# A General Method for Error Probability Computation of UWB Systems for Indoor Multiuser Communications

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Abstract: A general method for the evaluation of the symbol error probability (SER) of ultra wideband (UWB) systems with various kind of modulation schemes (N-PAM, M-PPM, Bi-Orthogonal), in presence of multipath channel, multiuser and strong narrowband interference, is presented. This method is shown to be able to include all the principal multiaccess techniques proposed so far for UWB, time hopping (TH), direct sequence (DS) and optical orthogonal codes (OOC). A comparison between the performance of these multiple access and modulation techniques is given, for both ideal Rake receiver and minimum mean square error (MMSE) equalizer.

It is shown that for all the analyzed multiple access schemes, a Rake receiver exhibits a high error floor in presence of narrow-band interference (NBI) and that the value of the error floor is influenced by the spectral characteristics of the spreading code. As expected, an MMSE receiver offers better performance, representing a promising candidate for UWB systems.

When the multiuser interference is dominant, all multiple access techniques exhibit similar performance under high-load conditions. If the number of users is significantly lower than the spreading factor, then DS outperforms both TH and OOC. Finally 2PPM is shown to offer better performance than the other modulation schemes in presence of multiuser interference; increasing the spreading factor is proposed as a more effective strategy for SER reduction than the use of time diversity.

Index Terms: Ultra wideband, multiple access, multipath channel, performance evaluation.

## I. INTRODUCTION

The successful deployment of ultra wideband (UWB) systems for high speed indoor communications strongly depends on the development of efficient multiaccess and modulation techniques and of low complexity receivers, robust against narrowband and multiuser interference. As far as the first issue is concerned, several proposals for UWB air interface are available in the literature, the principal ones being time hopping (TH) [1], direct sequence (DS) [2] and optical orthogonal codes (OOC) [3]. These three multiple access strategies were compared in [3] on AWGN channel and under equal power users assumption. The comparison highlighted the robustness of DS compared to TH against multiuser interference and pointed out the tradeoff between time diversity and spreading gain.

As far as the modulation choice is concerned, 2PPM is the

Manuscript received July 3, 2003.

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This work has been partially sponsored by MIUR (Italian Ministry of Education and Research) under the projects CERCOM and PRIMO.

most common scheme employed in UWB communication systems (see for example [4]); however other modulation techniques are presented as well in the literature. In [5] various optimization criteria for M-PPM delays are presented, and their performance is evaluated in multipath channels. In [6] and [2] on-off keying (OOK) and 2PAM are proposed, respectively. Finally, in [7], the use of Bi-Orthogonal modulation (Bi-M-PPM) is suggested.

In this paper we present a general method for the evaluation of the SER for coherent, single band UWB systems, in the presence of dense multipath channel [8], multiuser and narrowband interference. The recently proposed multiband technique (see, for example, [9]) will not be addressed. As in [2], the SER is evaluated for both ideal Rake receivers [10] and MMSE equalizers. Furthermore, we tested the validity of the Gaussian assumption, employed in [2] to model the multiuser-intersymbol (MUI-ISI) interference, every time its use was questionable. In addition, the equal-power user assumption is removed through the introduction of a path loss attenuation, related to the transmitterreceiver distance. The method is shown to be able to include all multiple access techniques proposed so far for UWB. Furthermore, SER formulas are given for three modulation schemes: N-PAM, M-PPM and Bi-M-PPM. This allows a comparison under various working hypotheses.

At the receiver, we assume to have perfect channel state information. The problem of channel estimation in UWB systems is addressed in [11]. In [6] some suboptimal reception techniques that simplify or overcome the problem of channel estimation are presented.

The paper is organized as follows. In Section II and III we briefly describe the general system model and its time-discrete version. In Section IV the problem of SER computation is presented and analytical formulas are given for N-PAM (exact), M-PPM (upper bound) and Bi-M-PPM (upper bound), using the Gaussian assumption and adopting a semi-analytical method. In Section V the Rake and MMSE receiver structures are analyzed and finally, after having introduced a reference scenario in Section VI, a comparison between the multiaccess and the modulation techniques is presented in Section VII.

## II. SYSTEM MODEL

In this section, the principal characteristics of the system will be described. The time axis is divided in symbol intervals of  $T_s$  seconds, each one further subdivided into smaller intervals, called *chips* of duration  $T_c$  seconds. The signature sequence assigned to each user for multiple access purpose is a periodic signal, with a period equal to T. For simplicity, we assume that  $T=N_bT_s$  and  $T_s=N_cT_c$ , with  $N_b,N_c\in\mathbb{N}$ . The period T is longer than the symbol time  $T_s$ , whenever the multiaccess code

Table 1. DS and TH signature sequences for user 1;  $j \in [0, N_b - 1]$ .

Code type	$N_b$	$N_c$	$N_p$	$oldsymbol{c_j^{(1)}}$
DS	1	4	4	$[1, 1, -1, 1]^T$
TH	3	4	1	$[0,0,0,1]^T$
				$[0, 1, 0, 0]^T$
				$[0,0,1,0]^T$

Table 2. DS and TH signature sequences as in the previous example. A repetition code of length 2 is this time considered.

Code type	$N_b$	$N_c$	$N_p$	$\mathbf{c_{j}^{(1)}}$
DS	1	8	8	$[1, 1, -1, 1, 1, 1, -1, 1]^T$
TH	3	8	2	$[0,0,0,1,0,1,0,0]^T$
				$[0,0,1,0,0,0,0,1]^T$
				$[0, 1, 0, 0, 0, 0, 1, 0]^T$

spans more than a single symbol.

The signal transmitted by user k can then be described as follows:

$$s^{(k)}(t) = \sum_{i} p_i^{(k)}(t - iT), \tag{1}$$

where

$$p_i^{(k)}(t) = \sum_{j=0}^{N_b - 1} \sum_{h=0}^{N_c - 1} c_{j,h}^{(k)} g\left(t - hT_c - jT_s; a_{iN_b + j}^{(k)}\right). \tag{2}$$

 $a_l^{(k)}$  is the l-th symbol, transmitted by user k, chosen independently and with uniform distribution from the alphabet  $\mathcal{A}_a$ . With the notation  $c_{j,h}^{(k)}$  we indicate the value assumed by the spreading code assigned to the user k in the chip h of the  $j^{th}$  symbol interval, in each period T. We will assume  $c_{j,h}^{(k)} \in \{0,\pm 1\}$ ; in this way all the principal multiaccess techniques proposed so far for UWB (TH, DS, OOC) are included. Note that the multiple access code  $\{c_{j,l}^{(k)}\}$  can be also constructed as the combination of a signature sequence and a repetition code, as often proposed in UWB literature (see for example [4]). The code associated with the symbol j transmitted by user k can be written in vector form as

$$\mathbf{c}_{j}^{(k)} = \left[ c_{j,0}^{(k)}, c_{j,1}^{(k)}, \cdots, c_{j,N_{c}-1}^{(k)} \right]^{T}.$$
 (3)

The number of pulses transmitted in each frame is given by

$$N_p = (\mathbf{c}_i^{(k)})^T (\mathbf{c}_i^{(k)}). \tag{4}$$

In our analysis we assume that the number of pulses per frame does not vary from frame to frame and is equal for all users. In Table 1 and 2, two examples of signature sequences are given, with and without repetition code.

The signal g(t,a) in (2) represents the random process at the output of the modulator and therefore its expression depends on the modulation format. In order to be as general as possible, let us assume that the transmitted signal is constituted by a sequence of basic waveforms that can be both pulse amplitude and pulse position modulated. In particular, assuming that the possible PPM positions are M, each of them univocally identified by its PPM delay  $\tau_l$ ,  $l = 0, \dots, M-1$ ,

Table 3. Modulation formats: Examples.

Modulation	M	Alphabet for $d_{i,j}^{(k)}$	Example
2PAM	1	$\mathcal{A}_a = \mathcal{A}_2 - \{0\}$	$\mathbf{d}_i^{(k)} = [1]$
2PPM	2	$\mathcal{A}_a = \mathcal{A}_2 \!-\! \{-1\}$	$\mathbf{d}_i^{(k)} = [1, 0]^T$
2PPM+2PAM	2	$A_a = A_2$	$\mathbf{d}_i^{(k)} = [0, -1]^T$

then it is possible to associate each symbol  $a_i^{(k)}$  with a vector  $\mathbf{d}_i^{(k)} = [d_{i,0}^{(k)}, \cdots, d_{i,M-1}^{(k)}]^T$ , containing the amplitude of the pulses transmitted in each PPM position. If an N-level amplitude modulation is employed, then, in the most general case,  $d_{i,j}^{(k)}$  belongs to a subset  $\mathcal{A}_a$  of  $\mathcal{A}_N = \{2p-1-N\}_{p=1}^N \cup \{0\}$ , as shown in Table 3, for some modulation formats.

Terming with x(t) the transmitted waveform with time duration  $T_x$ , chosen such as  $T_x + \max_l \tau_l \leq T_c$ , and with unitary energy, then

$$g\left(t; a_{i}^{(k)}\right) = \sqrt{E_{x}} \sum_{l=0}^{M-1} d_{i,l}^{(k)} x\left(t - \tau_{l}\right), \tag{5}$$

where  $E_x$  is the energy per pulse; defining  $x_l(t) = x(t - \tau_l)$ , for  $l = 0, \dots, M - 1$ , then

$$g\left(t; a_{i}^{(k)}\right) = \sqrt{E_{x}} \sum_{l=0}^{M-1} d_{i,l}^{(k)} x_{l}\left(t\right). \tag{6}$$

It is worth noting that, through (6), a general non-linear modulation scheme (the non-linearity is determined by the use of M-PPM modulation) is simplified to the sum of M linear amplitude modulated systems, each one employing a different waveform  $x_l(t)$ . This approach is similar to the one described in [1].

The parameter  $N_p$ , together with M and N, determines the relation between  $E_x$  and the energy per bit  $E_b$ . An explicit formula will be given in the next sections, for some modulation schemes. Assuming that  $N_u$  users are active, then the received signal can be written as

$$r(t) = \sum_{k=1}^{N_u} \sqrt{A_k} s^{(k)}(t) * h^{(k)}(t) + \tilde{n}(t), \tag{7}$$

$$\tilde{n}(t) = \sqrt{A_b} n_b(t) + n_G(t), \tag{8}$$

where  $n_G(t)$  is a white Gaussian noise process with two-sided power spectral density  $N_0/2$  and  $n_b(t)$  is the narrowband interference, modeled as an ergodic, zero mean, Gaussian random process. Furthermore,  $A_k$  and  $A_b$  represent the attenuations due to path loss, which are a function of the transmitter receiver (TX-RX) distance. Finally,  $h^{(k)}(t)$  is the time-invariant, asynchronous multipath channel impulse response for user k. Each asynchronous channel impulse response  $h^{(k)}(t)$  is assumed to have a maximum delay of  $t_{max}$  seconds. In the rest of the paper we will term with  $q_l^{(k)}(t)$  the convolution between the transmitted pulse  $x_l(t)$  and  $h^{(k)}(t)$ .

## III. DISCRETE TIME EQUIVALENT MODEL

In order to analytically evaluate the SER, we will construct a discrete time equivalent model, obtained by sampling r(t) ev-

ery  $T_r$  seconds, with  $T_r \ll T_c$ . Furthermore, the receiver is supposed to be constituted by a bank of M digital filters, active on an observation window  $T_w \geq T_s$ . The following parameters need also to be defined: 1

- $N_w = T_w/T_r$ , the number of samples in the observation window  $T_w$ , over which the receiver operates.
- $N_r = T_c/T_r$ , the number of samples per chip.
- $N_L = \lceil t_{max}/T_c \rceil + 1$ , the minimum number of chip intervals in which  $q_l^{(k)}(t)$  is contained,  $\forall k$  and  $\forall l$ .
- $L_L = \lceil N_L/N_c \rceil$ , the minimum number of symbol intervals in which  $q_i^{(k)}(t)$  is contained,  $\forall k$  and  $\forall l$ .  $N_R = \lceil T_w/T_c \rceil$ , the minimum number of chip intervals
- in which the observation window is contained.
- $L_R = \lceil N_R/N_c \rceil 1$ , the minimum number of symbol intervals in which the portion of the observation window that exceeds a symbol interval is contained.

For error probability computation, the system under analysis is identical to the superposition of  $N_b$  subsystems, transmitting in a round-robin fashion on successive slots of duration  $T_s$ . However, the presence of multipath causes the received signals to lose their mutual orthogonality, with the consequence of intersymbol interference in the overall system. The error probability is obtained by averaging on the performance of each subsystem. This task is performed if the error probability is computed averaging over  $N_b$  successive symbols.

Without loss of generality, let us assume that user 1 is the reference user and that the symbol  $a_n^{(1)}$  is transmitted, with  $n \in$  $[0, \cdots, N_b - 1]$ . The vector containing the discrete samples of the received signal will be

$$\mathbf{r}_n = [r(nT_s), r(T_r + nT_s), \cdots, r((N_w - 1)T_r + nT_s)]^T$$
. (9)

With the same notation, we can also introduce a overall Gaussian noise vector as follows:

$$\tilde{\mathbf{n}}_n = \left[\tilde{n}(nT_s), \tilde{n}(T_r + nT_s), \cdots, \tilde{n}((N_w - 1)T_r + nT_s)\right]^T.$$
(10)

Furthermore, it is also necessary to define the spreading block diagonal matrices  $\mathbf{S}_n^{(k)}$ ,

$$\mathbf{S}_{n}^{(k)} = \begin{bmatrix} \tilde{\mathbf{c}}_{-L_{L}+n}^{(k)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{c}}_{-L_{L}+n+1}^{(k)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \tilde{\mathbf{c}}_{L_{R}+n}^{(k)} \end{bmatrix}$$
(11)

where

$$\tilde{\mathbf{c}}_i^{(k)} = \mathbf{c}_{(i \bmod N_b)}^{(k)} \tag{12}$$

and 0 is a  $N_c$  sized zero vector. Finally, the channel matrices  $\mathbf{Q}_{l}^{(k)} \in \mathbb{R}^{N_{w},(L_{L}+L_{R}+1)N_{c}}$  is introduced as in (13). Using the previous definitions and rewriting  $\mathbf{S}_n^{(k)}$  as

$$\mathbf{S}_n^{(k)} = \left[\mathbf{s}_{-L_L+n}^{(k)}, \cdots, \mathbf{s}_n^{(k)}, \cdots, \mathbf{s}_{L_R+n}^{(k)}\right]$$
(14)

with  $\mathbf{s}_{i}^{(k)}$  being the  $(i+L_{l})^{th}$  column of  $\mathbf{S}_{n}^{(k)}$ , then

$$\mathbf{r}_{n} = \sum_{k=1}^{N_{u}} \sum_{i=-L_{L}+n}^{L_{R}+n} \sum_{l=0}^{M-1} \sqrt{A_{k} E_{x}} \mathbf{Q}_{l}^{(k)} \mathbf{s}_{i}^{(k)} d_{i,l}^{(k)} + \tilde{\mathbf{n}}_{n}.$$
 (15)

The receiver consists of a set of M digital filters  $\mathbf{w}_n^m$ , m = $0, \cdots, M-1$ , fully characterized by their  $N_w$  coefficients

$$\mathbf{w}_{n}^{m} = [w_{n,0}^{m}, w_{n,1}^{m}, \cdots, w_{n,N_{w}-1}^{m}]^{T}.$$
 (16)

The outputs of the M filters give the decision statistics vector

$$\alpha_n^{(1)} = [\alpha_{n,0}^{(1)}, \alpha_{n,1}^{(1)}, \cdots, \alpha_{n,M-1}^{(1)}]^T, \tag{17}$$

with

$$\alpha_{n,m}^{(1)} = (\mathbf{w}_n^m)^T \mathbf{r}_n, \ m = 0, \cdots, M - 1.$$
 (18)

Each component of the decision statistics vector  $\alpha_n^{(1)}$  can be rewritten to highlight the contributions of the useful signal, the multiuser and intersymbol interference terms, and the Gaussian noise to the total output. The final result is

$$\alpha_{n,m}^{(1)} = \sqrt{E_x} P_{n,m} + n_{MI,n,m} + n_{n,m}$$
 (19)

where

$$P_{n,m} = \sqrt{A_1} \left( \mathbf{w}_n^m \right)^T \mathbf{Q}_m^{(1)} \mathbf{s}_n^{(1)} d_{n,m}^{(1)}$$
 (20)

is the useful signal term,

$$n_{MI,n,m} = (\mathbf{w}_n^m)^T \sum_{k=1}^{N_u} \sum_{\substack{i=-L_L+n\\(k,i,l)\neq(1,n,m)}}^{L_R+n} \sum_{l=0}^{M-1} \sqrt{A_k E_x} \mathbf{Q}_l^{(k)} \mathbf{s}_i^{(k)} d_{i,l}^{(k)}$$
(21)

is the multiuser and intersymbol interference and

$$n_{n,m} = (\mathbf{w}_n^m)^T \tilde{\mathbf{n}}_n \tag{22}$$

is the narrowband interference and the thermal noise term at the output of the m-th digital filter.

$$\mathbf{Q}_{l}^{(k)} = \begin{bmatrix} q_{l}^{(k)}(L_{L}N_{c}T_{c}) & q_{l}^{(k)}((L_{L}N_{c}-1)T_{c}) & \dots & q_{l}^{(k)}(0) & 0 & 0 & \dots & 0 \\ q_{l}^{(k)}(L_{L}N_{c}T_{c}+T_{r}) & q_{l}^{(k)}((L_{L}N_{c}-1)T_{c}+T_{r}) & \dots & q_{l}^{(k)}(T_{r}) & 0 & 0 & \dots & 0 \\ \vdots & \vdots \\ q_{l}^{(k)}(L_{L}N_{c}T_{c}+(N_{r}-1)T_{r}) & q_{l}^{(k)}((L_{L}N_{c}-1)T_{c}+(N_{r}-1)T_{r}) & \dots & q_{l}^{(k)}((N_{r}-1)T_{r}) & 0 & 0 & \dots & 0 \\ 0 & q_{l}^{(k)}(L_{L}N_{c}T_{c}) & \dots & q_{l}^{(k)}(T_{c}) & q_{l}^{(k)}(0) & 0 & \dots & 0 \\ 0 & q_{l}^{(k)}(L_{L}N_{c}T_{c}+T_{r}) & \dots & q_{l}^{(k)}(T_{c}+T_{r}) & q_{l}^{(k)}(T_{r}) & 0 & \dots & 0 \\ \vdots & \vdots \end{bmatrix}$$

$$(13)$$

<sup>&</sup>lt;sup>1</sup> Note that the notation  $\lceil x \rceil$  indicates the nearest integer to x towards  $+\infty$ .

#### IV. COMPUTATION OF THE ERROR PROBABILITY

#### A. Introduction

In this section the error probability is computed for three different modulation schemes, N-PAM, M-PPM and Bi-M-PPM. For all but the first case, for which an exact formula for the SER can be easily computed, an upper bound based on the union bound [12] is given.

Assuming that the channel is deterministic, then the SER only depends on the cumulative distribution of the multiuser, intersymbol, thermal noise and narrowband interference. According to the assumption introduced in Section II, the last two distributions are Gaussian. Assuming that the symbol  $a_n^{(1)}$  is transmitted, let us define collectively with  $\mathcal{B}_n$  the set of the transmitted symbols that cause intersymbol and multiuser interference, that is

$$\mathcal{B}_{n} = \left\{ a_{i}^{(k)} | i = -L_{L} + n, \cdots, L_{R} + n; \\ k = 1, \cdots, N_{u}; (k, i) \neq (1, n) \right\}.$$
 (23)

If the union bound is employed, the error probability (both SER and BER) conditioned on  $\mathcal{B}_n$  and  $a_n^{(1)}$  can be expressed as the sum of Gaussian error functions whose arguments depend on the useful signal, the MUI-ISI and the variance of the Gaussian noise (both thermal and narrowband). Let us include all these contributions in a general set called  $\mathcal{D}_n$  of cardinality  $N_D$ . In formulas, having introduced a real constant  $c_0$ , related to the modulation and multiple access format, and a real function  $f_m: \mathbb{R}^{N_D} \to \mathbb{R}$ , then

$$P_n\left(e|a_n^{(1)}, \mathcal{B}_n\right) \le c_0 \sum_m \operatorname{erfc}\left(f_m(\mathcal{D}_n)\right).$$
 (24)

If the channel and the signature sequences are deterministic, then the distribution of the multiuser and intersymbol interference depends only on the discrete distribution of the transmitted symbols,

$$\begin{split} P_{n}\left(e|a_{n}^{(1)}\right) &= E_{\mathcal{B}_{n}}\left\{P_{n}\left(e|a_{n}^{(1)},\mathcal{B}_{n}\right)\right\} = \\ &= \sum_{a_{-L_{L}+n}^{(1)} \in \mathcal{A}_{a}} \dots \sum_{a_{n-1}^{(1)} \in \mathcal{A}_{a}} \sum_{a_{n+1}^{(1)} \in \mathcal{A}_{a}} \dots \\ &\sum_{a_{L_{R}+n}^{(1)} \in \mathcal{A}_{a}} \dots \sum_{a_{-L_{L}+n}^{(2)} \in \mathcal{A}_{a}} \dots \sum_{a_{L_{R}+n}^{(2)} \in \mathcal{A}_{a}} \dots \\ &\sum_{a_{-L_{L}+n}^{(N_{n})} \in \mathcal{A}_{a}} \dots \sum_{a_{L_{R}+n}^{(N_{n})} \in \mathcal{A}_{a}} P_{n}\left(e|a_{n}^{(1)},\mathcal{B}_{n}\right). \end{split}$$

Equation (25) points out that the SER is given by a sum of Gaussian error functions, that grows exponentially with the number of users  $N_u$  and with the number of symbol intervals  $L_L + L_R + 1$ . The evaluation of (25) is therefore not feasible even for systems with moderate size [13].

In this paper we adopt two well-known techniques to overcome the problem of the exact computation of (25). The first is based on the use of semi-analytical method. This method consists on approximating the calculus of the expected value in (25), with an average over  $N_{sim}$  independent realizations of the random variables in the set  $\mathcal{B}_n$ . In formulas,

$$P_{n}(e|a_{n}^{(1)}) = E_{\mathcal{B}_{n}} \left\{ P_{n} \left( e|a_{n}^{(1)}, \mathcal{B}_{n} \right) \right\}$$

$$\simeq \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} P_{n} \left( e|a_{n}^{(1)}, \mathcal{B}_{n,i} \right). \tag{26}$$

A different approach consists of modeling the MUI-ISI term as a normal random variable, employing the so called "Gaussian assumption." This approximation drastically simplifies (25) and it is asymptotically valid for  $N_u \to \infty$ , due to the central limit theorem. However, if the system size is not large enough, it can lead to incorrect estimation of the error probability, as shown for example in [14] for a matched filter receiver.

## B. Evaluation of the Second Order Statistical Parameters of the Noise Terms

In this section, the second order statistical properties of the noise terms will be evaluated. For error probability computation, in fact, it is necessary to statistically characterize the terms  $\alpha_{n,i}^{(1)}$ , for  $n=0,\cdots,N_b-1$  and  $i=0,\cdots,M-1$ . If the union bound is employed, then also the second order statistical description of  $\alpha_{n,i}^{(1)}-\alpha_{n,j}^{(1)}, i\neq j$  is required.

Let us assume that the narrowband interference  $n_b(t)$  has a power spectral density  $S_{n_b}(f)$  with the following characteristics

$$S_{n_b}(f) = \begin{cases} \frac{N_b}{2}, & f_c - \frac{B_b}{2} \le |f| \le f_c + \frac{B_b}{2} \\ 0, & \text{otherwise} \end{cases}$$
 (27)

where  $f_c$  and  $B_b$  are, respectively, its central frequency and bandwidth. The autocorrelation function of  $n_b(t)$  is therefore given by

$$R_b(\tau) = N_b B_b \cos(2\pi f_c \tau) \operatorname{sinc}(\pi B_b \tau)$$
$$= \frac{N_b}{2} \tilde{R}_b(\tau), \tag{28}$$

$$\tilde{R}_b(\tau) = 2B_b \cos(2\pi f_c \tau) \operatorname{sinc}(\pi B_b \tau).$$
 (29)

In matrix form,

$$\tilde{\mathbf{R}}_{b,n} = \begin{bmatrix} \tilde{R}_b(nT_s) & \dots & \tilde{R}_b((N_w-1)T_r + nT_s) \\ \tilde{R}_b(T_r + nT_s) & \dots & \tilde{R}_b((N_w-2)T_r + nT_s) \\ \vdots & \vdots & \vdots \\ \tilde{R}_b(nT_s + (N_w-1)T_r) & \dots & \tilde{R}_b(nT_s) \end{bmatrix},$$
(30)

with  $\tilde{\mathbf{R}}_{b,n} \in \mathbb{R}^{N_w,N_w}$ , is the Toeplitz narrowband interference autocorrelation matrix. Using the previous definitions, the mean and variance of the overall Gaussian noise term  $n_{m,n}$  can be expressed as:

$$E\{n_{n,m}\} = 0, (31)$$

$$\sigma_{n,m}^2 = \frac{N_0}{2} |\mathbf{w}_n^m|^2 + A_b \frac{N_b}{2} (\mathbf{w}_n^m)^T \tilde{\mathbf{R}}_{b,n} \mathbf{w}_n^m.$$
 (32)

Furthermore, the variance of the difference between the two Gaussian noise contributions at the output of the filters m and m' is given by

$$-\sigma_{n,m,m'}^{2} = E\{(n_{n,m} - n_{n,m'})^{2}\} = \frac{N_{0}}{2} \left| \mathbf{w}_{n}^{m} - \mathbf{w}_{n}^{m'} \right|^{2} + A_{b} \frac{N_{b}}{2} \left( \mathbf{w}_{n}^{m} - \mathbf{w}_{n}^{m'} \right)^{T} \tilde{\mathbf{R}}_{b,n} \left( \mathbf{w}_{n}^{m} - \mathbf{w}_{n}^{m'} \right).$$
(33)

If the Gaussian approximation is employed, then the second order statistical parameters of the MUI-ISI need to be known as well. The mean value of  $n_{MI,n,m}$  can be evaluated as:

$$E\{n_{MI,n,m}\} = (\mathbf{w}_{n}^{m})^{T} \cdot \sum_{k=1}^{N_{u}} \sum_{\substack{i=-L_{L}+n\\(k,i,l)\neq(1,n,m)}}^{L_{R}+n} \sum_{l=0}^{M-1} \sqrt{A_{k}E_{x}} \mathbf{Q}_{l}^{(k)} \mathbf{s}_{i}^{(k)} E\{d_{i,l}^{(k)}\}.$$

Evidently, the mean is equal to zero if  $E\{d_{i,l}^{(k)}\}=0$ . This is the case for the N-PAM scheme defined in Section II and for combinations of N-PAM and M-PPM. However, if M-PPM is employed, the result is not valid anymore. In fact, if the symbols are assumed to be equiprobable, then

$$E\{d_{i,l}^{(k)}\} = \frac{1}{M}, \ \forall k, \forall n, \forall l. \tag{35} \label{eq:35}$$

For simplicity, we will also in this case assume that the average value is zero. This assumption is reasonable if the error probability is computed averaging on different realizations of the channel impulse responses, characterized by asynchronous delays uniformly distributed over T. In this case, in fact, if the transmitted waveform is chosen to have zero DC content,

$$E_Q\left\{ (\mathbf{w}_n^m)^T \mathbf{Q}_l^{(k)} \mathbf{s}_i^{(k)} \right\} = 0, \ \forall m, n, k,$$
 (36)

where the expectation is taken over the asynchronous channel impulse response ensemble [4]. With this assumption,

$$\sigma_{MI,n,m}^{2} = \sum_{k=1}^{N_{u}} \sum_{\substack{i=-L_{L}+n\\(k,i,l)\neq(1,n,m)}}^{L_{R}+n} \sum_{l=0}^{M-1} A_{k} E_{x} E\left\{ \left(d_{i,l}^{(k)}\right)^{2} \right\}$$

$$\cdot \left| (\mathbf{w}_{n}^{m})^{T} \mathbf{Q}_{l}^{(k)} \mathbf{s}_{i}^{(k)} \right|^{2},$$
(37)

and

$$^{\pm}\sigma_{MI,n,m,m'}^{2} = \sum_{k=1}^{N_{u}} \sum_{\substack{i=-L_{L}+n\\(k,i,l)\neq(1,n,m)}}^{L_{R}+n} \sum_{l=0}^{M-1} A_{k} E_{x} E\left\{\left(d_{i,l}^{(k)}\right)^{2}\right\}$$

$$\cdot \left|\left(\mathbf{w}_{n}^{m} \pm \mathbf{w}_{n}^{m'}\right)^{T} \mathbf{Q}_{l}^{(k)} \mathbf{s}_{i}^{(k)}\right|^{2}$$
(38)

are, respectively, the variance of the MUI-ISI at the output of the m-th filter and of the sum/difference between the m-th, and the m'-th filter outputs.

#### C. N-PAM

In this case M=1 and the sum over l in (6), (37) and (38) can be omitted because  $a_n^{(k)}$  can be fully described by the scalar quantity  $d_n^{(k)}$ . Furthermore, as only one filter is necessary at the receiver to obtain the sufficient statistics, the index m can be also dropped. Adopting standard techniques [12], the symbol error probability, conditioned on the multiuser interference, can be evaluated as

$$P_n(e|\mathcal{B}_n) = \frac{M-1}{M} \operatorname{erfc} \left\{ \frac{\sqrt{E_b} P_n c - n_{MI,n}}{\sqrt{2\sigma_n^2}} \right\}, \quad (39)$$

where

$$c = \sqrt{\frac{3\log_2 M}{(M^2 - 1)N_p}},\tag{40}$$

and

$$E_b = E_x/c^2. (41)$$

The unconditional error probability can be evaluated numerically by adopting the semi-analytical method as in (26), or analytically by using the Gaussian assumption:

$$P_n(e) = \frac{M-1}{M} \operatorname{erfc} \left\{ c \frac{\sqrt{E_b} P_n}{\sqrt{2 \left(\sigma_{MI,n}^2 + \sigma_n^2\right)}} \right\}.$$
 (42)

Finally,

$$P(e) = \frac{1}{N_b} \sum_{n=0}^{N_b - 1} P_n(e).$$
 (43)

#### D. M-PPM

In this case  $\mathbf{d}_i^{(k)}$  is an M dimensional vector with  $d_{i,l}^{(k)} \in \{0,1\}$ , constructed such that  $\forall \mathbf{d}_i^{(k)}$  only one value of  $l \in [0,M-1]$  exists, such that  $d_{i,l}^{(k)}=1$ . Let us assume, without loss of generality, that the symbol  $a_n^{(1)}$  is transmitted and that, in its associated vector,  $d_{n,p}^{(k)}=1$ . The decision rule, whose aim is to establish in which position  $q \in [0,M-1]$  of the decision vector  $\alpha_n^{(1)}$  the element "1" is situated, can be described as follows:

$$q = \underset{m \in [0, M-1]}{\arg \max} \ \alpha_{n,m}^{(1)}. \tag{44}$$

Employing the union bound technique, the conditional error probability can be evaluated as

$$P_n(e|\mathcal{B}_n) \le \frac{1}{M} \sum_{p=0}^{M-1} \sum_{\substack{j=0\\j \ne p}}^{M-1} P\left(\alpha_{n,p}^{(1)} < \alpha_{n,j}^{(1)} \middle| \mathcal{B}_n\right), \quad (45)$$

where

$$P\left(\alpha_{n,p}^{(1)} < \alpha_{n,j}^{(1)} | \mathcal{B}_n\right) =$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{\sqrt{E_b} P_{n,p} c + n_{MI,n,p} - n_{MI,n,j}}{\sqrt{2 \sigma_{n,j,p}^2}}\right), \quad (46)$$

with

$$c = \sqrt{\frac{\log_2 M}{N_p}},\tag{47}$$

and  $E_b$  like in (41).

As in the previous subsection, the equation (46) can be used together with a semi-analytical method to obtain  $P_n(e)$ . Alternatively, the Gaussian assumption can be adopted, leading to

$$P\left(\alpha_{n,p}^{(1)} < \alpha_{n,j}^{(1)}\right) = \frac{1}{2}\operatorname{erfc}\left(c\frac{\sqrt{E_b}P_{n,p}}{\sqrt{2\left(\neg\sigma_{n,j,p}^2 + \neg\sigma_{MI,n,j,p}^2\right)}}\right). \tag{48}$$

#### E. Bi-M-PPM

Also in this case,  $\mathbf{d}_i^{(k)}$  is an M dimensional vector, but this time  $d_{i,l}^{(k)} \in \{0,\pm 1\}$ . The vector is constructed such that  $\forall \mathbf{d}_i^{(k)}$  only one value of  $l \in [0,M-1]$  exists, for which  $d_{i,l}^{(k)} = \pm 1$ . Let us assume that the symbol  $a_n^{(1)}$  is transmitted and that its corresponding vector has  $d_{n,p}^{(1)} = 1$ . The decision rule operates in two steps; first the index q is chosen such that

$$q = \underset{m \in [0, M-1]}{\arg \max} |\alpha_{n,m}^{(1)}|, \tag{49}$$

and then a standard zero threshold comparison is adopted, like in the conventional 2-PAM scheme. Using the union bound technique,

$$P_n(e|\mathcal{B}_n) \le \frac{1}{M} \sum_{p=0}^{M-1} \sum_{\substack{j=0\\ i \ne p}}^{M-1} (1 - P_{p,j}(c)), \tag{50}$$

where  $P_{p,j}(c)$  is the probability of correct decision of an equivalent QPSK system. Using standard techniques [12], it can be shown that

$$P_{p,j}(c) = \frac{1}{\sqrt{\sigma_{n,p}^2}} \int_{-y_0}^{+\infty} \left[ 1 - \frac{1}{2} \operatorname{erfc} \left( \frac{\rho_{p,j}z + x_0 \frac{\sigma_{n,j}^2}{\sigma_{n,p}^2}}{\sqrt{2\sigma_{n,j}^2 (1 - \rho_{p,j}^2)}} \right) \right] \cdot \exp\left( -\frac{z^2}{2\sigma_{n,p}^2} \right) dz, \tag{51}$$

where

$$\begin{split} x_0 &= \frac{1}{\sqrt{2}} \left( \sqrt{E_b} P_{n,p} c + n_{MI,n,p} + n_{MI,n,j}^{\parallel} - n_{MI,n,j}^{\perp} \right), \\ y_0 &= \frac{1}{\sqrt{2}} \left( \sqrt{E_b} P_{n,p} c + n_{MI,n,p} + n_{MI,n,j}^{\parallel} + n_{MI,n,j}^{\perp} \right). \end{split}$$

with

$$c = \sqrt{\frac{1 + \log_2 M}{N_p}},\tag{52}$$

and  $E_b$  like in (41). The coefficient

$$\rho_{p,j} = \frac{E\left[n_{n,p}n_{n,j}\right]}{\sqrt{\sigma_{n,p}^2 \sigma_{n,j}^2}} \tag{53}$$

represents the correlation between the noise terms at the output of the two correlators and

$$\begin{split} n_{MI,n,j}^{\parallel} &= \rho_{p,j} n_{MI,n,j}, \\ n_{MI,n,j}^{\perp} &= \sqrt{(1-\rho_{p,j}^2)} n_{MI,n,j} \end{split}$$

are the projection of the multiuser-interference term at the output of the  $j^{th}$  correlator on the ortho-normal basis defined by a vector parallel to the noise component at the output of the p correlator and its orthogonal vector.

If the correlation coefficient  $\rho_{p,j}$  is equal to zero  $\forall p,j$ ,  $p \neq j$ , then the expression in (51) simplifies into a product of two complementary error functions. In particular, it can be easily verified that

$$1 - P_{p,j}(c) = a_{p,j} + b_{p,j} - a_{p,j}b_{p,j}, \tag{54}$$

where

$$a_{p,j} = \frac{1}{2} \operatorname{erfc}\left(\frac{x_0}{\sqrt{2\sigma_{n,p}^2}}\right),$$
 (55)

$$b_{p,j} = \frac{1}{2} \operatorname{erfc}\left(\frac{y_0}{\sqrt{2\sigma_{n,j}^2}}\right). \tag{56}$$

If the Gaussian assumption is adopted and  $\rho_{p,j} = 0$ ,

$$P_n(e) \le \frac{1}{M} \sum_{p=0}^{M-1} \sum_{\substack{j=0\\j \ne p}}^{M-1} a_{p,j}^{(G)} + b_{p,j}^{(G)} - a_{p,j}^{(G)} b_{p,j}^{(G)}, \tag{57}$$

where

$$a_{p,j}^{(G)} = \frac{1}{2} \operatorname{erfc} \left( c \frac{\sqrt{E_b} P_{n,p}}{\sqrt{2 \left( 2\sigma_{n,p}^2 + -\sigma_{MI,n,j,p}^2 \right)}} \right),$$
 (58)

$$b_{p,j}^{(G)} = \frac{1}{2} \operatorname{erfc} \left( \frac{c\sqrt{E_b} P_{n,p}}{\sqrt{2 \left( 2\sigma_{n,j}^2 + {}^{+}\sigma_{MI,n,j,p}^2 \right)}} \right).$$
 (59)

It is interesting to note that if the narrowband interference is absent, the independence between the noise components  $n_{n,m}$  requires the orthogonality between the receiver vectors  $\mathbf{w}_n^m$ ,  $\forall m$ . This condition is verified for linear receivers (matched filter, Rake or MMSE receiver, for example), if  $\left(\mathbf{Q}_j^{(k)}\right)^T\mathbf{Q}_i^{(k)}=\mathbf{0}, \forall i\neq j$ . This implies that the PPM delays are chosen such that the autocorrelation function of the channel, evaluated at those time instant, is zero. This is the average case for UWB channels as they are characterized by a rather impulsive autocorrelation function.

## V. RAKE AND MMSE RECEIVERS

In this section, two examples of digital filters are presented. The first one is the standard ideal Rake receiver [10], which can

combine all the resolvable multipath components. According to the model presented in the previous section, the set of digital filters  $\mathbf{w}_n^m$ ,  $m=0,\cdots,M-1$ , defines a Rake receiver, if

$$\mathbf{w}_n^m = \mathbf{Q}_m^{(1)} \mathbf{s}_n^{(1)},\tag{60}$$

and  $T_w = T_s + t_{max}$ . It is evident that this kind of receiver corresponds to a filter matched to the convolution of the transmitted signal and the channel impulse response.

The second receiver analyzed in this paper is the linear MMSE multiuser receiver. This receiver can better cope with multiuser and narrowband interference, maximizing the signal to interference ratio at the output of the digital filters. Even if this maximization does not lead to a minimization of the error probability, for the interference is not white Gaussian, however, it leads to an improvement of the error probability of the systems [14] compared to the matched filter, at the expense of complexity.

Setting the observation window to a value  $T_w \geq T_s$  (in our analysis we will assume  $T_w=2T_s$ ), then the digital filter is an MMSE receiver if

$$\mathbf{w}_{n}^{m} = \operatorname*{arg\ min}_{\mathbf{z} \in \mathbb{R}^{N_{w}}} \mathrm{E} \left\{ \left| \sqrt{\mathbf{A}_{1} \mathbf{E}_{x}} \mathbf{d}_{\mathbf{n}, \mathbf{m}}^{(1)} - \mathbf{z}^{\mathrm{T}} \mathbf{r}_{\mathbf{n}} \right|^{2} \right\}. \tag{61}$$

Defining

$$\mathbf{X}_{n} = E\{\mathbf{r}_{n}^{T}\mathbf{r}_{n}\} =$$

$$= \sum_{k=1}^{N_{u}} \sum_{-L_{L}+n}^{L_{R}+n} \sum_{l=0}^{M-1} A_{k} E_{x} \mathbf{Q}_{l}^{(k)} \mathbf{s}_{i}^{(k)} \left(\mathbf{s}_{i}^{(k)}\right)^{T} \left(\mathbf{Q}_{l}^{(k)}\right)^{T} \cdot E\left\{\left(d_{i,l}^{(k)}\right)^{2}\right\} + \frac{N_{0}}{2} \mathbf{I} + A_{b} \frac{N_{b}}{2} \tilde{\mathbf{R}}_{b,n}, \tag{62}$$

where I is a  $N_w \times N_w$  identity matrix, and

$$\mathbf{v}_n^m = A_1 E_x \mathbf{Q}_m^{(1)} \mathbf{s}_n^{(1)}, \tag{63}$$

then, it can be demonstrated [12] that equation (61) leads to

$$\mathbf{w}_n^m = \mathbf{X}_n^{-1} \mathbf{v}_n^m. \tag{64}$$

The advantage of the Rake receiver over the MMSE filter is that its implementation requires only the knowledge of the signature sequence and the channel impulse response of the reference user (see equation (60)). On the contrary, according to our model, the MMSE receiver requires the knowledge of the signature sequences and the channel impulse responses of all the users, together with the signal to noise ratio and the TR-RX attenuation factors. However, as noted in [14], the minimization of (61) can be carried out through an adaptive implementation, in which all the quantities that are needed are learned, provided that a preamble of symbols known to the receiver is transmitted (transmission of training sequences). Furthermore, the adaptive implementation overcomes the problem of matrix inversion in (64).

### VI. REFERENCE SCENARIO

#### A. Transmitters Positions

We consider a propagation environment delimited by a circumference of  $10\ \mathrm{m}$  radius, with the receiver in the center. All

the active users are inside this area, with a distance of at least one meter from the receiver. The position of all the transmitters is randomly chosen, assuming a uniform distribution over the surface delimited by the 1 and 10 m radius circumferences. Both line-of-sight (LOS) and non-line-of-sight (NLOS) cases are considered.

#### B. Channel Model

In order to compare the performance of the previously described multiple access schemes, an adequate indoor UWB channel should be introduced. In this paper we will employ the model proposed by the IEEE 802.15.3a working group [8], which is based on a modification of the Saleh-Valenzuela [15]. This model takes into account the clustering phenomena observed in several UWB channel measurements (see for example [16]). According to [8], the channel impulse response can be modeled as

$$h^{(k)}(t) = \sum_{l=0}^{L} \sum_{h=0}^{H} \alpha_{l,h}^{(k)} \delta(t - T_l^{(k)} - \tau_{l,h}^{(k)} - \tau_a^{(k)}),$$
 (65)

where  $\left\{\alpha_{l,h}^{(k)}\right\}$  are the multipath gain coefficients,  $\left\{T_l^{(k)}\right\}$  and  $\left\{\tau_{l,h}^{(k)}\right\}$  represent the delay of the  $l^{th}$  cluster and of the  $k^{th}$  multipath ray relative to the  $l^{th}$  cluster arrival time. The distribution of clusters and rays interarrival time is exponential. The average power delay profile shows a double exponential decay (for cluster average power and for rays average power in each cluster), and the fading statistics is lognormal. Finally the sign of each multipath replica is either positive or negative, each with the same probability. In our analysis, we will introduce another random variable  $\tau_a^{(k)}$ , modeling the delay due to asynchronism between users. In particular,  $\tau_a^{(k)}$  is assumed to be uniformly distributed over the interval T.

In [8] four sets of parameters are given, to characterize the statistical properties of different channels. In particular, the following propagation conditions are considered:

- 1. LOS channel with a TX-RX distance between 0 and 4 m.
- 2. NLOS channel with a TX-RX distance between 0 and 4 m.
- 3. NLOS channel with a TX-RX distance between 4 and 10 m.
- 4. Extreme NLOS channel (RMS delay spread of 25 ns).

In our model, a randomly generated channel will be assigned to each user according to the following rule: if the TX-RX distance is less than 4 m, then a channel impulse response of type 1 or 2 (with the same probability) is considered, otherwise one of type 3 or 4.

Finally, the path loss attenuations  $\{A_k\}$  and  $\{A_b\}$  are assumed proportional to  $d^{-\gamma}$ , where d is the TX-RX distance. The parameter  $\gamma$  is set equal to 2 for LOS channel and 3.5 for NLOS ones.

#### C. Narrowband Interference

For narrowband interferers we consider an IEEE 802.11a system, a possible competitor for WPAN applications. As shown in [2], this signal can be approximated with a Gaussian narrowband process. The central frequency and the bandwidth of the

interferer will be then set to 5 GHz and 200 MHz, respectively. Assuming that the UWB system, which has a bandwidth of approximately 3 GHz, operates at FCC part 15 limits of -41 dBm per MHz and that the narrowband interferer transmitted power is 100 mW, we obtain a signal to interference ratio of -26 dB, given that the two transmitters experience the same attenuation [2].

#### D. Transmitted Pulse and Multiuser Codes

The transmitted pulse x(t) is the second derivative of a Gaussian pulse, like in [4], with the time duration  $T_x$  equal to 0.7 ns. The multiuser codes are chosen respectively from the Gold codes for the DS system and from codes based on quadratic congruence [17] for the TH one. The optical orthogonal codes are designed such that two pulses are transmitted inside each frame [3].

## VII. PERFORMANCE EVALUATION

#### A. Introduction

In this section we present some BER curves obtained employing the methods described previously. All the curves are obtained from an average of 1000 realizations of the scenario described in section VI. In all situations, the multipath channel associated with the reference user is assumed to be LOS. In the first set of plots, we will compare the performance of the three proposed multiple access schemes over different conditions (system load, Rake/MMSE reception, Gaussian assumption for MUI-ISI interference, and narrowband interferer). In the second set, the impact of the modulation techniques (2PAM, 2PPM, Bi2PPM) is illustrated.

## B. Multiple Access Techniques

In Fig. 1 the BER curves for the three multiple access schemes are illustrated. In the left column plots, a system characterized by high bit rate is analyzed. In this case the number of chip per frame  $N_c$  is equal to 7, leading to a bit rate of 204 Mbit/s per user. The chip length is chosen equal to the time duration of the pulse. In the right column plots, a lower rate version of the same system is considered, with  $N_c=31$  and a bit rate of 46 Mbit/s per user. In both situations, the number of active users  $N_u$  is equal to 6. The first system is therefore nearly fully loaded, while the second one can be considered to be in a rather low load condition.

Fig. 1(a) and 1(b) show the performance of the systems in the presence of strong NBI. Rake receivers exhibit high error floor for all the multiple access techniques, due to its lack of robustness against strong narrowband interference. On the contrary, MMSE receivers offer, as expected, much better performance at the cost of higher computational complexity. The Gaussian approximation was successfully adopted to derive these curves; in fact the dominant noise in this situation is the narrowband one, that was modeled as a colored Gaussian random process.

The comparison of the two plots give rise to some interesting considerations. Increasing the number of chip per frame (with a reduction of the bit rate) leads to an improvement in the crosscorrelation and autocorrelation properties of the multiaccess codes and therefore to a reduction of the multiuser interference. This fact justifies the average 1 dB gain in performance of the MMSE receivers, in the lower bit rate case depicted in Fig. 1(b), compared to Fig. 1(a). For the Rake receiver however, the high error floor is uniquely determined by the effect of the strong narrowband interference, therefore a mitigation of the multiuser interference does not lead to an improvement in performance. The difference between the position of the BER floors for the different schemes at both rates are strongly influenced by the shaping effect of the multiuser code on the power spectrum of the transmitted signal. For example, the better performance of the OOC scheme with a Rake receiver in Fig. 1(a) can be justified by the spectral analysis, noting that the code assigned to the reference user introduces a spectral attenuation in the vicinity of the central frequency of the narrowband interference. Furthermore, when the code does not introduce any shaping (like in uncoded TH-PAM [18] or DS-PAM with a sufficiently high number of chips per frame, such that the autocorrelation function is nearly impulsive), the floor position is independent of the value of  $N_c$ .

This consideration suggests that an effective strategy to optimize the performance of UWB systems with Rake reception, whenever the narrowband interference is the limiting factor, could be based on the design of spreading codes with desired spectral characteristics, rather than on the optimization of their auto and crosscorrelation properties.

On the contrary, when the narrowband interference is absent (see Fig. 1(c) and Fig. 1(d)), the cross and auto correlation characteristics of the codes play an important role in determining the position of the BER floor for Rake receivers. It is interesting to note that for the high rate system, the difference between the multiuser techniques are negligible, showing that the effect of dense multipath channel substantially deteriorates the higher MUI robustness of DS compared to TH and OOC observed in [3] in the AWGN channel. In the lower rate system case, instead, a difference in performance can be still appreciated.

In Figs. 1(e) and 1(f) the BER curves obtained with the Gaussian assumption are plotted for Rake receivers in the absence of NBI. Note that for the high bit rate, fully loaded system, the MUI-ISI term can be reasonably considered Gaussian, as no significative differences can be appreciated comparing Fig. 1(e) with 1(c). However, for the lower rate system, the use of this hypothesis leads to an overestimation of the error probability, as can be verified by comparing Fig. 1(f) with 1(d).

As far as the MMSE filter is concerned, it has been shown in [19] that its output can be considered Gaussian under general conditions, so this assumption is fruitfully adopted to derive all the BER curves with this type of receivers.

## C. Modulation Techniques

In Fig. 2 we compare the impact of different modulation strategies for both TH (left column plots) and DS (right column plots). Ideal Rake reception,  $N_c=7$ , and no narrowband interference are assumed. Both single user and a multiple-access cases are considered for two target bit rates, 204 Mbit/s and 102 Mbit/s per user, respectively.

For the first rate, 2PAM and Bi2PPM with Gray mapping are

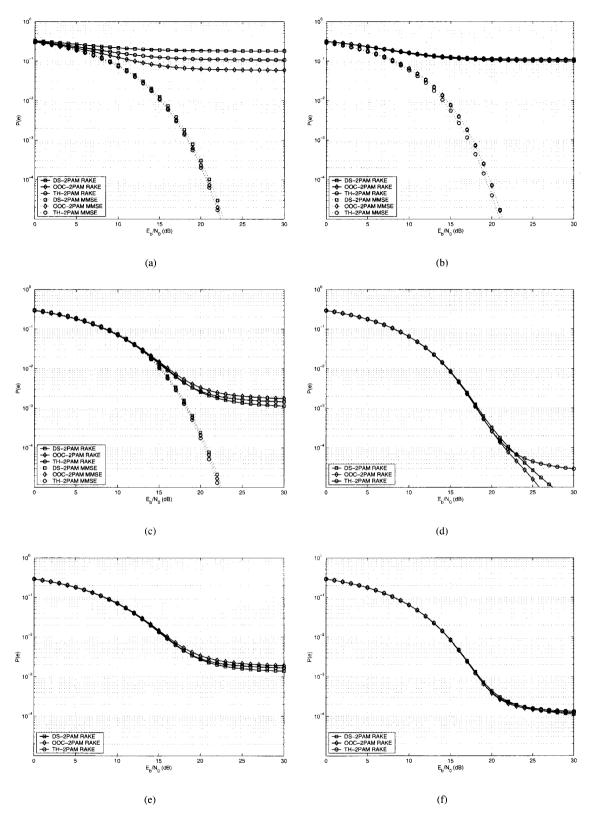


Fig. 1. Multiple access techniques comparison: (a)  $N_c=7$ ,  $E_b/N_b=-26$  dB, (b)  $N_c=31$ ,  $E_b/N_b=-26$  dB, (c)  $N_c=7$ , no NBI, (d)  $N_c=31$ , no NBI, (e)  $N_c=7$ , no NBI, Gaussian assumption.

compared in terms of BER. The independence between the noise components is assumed for the last modulation technique. As shown in Fig. 2(a) and 2(b), the second technique leads to an

improvement in the performance of the system in the single user case, due to its high robustness against ISI. As a matter of fact, as two bits are transmitted for each symbol, the frame length must

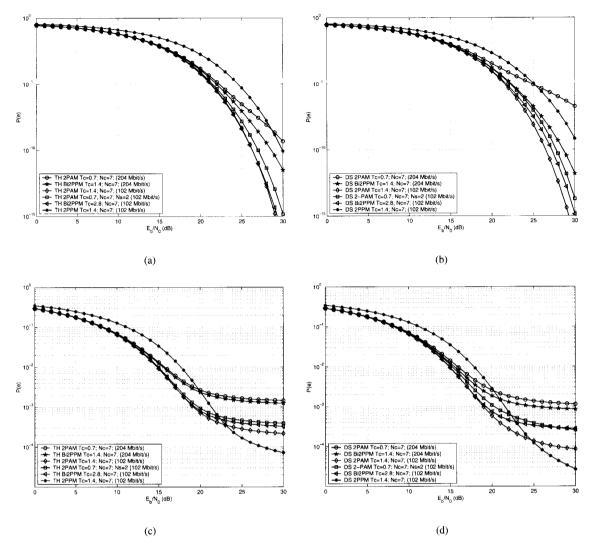


Fig. 2. Modulation techniques comparison: (a)  $N_u=1$ , TH, (b)  $N_u=1$ , DS, (c)  $N_u=6$ , TH, (d)  $N_u=6$ , DS. Note that  $N_s$  is the repetition code length.

be doubled in order to maintain the same bit rate as in the 2PAM case. This holds for both TH and DS. However, in the presence of multiuser interference (Fig. 2(c), Fig. 2(d)), the advantage of the second technique over the first one in terms of BER floor becomes rather small.

In the lower rate case, four modulation techniques are compared: 1) a 2PAM scheme in which the chip length is doubled ( $T_c=1.4~\mathrm{ns}$ ), compared to the fast rate case, 2) a 2PAM scheme with  $T_c=0.7~\mathrm{ns}$  and a repetition code of length  $N_s=2$ , 3) a Bi2PPM modulation technique with  $T_c=2.8~\mathrm{ns}$ , and 4) a 2PPM scheme with  $\tau_0=0$  and  $\tau_1=0.7~\mathrm{ns}$ .

When only one user is active (Fig. 2(a), Fig. 2(b)), the most effective techniques are 2PAM and Bi2PPM, which show similar performance. The use of a repetition code does not lead to an improvement of performance, as noted in [3] in the AWGN channel. Evidently, the time diversity gain is smaller than the ISI enhancement due to the reduction of frame duration. Finally 2PPM exhibits a poor behavior due to the 3 dB orthogonal modulation loss.

The situation changes when MUI is considered (Fig. 2(c) and Fig. 2(d)). While the first three techniques have a similar error

floor (around  $2-4\cdot 10^{-4}$  for TH), 2PPM substantially outperforms the other techniques for sufficiently high SNR values. This behaviour could be justified hypothesizing a higher robustness of 2PPM against the multiuser and intersymbol interference, due to the correlation characteristics of the transmitted pulse.

Finally it is interesting to note that, as in the previous section, when the system operates at a bit rate smaller than the one given by the fully loaded condition (204 Mbit/s), DS is more robust than TH against MUI-ISI interference, for all considered modulation schemes.

## VIII. CONCLUSIONS

In this paper we presented a general method for the evaluation of the error probability of UWB systems employing different multiaccess, modulation and reception techniques. An indoor system characterized by dense multipath and narrowband transmitters was considered.

It is shown that for all the multiple access schemes considered, the Rake receiver exhibits a high error floor in the pres-

ence of NBI and that the error floor position is influenced by the spectral characteristics of the spreading code. As expected, an MMSE receiver offers better performance, representing a promising candidate for UWB systems. However, its complexity and feasibility for practical implementation must still be assessed.

When the multiuser interference is dominant, all the multiuser techniques exhibit similar performance under high load conditions. On the contrary, when the number of users is significantly less than the spreading factor of the system (defined as the ratio between the frame and the pulse duration), DS outperforms both TH and OOC.

For a fixed target bit rate, 2PPM exhibits better performance than the other modulation schemes in the presence of multiuser interference. Finally, increasing the spreading factor is proposed as a more effective strategy for system BER reduction than the use of time diversity.

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