 0.7$ and $\beta_2 = 0.4$ and leave the determination of the two parameters as future tasks. Similarly, we could apply the measures $UFree$ and $Free$ to measure the difficulty of each cluster and hence we propose another coupling rule as follows, where

$$UFree(\pi) = \sum_{l\in\pi} UFree(l|\pi - \{l\}). \qquad (32)$$

**Freedom-based coupling rule**
Select $\pi \in \Pi_u$ satisfying

$$UFree(\pi) + \beta_1 \times Free(\pi| \bigcup_{X\in\Pi_d} X)$$

$$+\beta_2 \times UFree(\pi | \bigcup_{Y \in \Pi_u - \{\pi\}} Y)$$

$$\geq UFree(\zeta) + \beta_1 \times Free(\zeta | \bigcup_{X \in \Pi_d} X)$$

$$+\beta_2 \times UFree(\zeta | \bigcup_{Y \in \Pi_u - \{\zeta\}} Y) \quad \text{for all } \zeta \in \Pi_u.$$

### C. The Assigning Algorithm

Let $X = \{X_1, X_2, \cdots, X_m\}$be a set of undetermined cells and let $X_{i,p}$ denote the status of the $p^{th}$ frequency in the cell $X_i$, where the domain of each $X_{i,p}$ are defined in (14). According to (14), $X_{i,p} < 0$ implies the $p^{th}$ frequency is a restricted frequency in cell $X_i$. This information reveals that the assigning algorithm should not assign the $p^{th}$ frequency to $X_i$. Otherwise, $X_i$ would interfere with some determined cells. Let $Y_i$ denote the number of restricted frequencies in $X_i$. The principle behind the assigning algorithm are as follows: 1) If $Y_i$ is large, then $X_i$ should be determined earlier; otherwise determining $X_i$ may become more difficult (Step 3 below), 2) Each assignment should not increase too much cost from interference (Step 7 below) and 3) Reduce the number of channel resources in each cell if necessary (Step 8 below). Let $\gamma_i$ denote the number of required frequencies of $X_i$, where $\gamma_i$ could be calculated by equation (18), and let $\varphi_i$ denote the minimum frequency separation between the frequencies used simultaneously in $X_i$. We propose an assigning algorithm as follows.

Step 1 : Let $X' = \phi$.
Step 2 : If $X' = X$, terminate this algorithm.
Step 3 : Select a $X_i \in X - X'$ such that $Y_i \geq Y_j$ for all $X_j \in X - X'$
Step 4 : If $\gamma_i < \lfloor \frac{Z}{\varphi_i} \rfloor$ , choose an integer $\lambda \in [\gamma_i, \lfloor \frac{Z}{\varphi_i} \rfloor]$ randomly, otherwise, choose $\lambda = \lceil \frac{Z}{\varphi_i} \rceil$.
Step 5 : Randomly choose an integer sequence $1 = d_0 \leq d_1 < d_2 < \cdots < d_\lambda \leq d_{\lambda+1} = Z$ such that $d_{h+1} - d_h$ is about $Z/\lambda$ for all $1 \leq h < \lambda$. Let $k = 1$ and $\lambda' = \lambda$.
Step 6 : Let $j = d_k$
Step 7 : If $X_{i,j} = *$, then $y = j$. Otherwise, find $y \in (d_{k-1}, d_{k+1})$ such that $X_{i,y} - f(X_i, d_{k-1}, y) - f(X_{i,y}, d_{k+1}) \geq X_{i,y'} - f(X_i, d_{k-1}, y') - f(X_{i,y'}, d_{k+1})$ for all $y' \in (d_{k-1}, d_{k+1})$, where

$$f(X_i, a, b) = \begin{cases} \Psi_c(\varphi_i - |b-a|) & \text{if } |b-a| < \varphi_i \\ 0 & \text{Otherwise.} \end{cases} \tag{33}$$

Step 8 : Let $d_k = y$. If

$$\frac{\alpha}{\sqrt{2\pi}\sigma_i'}\left(\int_{T\times(\lambda-1)}^{\infty}(w - T\times(\lambda-1))e^{-\frac{1}{2}(\frac{y-\mu_i'}{\sigma_i'})^2}dw - \int_{T\times\lambda}^{\infty}(w - T\times\lambda)e^{-\frac{1}{2}(\frac{y-\mu_i'}{\sigma_i'})^2}dw\right)$$
$$+2(X_{i,y} - f(X_i, d_{k-1}, y) - f(X_i, y, d_{k+1})) < 0$$

, set $X_{i,y} = 0$ and $\lambda = \lambda - 1$. Otherwise, set $X_{i,y} = 1$. If $k < \lambda'$, let $k = k + 1$ and go to Step 6.

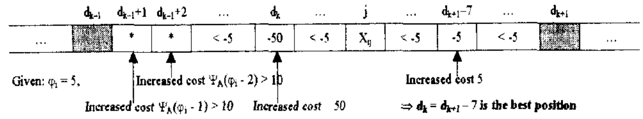


Fig. 7. A local search for finding better frequencies.

Step 9 : For all $1 \leq j \leq Z$, If $X_{i,j} = *, X_{i,j} = 0$. Whenever $X_i$ becomes a determined cell, let $X' = X' \cup X_i$.
Step 10 : Recalculate the values of frequency states (14) of each undetermined cell which is adjacent to $X_i$ and go to Step 2.

Fig. 7 explains the process in Step 7 above. In Fig. 7, the initial assignment assigns the $d_k^{th}$ frequency to the cell $X_i$. However, this assignment increases the cost to 100 (50 in this cell and 50 in other cells). However, if we replace the $d_k^{th}$ frequency by the $(d_{k+1} - 7)^{th}$ frequency, we could reduce cost to 90 (45 in this cell and 45 in other cells). For each $y \in (d_{k-1}, d_{k+1})$, if the assigning algorithm would like to assign the $y^{th}$ frequency to cell $X_i$, then the cost will increase $2(f(X_i, d_{k-1}, y) + f(X_i, y, d_{k+1}) - X_{i,y})$. Thus, if the initial value of $d_k$ is not good enough, we can improve the assignment through finding a better value for $d_k$ in the interval $(d_{k-1}, d_{k+1})$ in Step 7. If the increased cost of assigning the $y^{th}$ frequency to $X_i$ is large, assigning fewer frequencies may be a better choice. Whenever we are aware of that, for all $y' \in (d_{k-1}, d_{k+1})$, the increased cost $2(f(X_i, d_{k-1}, y') + f(X_i, y', d_{k+1}) - X_i, y')$ is larger then the increased new call blocking cost, then we consider assigning no frequencies between the $d_{k-1}$ and the $d_{k+1}$ frequency to the cell $X_i$ in Step 8. Finally, whenever $X_i$ becomes determined in step 10, for each undetermined cell $l$, we reset $l_k$, the status of the $k^{th}$ frequency of cell $l$, as follows.

$$l_k = \begin{cases} -\sum_{y=max(0,k-\varphi_i+1)}^{min(k+\varphi_i-1,Z)} X_{i,y} \times \Psi_A(\varphi_i - |y-k|) \\ \quad if\ l_k = * \\ l_k - \sum_{y=max(0,k-\varphi_i+1)}^{min(k+\varphi_i-1,Z)} X_{i,y} \times \Psi_A(\varphi_i - |y-k|) \\ \quad if\ l_k < 0. \end{cases} \tag{34}$$

## V. EXPERIMENT RESULTS AND COMPARISONS

In this section, two experiments were performed to demonstrate the performances of our algorithms. In the first experiment, a layout of a cellular network problem was proposed to be the benchmark for testing the performance of our algorithms under different interference radius, which decides the compatibility matrices of the problems. We then applied our algorithms to solve the frequency assignment problem in the cellular network of the Taiwan Cellular Network (TCC) in the Chung-Li city and showed the improvements in the second experiment.

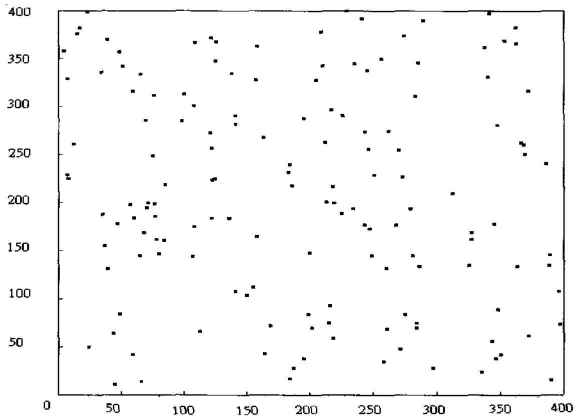


Fig. 8. The layout of the base stations.

### A. The Benchmark Problems

According to the cost model (6) – (13), we need to specify a compatibility matrix and a loading table for each benchmark problem. Specifying the details of each compatibility matrix is difficult, especially when the scale of the corresponding network is large. To overcome this difficulty, we specify the position of all the base stations of the corresponding network and use the interference radius to calculate the corresponding compatibility matrix. In this paper, an interference radius is defined to be a triple $(a, b, c)$ and the calculation of the minimal frequency separation is defined as follows.

$$C_{i,j} = \begin{cases} 3, & \text{if } 0 < \text{dist}(i,j) \le a \\ 2, & \text{if } a < \text{dist}(i,j) \le b \\ 1, & \text{if } b < \text{dist}(i,j) \le c \\ 0, & \text{if } c < \text{dist}(i,j), \end{cases} \tag{35}$$

where $\text{dist}(i,j)$ is the distance between the $i^{th}$ and the $j^{th}$ base station, and $C_{i,j} = 7$ if $i = j$.

Fig. 8 below shows the layout of all the base stations of the benchmark problems. All the base stations were placed randomly in a $400 \times 400$ $unit^2$ area. The positions of all the base stations and the loading table are specified in appendix 1. The new call blocking rate $P_{BR}$ is set to be 0.1. All the parameters of the cost model (6) - (13) are $Z = 66^+$, $\alpha = 1000$, $T = 7$ and $\psi_A(x) = \psi_C(x)^* = 5^{x-1}$, and the parameters $\beta_1$ and $\beta_2$ in the two coupling rules in Section IV-B are 0.7 and 0.4, respectively.

### B. Experimental Results and Comparisons

To show the performance of our algorithms, we compare the costs of 100 solutions generated by the following four algorithms under different interference radius. The first algorithm is denoted as "Non Clustered". In this experiment, only the assignment algorithm is applied to the whole network. The second is denoted as "Random". In this experiment, we use strait-based decomposition algorithm to partition the whole network into several clusters, each one of which contains no more than 21 cells. The coupling sequences of the corresponding 100 solutions are decided randomly and then the assignment algorithm are applied to each cluster. The third and the fourth are denoted as "Strait" and "Free", respectively. The Strait-based decomposition algorithm and coupling rule are adopted in the algorithm "Strait", while the Freedom-based decomposition algorithm and coupling rule are used for the algorithm "Free".

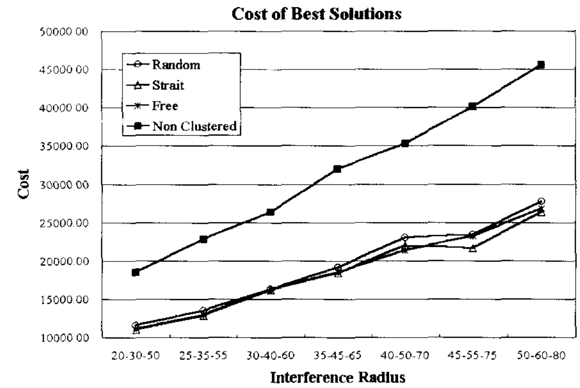


Fig. 9. The costs of the best solutions.

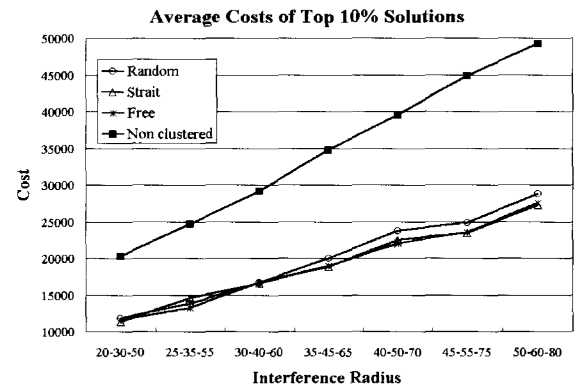


Fig. 10. The average cost of the top 10% solutions.

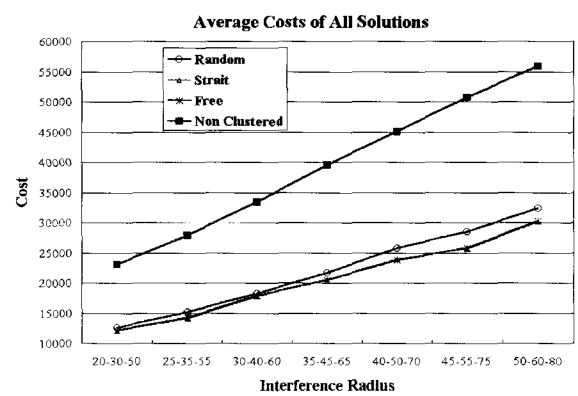


Fig. 11. The average costs of all the solutions.

Table 2 in appendix 2 shows the experimental results. In table 2, the costs of best solutions, the average of top 10 solutions and the average of all the solutions generated by the four algorithms under different interference radius were presented. We compare the performances of the four algorithms by Fig. 9, 10 and 11 below.

Fig. 9, 10 and 11 show the improvement from our decomposition algorithm is remarkable. The costs of all the solutions are reduced significantly under the help of our decomposition algorithm. It should be noticed that the low values of SDV (standard deviation) in the column "Random" of table 2 shows that the clusters are almost independent. This is the reason why the solution quality has not improved significantly under the help

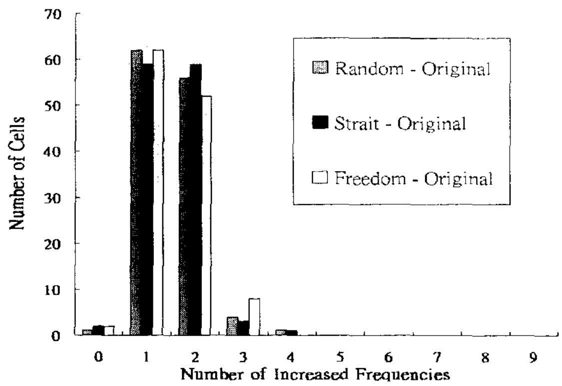


Fig. 12. The improvement of Chung-Li city problem.

of our coupling rule. To show the effect of our coupling rule, we increase the dependencies between the clusters through increasing the interference radius. According to Fig. 9 - 11, we observe that our coupling rules do improve the costs of the solutions when the dependences of the clusters increase. Based on the experimental results, we conclude that our decomposition algorithm and coupling rule do improve the solution quality of large-scale channel assignment problems significantly.

### C. The Chung-Li City Problem

We have applied our algorithms to improve the quality of frequency assignments in a sub-network, which contains 150 base stations, of TCC in the Chung-Li city. According to the operator's suggestion, we set $C_{i,j} = 5$ if $i = j$ and $C_{i,j} = 2$ if the $i^{\text{th}}$ cell is adjacent or opposite to the $j^{\text{th}}$ cell. Fig. 12 shows an improvement of the Chung-Li city problem, which satisfies a new constraint that no channel interference is allowed, with $P_{BR} = 0.05$.

In Fig. 12, there are about 60 base stations, which could get one more frequency, and there are about 55 base stations, which could increase two more frequencies under the help of our algorithms. Compared to the original assignment, Fig. 12shows that our algorithm improves the solution quality of the Chung-Li city problem significantly.

## VI. CONCLUSION AND FUTURE WORKS

Due to the difficulty of solving the fixed channel assignment for large-scale cellular network directly, we propose a decomposition algorithm to decompose the whole network into several smaller clusters and then solve them individually. Because the clusters are not independent and assigning frequencies to one cluster will affect the assignment in other clusters, minimizing the dependences between the clusters and finding appropriate coupling sequence become crucial for frequency assignment algorithms.

In this paper, we analyze and then propose several measures for measuring the dependences between clusters of base stations. Through the help of the measures, we define the weights between each two clusters and then propose two decomposition algorithm. Besides this, two coupling rules, which are helpful in finding better coupling sequences, were proposed based on the measures. The experimental results show that the decomposition algorithms do improve the solution quality significantly and the coupling rules are helpful in finding better coupling sequences.

Because the EMC and demand constraints were relaxed in the cost model (6) – (13), solving the frequency assignment problem in each cluster becomes more difficult. The difficulties arise from the boundary cells, which conceal hidden interference cost. Instead of forbidding the restricted frequencies to be assigned in the corresponding cells, we propose a transient solution format for recording the hidden interference cost of each restricted frequency. Although the transient solution format proposes suggestions to the assigning algorithm, however, it is insufficient because we provide no means for predicting the cost of interfering with the undetermined clusters. This inspires us to propose a more appropriate approach for determining each cluster.

Besides designing better decomposition algorithm, coupling rule and assigning algorithm, determining the related parameters such as $\alpha, \beta_1$ and $\beta_2$ is also important for improving the quality of frequency assignment. We are trying to design algorithms for determining the related parameters.

## APPENDIX A: THE BENCHMARK PROBLEM SPECIFICATIONS

Table 1. The position and loading of each base station.

| ID | Position | Mean | SDV | ID | Position | Mean | SDV |
|---|---|---|---|---|---|---|---|
| 1 | (260, 268) | 20 | 6.57 | 39 | (39, 269) | 10 | 3.31 |
| 2 | (136, 216) | 21 | 7.01 | 40 | (123, 176) | 23 | 8.12 |
| 3 | (235, 55) | 22 | 8.16 | 41 | (163, 132) | 22 | 7.67 |
| 4 | (200, 252) | 20 | 6.41 | 42 | (243, 126) | 23 | 7.36 |
| 5 | (199, 316) | 19 | 5.94 | 43 | (217, 103) | 21 | 7.93 |
| 6 | (396, 292) | 6 | 2.13 | 44 | (141, 118) | 21 | 6.98 |
| 7 | (388, 265) | 6 | 1.79 | 45 | (205, 72) | 10 | 3.43 |
| 8 | (45, 389) | 11 | 3.48 | 46 | (386, 159) | 6 | 1.82 |
| 9 | (69, 114) | 19 | 6.85 | 47 | (24, 350) | 8 | 2.57 |
| 10 | (256, 50) | 21 | 6.5 | 48 | (271, 352) | 20 | 7.19 |
| 11 | (218, 183) | 20 | 7.33 | 49 | (187, 372) | 19 | 5.82 |
| 12 | (150, 296) | 21 | 7.76 | 50 | (78, 238) | 21 | 7.73 |
| 13 | (340, 69) | 18 | 6.88 | 51 | (35, 212) | 10 | 3.06 |
| 14 | (274, 26) | 23 | 8.15 | 52 | (37, 245) | 10 | 2.88 |
| 15 | (184, 160) | 20 | 7.5 | 53 | (285, 54) | 13 | 4.3 |
| 16 | (241, 8) | 24 | 8.22 | 54 | (284, 330) | 22 | 6.96 |
| 17 | (243, 223) | 23 | 7.54 | 55 | (108, 99) | 14 | 4.23 |
| 18 | (85, 181) | 12 | 4.52 | 56 | (389, 254) | 6 | 1.94 |
| 19 | (157, 72) | 24 | 9.13 | 57 | (249, 255) | 20 | 6.27 |
| 20 | (210, 57) | 10 | 3.44 | 58 | (215, 325) | 19 | 6.21 |
| 21 | (273, 173) | 22 | 8.05 | 59 | (347, 119) | 16 | 4.83 |
| 22 | (125, 52) | 23 | 7.95 | 60 | (219, 200) | 21 | 7.43 |
| 23 | (348, 311) | 15 | 4.95 | 61 | (7, 71) | 6 | 1.83 |
| 24 | (369, 149) | 10 | 2.86 | 62 | (75, 151) | 21 | 7.97 |
| 25 | (372, 83) | 9 | 3.06 | 63 | (281, 255) | 22 | 6.95 |
| 26 | (195, 362) | 20 | 6.17 | 64 | (247, 227) | 23 | 8.12 |
| 27 | (363, 266) | 10 | 3.59 | 65 | (100, 87) | 3 | 1.12 |
| 28 | (169, 328) | 21 | 6.8 | 66 | (251, 171) | 19 | 7 |
| 29 | (164, 357) | 21 | 6.66 | 67 | (390, 384) | 6 | 2.29 |
| 30 | (39, 30) | 12 | 4.39 | 68 | (158, 37) | 24 | 7.58 |
| 31 | (229, 0) | 23 | 8.66 | 69 | (80, 253) | 21 | 7.99 |
| 32 | (218, 341) | 19 | 6.97 | 70 | (107, 256) | 13 | 3.97 |
| 33 | (125, 175) | 23 | 8.22 | 71 | (372, 338) | 8 | 2.34 |
| 34 | (57, 202) | 16 | 5.18 | 72 | (213, 199) | 20 | 6.58 |
| 35 | (8, 175) | 6 | 2.02 | 73 | (368, 139) | 10 | 3.36 |
| 36 | (121, 127) | 24 | 8.32 | 74 | (71, 200) | 19 | 7.05 |
| 37 | (113, 334) | 22 | 8.35 | 75 | (362, 17) | 11 | 4.09 |
| 38 | (47, 222) | 12 | 3.75 | 76 | (186, 182) | 20 | 6.37 |

| ID | Position | Mean | SDV | ID | Position | Mean | SDV |
|---|---|---|---|---|---|---|---|
| 77 | (76, 201) | 21 | 7.06 | 115 | (212, 137) | 21 | 7.75 |
| 78 | (122, 28) | 14 | 4.02 | 116 | (343, 344) | 16 | 5.53 |
| 79 | (226, 109) | 21 | 7.08 | 117 | (345, 222) | 16 | 5.96 |
| 80 | (59, 84) | 17 | 5.92 | 118 | (261, 331) | 20 | 6.14 |
| 81 | (65, 255) | 18 | 6.12 | 119 | (155, 288) | 23 | 7.01 |
| 82 | (327, 231) | 19 | 6.84 | 120 | (17, 18) | 7 | 2.38 |
| 83 | (202, 330) | 19 | 5.42 | 121 | (15, 24) | 6 | 2.07 |
| 84 | (284, 325) | 22 | 7.88 | 122 | (366, 137) | 10 | 3.1 |
| 85 | (48, 43) | 13 | 4.06 | 123 | (258, 365) | 19 | 6.26 |
| 86 | (68, 231) | 19 | 6.41 | 124 | (268, 223) | 22 | 7.33 |
| 87 | (23, 1) | 8 | 2.87 | 125 | (66, 386) | 17 | 5.34 |
| 88 | (341, 3) | 18 | 6.28 | 126 | (44, 336) | 11 | 3.64 |
| 89 | (109, 33) | 3 | 1.11 | 127 | (34, 64) | 10 | 3.4 |
| 90 | (122, 143) | 23 | 7.12 | 128 | (246, 144) | 23 | 7.16 |
| 91 | (141, 109) | 21 | 6.89 | 129 | (77, 214) | 21 | 7.02 |
| 92 | (289, 10) | 3 | 0.86 | 130 | (286, 266) | 22 | 7.53 |
| 93 | (337, 38) | 19 | 6.01 | 131 | (234, 206) | 22 | 7.01 |
| 94 | (183, 168) | 20 | 6.51 | 132 | (4, 42) | 6 | 2.04 |
| 95 | (184, 383) | 19 | 6.24 | 133 | (225, 211) | 21 | 6.8 |
| 96 | (49, 316) | 12 | 3.66 | 134 | (76, 88) | 21 | 7.69 |
| 97 | (297, 372) | 22 | 6.81 | 135 | (126, 32) | 23 | 7.79 |
| 98 | (12, 139) | 6 | 1.71 | 136 | (350, 358) | 12 | 3.56 |
| 99 | (7, 171) | 6 | 1.72 | 137 | (51, 58) | 16 | 5.42 |
| 100 | (279, 206) | 23 | 8.72 | 138 | (158, 235) | 22 | 6.92 |
| 101 | (325, 265) | 19 | 5.82 | 139 | (195, 112) | 10 | 3.02 |
| 102 | (335, 376) | 17 | 5.88 | 140 | (270, 145) | 22 | 8.18 |
| 103 | (60, 216) | 17 | 5.39 | 141 | (108, 225) | 13 | 4.55 |
| 104 | (283, 89) | 13 | 4.05 | 142 | (275, 316) | 21 | 6.26 |
| 105 | (122, 216) | 23 | 7.12 | 143 | (84, 239) | 21 | 7.18 |
| 106 | (362, 34) | 11 | 4 | 144 | (70, 205) | 19 | 6.95 |
| 107 | (141, 292) | 19 | 5.85 | 145 | (216, 307) | 19 | 5.73 |
| 108 | (346, 362) | 15 | 5.6 | 146 | (59, 358) | 16 | 5.51 |
| 109 | (209, 22) | 10 | 2.85 | 147 | (65, 66) | 19 | 6.47 |
| 110 | (138, 65) | 22 | 6.28 | 148 | (327, 238) | 19 | 6.33 |
| 111 | (262, 125) | 21 | 7.23 | 149 | (312, 190) | 12 | 4.52 |
| 112 | (98, 115) | 13 | 3.86 | 150 | (397, 326) | 6 | 2.29 |
| 113 | (245, 62) | 23 | 6.6 | | | | |
| 114 | (353, 31) | 13 | 4.55 | | | | |

## APPENDIX B: THE EXPERIMENTAL RESULTS

Table 2. Experimental results.

| Interference Radius | | Non Clustered | Random | Strait | Free |
|---|---|---|---|---|---|
| 20-30-50 | Best | 18512.01 | 11552.88 | 11004.72 | 11152.65 |
| | Mean | 23037.16 | 12599.89 | 12153.32 | 12168.39 |
| | Top 10% | 20318.78 | 11847.77 | 11380.35 | 11539.41 |
| | SDV | 1616.663 | 455.11 | 474.26 | 356.41 |
| 25-35-55 | Best | 22883.13 | 13466.09 | 12838.09 | 12911.41 |
| | Mean | 27890.27 | 15160.15 | 14136.65 | 14221.36 |
| | Top 10% | 24692.33 | 13852.21 | 14546.52 | 13285.06 |
| | SDV | 1940.694 | 813.59 | 633.26 | 616.49 |
| 30-40-60 | Best | 26343.1 | 16311.27 | 16205.07 | 16307.5 |
| | Mean | 33365.25 | 18203.57 | 17821.21 | 17907.32 |
| | Top 10% | 29180.46 | 16682.55 | 16536.16 | 16630.82 |
| | SDV | 2503.65 | 866.05 | 802.98 | 796.29 |
| 35-45-65 | Best | 32017.05 | 19103.34 | 18365.86 | 18503.01 |
| | Mean | 39585.43 | 21695.66 | 20546.46 | 20477.76 |
| | Top 10% | 34747.12 | 19964.27 | 18869.26 | 18987.64 |
| | SDV | 2833.503 | 1019.06 | 982.99 | 1005.7 |
| 40-50-70 | Best | 35268.8 | 23026.39 | 21963.83 | 21406.46 |
| | Mean | 45084.51 | 25722.58 | 24002.71 | 23806.04 |
| | Top 10% | 39479.11 | 23710 | 22432.39 | 21997.23 |
| | SDV | 3234.811 | 1326.43 | 1049.11 | 1133.65 |
| 45-55-75 | Best | 40071.28 | 23357.64 | 21598.65 | 23173.13 |
| | Mean | 50705.28 | 28413.96 | 25604.42 | 25822.85 |
| | Top 10% | 44870.01 | 24841.19 | 23396.55 | 23547.72 |
| | SDV | 3415.387 | 2194.59 | 1286.66 | 1317.95 |
| 50-60-80 | Best | 45529.24 | 27747.28 | 26334.32 | 26850.39 |
| | Mean | 55827.82 | 32357.73 | 30062.83 | 30222.72 |
| | Top 10% | 49231.9 | 28794.62 | 27228.1 | 27522.62 |
| | SDV | 3907.319 | 2056.63 | 1703.29 | 1636.57 |

## APPENDIX C: THE EXPERIMENTAL DESIGN AND RESULT FOR UNDERMINED MEASURES

Table 3. The summary of the 2500 data from 500 experiments.

| S | Sample Size | E(US) | E(F) | E(UF) | E(\|S-US\|) | Var(\|S-US\|) | E(\|F-UF\|) | Var(\|F-UF\|) |
|---|---|---|---|---|---|---|---|---|
| 2 | 8 | 2.864 | 4.575 | 4.5102 | 0.863636 | 1.951E-08 | 0.064773 | 0.0230817 |
| 3 | 108 | 2.864 | 4.3019 | 4.3116 | 0.136364 | 0 | 0.009777 | 0.0037506 |
| 5 | 24 | 5.997 | 4.0278 | 3.9586 | 0.996827 | 2.1527E-08 | 0.069224 | 0.0240343 |
| 6 | 476 | 5.997 | 4.3828 | 4.383 | 0.003173 | 0 | 0.000244 | 0.0001242 |
| 8 | 92 | 9.435 | 3.9457 | 3.8427 | 1.434891 | 9.5171E-08 | 0.102938 | 0.0351221 |
| 9 | 898 | 9.442 | 3.9746 | 3.9421 | 0.445635 | 0.16224111 | 0.032728 | 0.0177981 |
| 10 | 6 | 14.53 | 3.975 | 3.635 | 4.533632 | 0 | 0.340023 | 0.1241583 |
| 11 | 73 | 13.22 | 4.8153 | 4.61 | 2.217707 | 0 | 0.205366 | 0.1019208 |
| 12 | 559 | 13.23 | 3.7491 | 3.659 | 1.227123 | 0.11101576 | 0.090144 | 0.0382336 |
| 13 | 10 | 18.42 | 6.5 | 5.7276 | 5.421701 | 1.55362098 | 0.772363 | 0.5260085 |
| 14 | 7 | 17.39 | 4.2 | 3.9095 | 3.389286 | 0 | 0.290511 | 0.0603882 |
| 15 | 141 | 17.05 | 3.6119 | 3.4516 | 2.15936 | 0.62919221 | 0.166734 | 0.0980915 |
| 16 | 4 | 20.67 | 3.1333 | 2.8218 | 4.673112 | 0 | 0.31154 | 0 |
| 17 | 17 | 19.16 | 3.2922 | 3.1459 | 2.159559 | 1.65477321 | 0.14621 | 0.1081361 |
| 18 | 28 | 18.44 | 3.4714 | 3.4183 | 0.929973 | 0.93534422 | 0.08415 | 0.0983713 |
| 20 | 4 | 20.67 | 4.3 | 4.2327 | 0.673112 | 0 | 0.067311 | 0 |
| 21 | 13 | 22.08 | 2.3692 | 2.3095 | 1.47868 | 1.73893126 | 0.08325 | 0.1086087 |
| 22 | 12 | 23.66 | 2.9611 | 2.8411 | 1.662274 | 0 | 0.120053 | 0.0360899 |
| 23 | 8 | 23.94 | 3 | 2.9364 | 0.941114 | 0.2980925 | 0.063613 | 0.002795 |
| 24 | 11 | 25.01 | 2.3636 | 2.314 | 1.258665 | 1.34866163 | 0.06598 | 0.0650549 |
| 27 | 1 | 28.6 | 2.4 | 2.2935 | 1.59688 | #DIV/0! | 0.106459 | #DIV/0! |

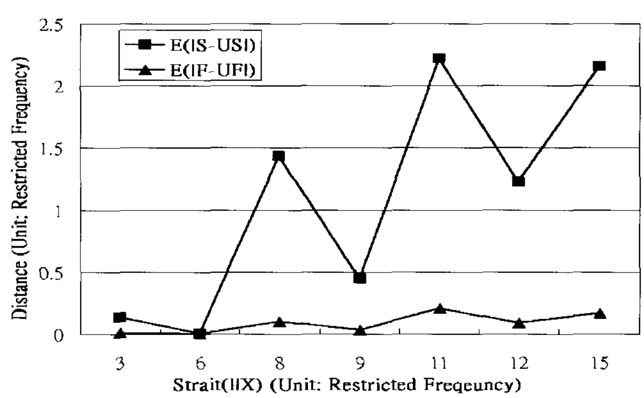


Fig. 13. The average distances.

In all the 500 experiments, the whole network contains 200 cells. For each $1 \leq i, j \leq 200$, the minimal frequency separation $C_{i,j}$ and communication demand $D_i$ in each experiment are randomly set by the following rules.

- $C_{i,i} = 5$ for all $i$
- $C_{i,j} = C_{j,i} \in \{0, 2\}$ and the probability that $C_{i,j} = 2$ is 0.03
- $D_i =$, where $a$ and $b$ are randomly selected from the interval [0, 6]

In each experiment, we randomly select 5 cells for $X$ and 5 cells for $Y$ satisfying . Then we calculate $Strait(l|X)$, $UStrait(l|X)$, $Free(l|X)$, $UFree(l|X)$ for each $l \in Y$. Table 3 is a summary of the 2500 results from the 500 experiments where S $=$ $Strait(l|X)$, US $=$ $Strait(l|X)$, F $=$ $Free(l|X)$ and UF $=$ $UFree(l|X)$. E(US) is the mean of $Strait(l|X)$ and Var(|S − US|) is the standard deviation of $|Strait(l|X) - UStrait(l|X)|$.

Some data are not reliable. For example, the group $Strait(l|X) = 2$ contains 8 samples whose $Ustrait(l|X)$ is almost the same with the group $Strait(l|X) = 3$. We believe that the group $Strait(l|X) = 2$ is a deviation from another group. Due to this reason, we consider only the groups which contain more than 50 samples and list their E(|S − US|) and

E(|F − UF|) in Fig. 13. In Fig. 13, we conclude that the two measures $UStrait(l|X)$ and $UFree(l|X)$ are close enough to $Strait(l|X)$ and $Free(l|X)$ in most cases.

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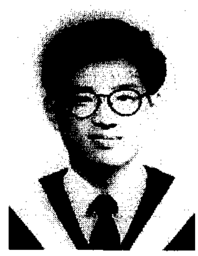

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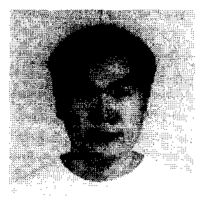

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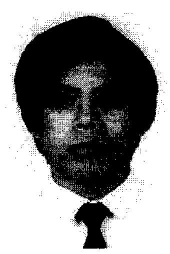

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