M진 비직교 신호를 위한 최적의 위상비동기 검출기

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The Optimum Noncoherent Detector for M-ary Nonorthogonal Signals

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ABSTRACT

This paper proposes an optimum noncoherent detector for M-ary nonorthogonal, unequal energies, unequal priori probabilities nonlinear modulation signals. Theoretical derivations are given in detail. Under above conditions, there is not any previous papers to explore the corresponding optimum noncoherent detector. The detector proposed in this paper can be regarded as a generic optimum detector which can be applied to nonorthogonal nonlinear M-ary communication systems.

Index Terms: M-ary modulation, nonlinear modulation, noncoherent detection, mobile communications, nonorthogonal modulation.

Key words: Detector

Ⅰ. Introduction

For M-ary nonorthogonal modulation scheme, the information contained in one symbol is sent by one of M nonorthogonal, unequal energy waveforms . It is meaningful to analyze this kind of systems, because for the real communication systems, especially mobile communication systems, it is impossible to design a signal set whose elements are orthogonal each other in the receiving end. Mobile communication channel exists multipath fading, phase instability, transmission time delay, and Doppler frequency shift, so noncoherent detection is preferred for this kind of application. Based on above considerations, it is significant to analyze and design nonorthogonal nonlinear modulation schemes.

The optimum coherent detection for M-ary orthogonal nonlinear modulation was introduced in [1], and the optimum coherent detectors for equal priori probabilities, equal energies and unequal energies modulations were proposed in [1]. The closed solution for the bit error probability of the optimum coherent detection under the binary, equal priori probabilities, unequal energies conditions was obtained in [1], but for M-ary situation, the closed solution for the bit error probability is still a problem. The optimum noncoherent detector for M-ary orthogonal nonlinear modulation was also presented in [1]. The optimum coherent detection for binary nonlinear modulation was discussed in [2]. Up to now, it is not possible to find the results about the optimum noncoherent detection for the M-ary nonorthogonal nonlinear modulation in previous research papers.

This paper discusses the optimum noncoherent detector for M-ary nonorthogonal nonlinear modulation under more general conditions. The waveforms of the transmitted signals are

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not orthogonal each other, and the signal energies and the symbol priori probabilities are not equal. The channel is white Gaussian. The optimum noncoherent detector under above constrains is presented in this paper. Compared with previous research results, our proposed detector is more generic, it can be applied to nonorthogonal, nonlinear M-ary communications systems.

Ⅱ. System modeling

The modulation scheme is M-ary, the M baseband waveforms are denoted as $s_m(t)$ $(m=1,2,...,M)$. The waveform set $\{ s_m(t)$ $(m=1,2,...,M) \}$ is nonorthogonal. The priori probabilities for $s_m(t)$ $(m=1,2,...,M)$ are not equal, and they are denoted as P_m respectively. The transmitted symbol is denoted as b_m. For one symbol time period, if the modulator transmits a specific $s_m(t)$, then $b_m=1$,otherwise $b_m=0$.The received signals at the receiving side have unequal energies,and are denoted as $E_m$respectively. Using noncoherent detection, then the complex envolope of the received equivalent baseband signal is $A_m=\sqrt{E_m}e^{j\phi_m}$,where ϕ_m is uniformly distributed on $[0,2\pi]$. The channel noise is white Gaussian, its mean is 0, variance is σ^2.

The equivalent complex baseband output of receiver is

$$r(t)=\sum_{m=1}^{M} b_m A_m s_m(t)+n(t) \text{ ,t} \in [0,T) \tag{1}$$

Denote

$S^T(t)=[\ s_1(t)\ s_2(t)\cdots\ s_M(t)]$,

$b^T=[\ b_1\ b_2\cdots\ b_M]$,

$A=diag(\ A_1\ A_2\cdots\ A_M)$,

The correlation matrix of the signal vector $S(t)$ is

$$R=\int_0^T S^*(t)S^T(t)dt$$
$$=\int_0^T \begin{bmatrix} s_1^*(t) \\ \vdots \\ s_M^*(t) \end{bmatrix} [s_1(t) \ \cdots \ s_M(t)]dt$$
$$=\begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & o_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & \rho_{MM} \end{bmatrix} \tag{2}$$

Using a bank of M matched filters, matched to each of the transmitted symbol waveforms $s_m(t)$, and the filters are driven by the received signal $r(t)$, then the output of the filter bank is a M-dimension complex vector given as

$$X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}=\int_0^T r(t)s^*(t)dt=RAb+\xi \tag{3}$$

where $\xi^T=[\ \xi_1\ \xi_2\cdots\ \xi_M]$ is a Gaussian vector.Its mean is 0, covariance matrix is $\sigma^2 R$(The superscript * represents complex conjugate operation).

Denote the Moore-Penrose generalized inverse of R as R^H, apply the transformation R^H to equation (3), then the new decision statistics becomes

$$Y=R^H X=Ab+\eta \tag{4}$$

where η is a zero-mean Gaussian vector with covariance matrix $E[\ \eta^*\ \eta^T]=\ \sigma^2\ R^H$. Y is a complex Gaussian vector, with mean vector $(0\cdots0\ \ A_i\ 0\cdots0)$,and covariance matrix $\sigma^2\ R^H$ (assume the modulator sends $S_i(t)$). Here we imply that the M signals $s_m(t)$ (m from 1 to M) are probably nonindependent, so the inverse of matrix R maybe do not exist. In the case of nonindependent sig nal set, Moore-Penrose generalized inverse of matrix R does exist. Our optimum detector in this paper does work when modulators signals are not independent.

According to [3], the minimum bit error probability criterion is equivalent to MAP criterion, so the optimum detector is

$$\hat{i} = \arg\max_i \{P_i f[Y| s_i(t)]\} \tag{5}$$

where $f[Y| s_i(t)]$ is the conditional probability density function under the hypothesis that modulator sends $s_i(t)$.

According to the general processing method for noncoherent detection, for getting the conditional probability density function $f[Y| s_i(t)]$, first for given ϕ_i, calculate the conditional probability density function $f(Y| s_i(t),\phi_i)$, then we obtain $f[Y| s_i(t)]$ according to the following formula

$$f[Y| s_i(t)] = \int_0^{2\pi} f[Y| s_i(t),\phi_i]\, f(\phi_i)d\phi_i$$
$$= \frac{1}{2\pi}\int_0^{2\pi} f(Y| s_i(t),\phi_i)d\phi_i \tag{6}$$

Denote the mean vector and covariance matrix of Y as m_Y, C_Y respectively, according to [3] we have

$$f(Y| s_i(t),\phi_i)$$
$$= \frac{1}{\pi^M |C_Y|}\exp[-(Y-m_Y)^* C_Y^H (Y-m_Y)] \tag{7}$$

Substitute concerned parameters into equation (7),we have (denote $B=\frac{1}{\pi^M| C_Y|}$,it is a constant.)

$$f(Y| s_i(t),\phi_i)$$
$$= B\exp\left[-(Y-m_Y)^{*T}(\sigma^2 R^H)^H (Y-m_y)\right]$$

$$= B\exp\left[\frac{-(Y^{*T}RY - Y^TRm_Y - m_Y^{*T}RY + m_Y^{*T}Rm_Y)}{\sigma^2}\right] \tag{8}$$

In above formula,

$$m_y^* Rm_Y = [0\cdots 0\ A_i^*\ 0\cdots 0]$$
$$\begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & \rho_{MM} \end{bmatrix} [0\cdots 0\ A_i\ 0\cdots 0]$$
$$= A_i^* \rho_{ii} A_i = | A_i |^2 \rho_{ii} = E_i \rho_{ii} \tag{9}$$

$$Y^{*T}Rm_Y = [y_1^*\ y_2^* \cdots y_M^*]$$
$$\begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & \rho_{MM} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ A_i \\ \vdots \\ 0 \end{bmatrix}$$
$$= A_i \sum_{m=1}^{M} y_m^* \rho_{mi} \tag{10}$$

$$m_Y^{*T} RY = [0\cdots 0\ A_i^*\ 0\cdots 0]$$
$$\begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & \rho_{MM} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}$$
$$= A_i^* \sum_{m=1}^{M} \rho_{im} y_m \tag{11}$$

Substitute above variables into formula (8), we obtain

$$f_Y(s_i(t),\phi_i)$$
$$= B\exp\left[\frac{-(Y^{*T}RY - A_i\sum_{m=1}^{M} Y_m^*\rho_{mi} - A_i^*\sum_{m=1}^{M}\rho_{im}y_m + E_i\rho_{ii})}{\sigma^2}\right]$$

$$= B\exp\left[\frac{-Y^{*T}RY + 2\mathrm{Re}(A_i^*\sum_{m=1}^{M}\rho_{im}y_m)^* - E_i\rho_{ii}}{\sigma^2}\right]$$

$$= \mathrm{B}\exp\left[-\frac{1}{\sigma^2}(Y^{*T}RY + E_i\rho_{ii}) + 2\frac{\sqrt{E_i}}{\sigma^2}\mathrm{Re}\left[e^{j\phi_m}\sum_{m=1}^{M}(\rho_{im}y_m)^*\right]\right] \tag{12}$$

Introduce equation (12) into MAP criterion, we get the decision rule as

$$\hat{i}=\arg\max P_i f_Y(Y|s_i(t),\phi_i)$$
$$=\arg\max\left\{P_i B\frac{1}{2\pi}\int_0^{2\pi}\exp\left(-\frac{1}{\sigma^2}\left(Y^T PY+E_i p_{ii}\right)\right.\right.$$
$$\left.\left.+2\frac{\sqrt{E_i}}{\sigma^2}\mathrm{Re}\left[\exp(j\phi_i)\sum_{m=1}^{M}(p_{im}y_m)^*\right]\right)d\phi_i\right\}$$
$$=\arg\max\left\{P_i B\frac{1}{2\pi}\int_0^{2\pi}\exp\left[-\frac{1}{\sigma^2}(Y^T PY)\right]\right.$$
$$\left.\exp\left(-\frac{E_i p_{ii}}{\sigma^2}\right).\exp\left\{2\frac{\sqrt{E_i}}{\sigma^2}\mathrm{Re}\left[\exp(j\phi_i)\sum_{m=1}^{M}(p_{im}y_m)^*\right]\right\}d\phi_i\right\}$$
(13)

In above equation, $\int\exp\left[-\frac{1}{\sigma^2}(Y^{*T}PY)\right]$ is not related to ϕ_i, and it is a positive number for all i, so we can get it out of $\int_0^{2\pi}$,and cancel it from above formula, this processing does not influe nce the decision of argmax. So does constant B. Also $\exp\left(-\frac{E_i P_{ii}}{\sigma^2}\right)$ in above formula i s not related to ϕ_i ,so we can take it to the outside of $\int_0^{2\pi}$.

In equation (13), $\sum_{m=1}^{M}(p_{im}y_m)^*=\left|\sum_{m=1}^{M}(p_{im}y_m)^*\right|e^{j\beta}$, where β is the phase angel of $\sum_{m=1}^{M}(p_{im}y_m)^*$, then

$$\exp\left\{2\frac{\sqrt{E_i}}{\sigma^2}\mathrm{Re}\left[\exp(j\phi_i)\sum_{m=1}^{M}(p_{im}y_m)^*\right]\right\}$$
$$=\exp\left\{2\frac{\sqrt{E_i}}{\sigma^2}\mathrm{Re}\left[\exp(j\phi_i)\left|\sum_{m=1}^{M}(p_{im}y_m)^*\right|\exp(j\beta)\right]\right\}$$
$$=\exp\left\{2\frac{\sqrt{E_i}}{\sigma^2}\left|\sum_{m=1}^{M}(p_{im}y_m)\right|\cos(\phi_i+\beta)\right\}$$
(14)

Substitute concerned results into (6), we ge t the optimum decision rule as

$$\hat{i}=\arg\max\{P_i\exp\left(-\frac{E_i p_{ii}}{\sigma^2}\right)$$
$$\left\{\int_0^{2\pi}\left[\exp\left(2\frac{\sqrt{E_i}}{\sigma^2}\left|\sum_{m=1}^{M}(p_{im}y_m)\right|\cos(\phi_i+\beta)\right)\right]\right\}\}$$
$$=\arg\max\{P_i\exp\left(-\frac{E_i p_{ii}}{\sigma^2}\right)I_0\left(\frac{2\sqrt{E_i}}{\sigma^2}\left|\sum_{m=1}^{M}(p_{im}y_m)\right|\right)\}$$
$$(\because I_0(x)=\int_0^{2\pi}\exp(x\cos\gamma)d\gamma\ )$$
$$=\arg\max\{-\frac{E_i p_{ii}}{\sigma^2}+\ln[I_0(2\frac{\sqrt{E_i}}{\sigma^2}\left|\sum_{m=1}^{M}(p_{im}y_m)\right|)]\}P_i$$
(15)

Equation (15) is the optimum noncoherent d etector for M-ary nonorthogonal nonlinear mo dulation. This is a general noncoherent detect or. If we let M signals be orthogonal and eq ual energies, then the detector proposed here is the same with the optimum detector in [1] for orthogonal nonlinear modulation. If we set that M signals are orthogonal, but with uneq ual energies, then the detector here is the sa me with the detector proposed in [2].

Ⅲ. Conclusion

This paper focuses on the noncoherent dete ction for nonorthogonal nonlinearly modulated M-ary communications systems. An optimum noncoherent detector has been derived and pr oposed in this paper for this kind of systems. Previous noncoherent optimum detectors are o nly for orthogonal M-ary systems, our nonco herent optimum detector is for nonorthogonal M-ary system. Compared with previous opti mum detectors, our proposed noncoherent det ector is more general, and can be used for all kinds of M-ary communications systems, wh ether or not the signals employed are orthogo nal, equal energies, and equal priori probabiliti es. Previous noncoherent detectors for M-ary orthogonal systems can be regarded as the s pecial case of the detector proposed in this p aper.

In this paper, we have proposed a novel no ncoherent optimum detector for nonorthogona l, nonlinear M-ary communications systems. I t can be applied to all kinds of M-ary modu lated communications systems. It is a generic noncoherent optimum detector. Previous corre sponding detectors assumed certain assumptio ns, our detector generalized previous research, it is applicable to all kinds of M-ary modulat ed communications systems.

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