

■ 論 文 ■

An Solution Algorithm for A Multi-Class Dynamic Traffic Assignment Problem

다계층운전자를 고려한 동적통행배정모형의 해법

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Key Words : dynamic traffic assignment, multi-class, fixed route, unguided, guided, variational inequality, variational equality

요 약

동적통행배정모형을 이용해서 교통정보를 제공하기 위해서는 다양한 여행자의 경로선택행태를 고려하는 것이 필요하다. 여행자계층은 일반적으로 3가지 형태로 분류된다: 1) 버스나 지하철과 같은 대중교통의 고정된 경로(fixed route class)를 이용하는 그룹, 2) 자신이 인지하는 경로비용을(perceived route, unguided class) 판단하여 경로를 선택하는 그룹, 3) 정확한 경로선택에 대한 정보를(guided class)기반으로 경로를 선택하는 그룹. 본 연구에서는 이 3그룹의 여행자를 포함하는 동적통행배정모형의 해법을 제안한다. 제안된 해법에서는 링크의 교통량과 유출교통류를 진입교통류 단일변수로 축소하여 시간과 공간을 확장하지 않고 실제의 네트워크에서 최단경로를 도출하는 방법을 적용한다. 따라서 시간종속적인 통행비용함수에 진입교통유율, 교통량, 유출교통유율 3가지 변수를 고려해야 하는 시공간확장방법에 비해 네트워크의 규모와 수행시간에 있어 유리하다.

I. Introduction

The development of the Advanced Traveler Information System(ATIS) will make it possible that travelers are furnished with real-time(instantaneous and predictive) traffic information. The DTA(dynamic traffic assignment) models, both simulation and analytical approaches, should demonstrate the capability to differentiate travelers' route choice behavior based on their realization of traffic conditions. The route choice behavior such as fixed route, stochastic dynamic user optimum(SDUO), and dynamic user optimum(DUO) should be considered as a whole.

In this paper, travelers are classified into three classes according to respective route choice behavior, who follow : (1) predetermined or fixed routes, (2) stochastic dynamic user-optimal(SDUO) principle, and (3) dynamic user-optimal(DUO) principle. Class 1 travelers are those who have no access to real-time traffic information and travel on predetermined routes. Class 2 travelers make their route choices based on perceived route travel times because of their limited accessibility to real-time traffic information. They determine routes based on the perceived lowest travel times when they start their trips. Class 2 travelers' route choice behavior is referred as the so-called stochastic dynamic user optimum(SDUO). One can model these perceived travel times by adequately introducing a distribution to account for the errors between actual travel times and travelers' perceived travel times. Typically, the notion of perceived travel times within the static context is modeled through a stochastic generalization of Wardrop's first principle (Daganzo and Sheffi, 1977; Sheffi and Powell, 1982).

Class 3 travelers are perfectly aware of real-time traffic condition. Their route choices are delineated by the so-called dynamic user optimum (DUO) principle, which is the temporal generalization of Wardrop first principle(Wardrop, 1952).

An assumption of Class 3 travelers is that they possess perfect knowledge of dynamic traffic conditions so that their perceptions of travel time are free of errors.

In this paper, the continuous-time variational equality(VE) formulation is presented first, followed by the transformed discrete-time VE formulation and nonlinear programming(NLP) problems. The solution algorithm developed to solve the formulated model is a combination of relaxation procedure, Frank-Wolfe algorithm, and Method of Successive Averages. This paper analytically proves that the proposed solution algorithm can be carried out on physical networks without performing the time-space network expansion, which is a remarkable contribution of this paper.

II. Route Choice Condition of Multiple User Classes

1. Definitions of Variation Equality

Define G as a given closed convex set of the decision variables \mathbf{x} : \mathbf{f} is a vector of given continuous functions defined on \mathbf{R}^n . In the dynamic problem, we are concerned with a vector of control variable $\mathbf{u}(t) = [u_1(t), u_2(t), \dots, u_n(t)]$ and state variable $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_n(t)]$. Associated with the dynamic processes, there is a vector of cost functions $\mathbf{F}(t) = [F_1(t), F_2(t), \dots, F_n(t)]$. Each element of the cost function vector is a function of state and control variables, i.e., (1)

$$F_i(t) = F_i[\mathbf{x}(t), \mathbf{u}(t)] \quad i = 1, 2, \dots, n \quad (1)$$

Since the state variables $\mathbf{x}(t)$ can be determined by the state equations when the control variables $\mathbf{u}(t)$ are given, the vector of cost functions can be further simplified as $\mathbf{F}(t) = [\mathbf{u}(t)]$. Define $\mathbf{G}(t)$ as a given closed convex set of the control variables $\mathbf{u}(t)$. We assume $\mathbf{F}(t)$ is a set of given continuous

functions from $G(t)$ to $R^n(t)$. Then, we give the following definition of the dynamic variational equality problem.

[Definition 1]

The infinite-dimensional "variational equality" problem is to determine a control vector $u(t) \in G(t) \subset R^n(t)$, such that

$$F[u^*(t)] = 0 \quad \forall u^*(t) \in G(t) \quad (2)$$

The following definition is also useful where continuous time problems need to be transformed to discrete time problems.

[Definition 2]

The infinite-dimensional "variational equality" problem is to determine a control vector $u(t) \in G(t) \subset R^n(t)$, such that

$$\int_0^T F[u^*(t)] dt = 0 \quad \forall u^*(t) \in G(t) \quad (3)$$

2. Underlying Principle

Wardrop's first principle is known as the basis of user equilibrium traffic assignment, which states:

The route cost for any used routes between a given origin-destination pair must equal to the minimum route cost, and no unused route has a lower cost.

In order to consider temporal variations in the traffic flows in a network, the *Dynamic User-Optimal*(DUO) principle is applied as the temporal generalization of Wardrop's first principle which states:

For each O-D pair at each interval of time, if the actual travel times experienced by travelers departing at the same time are equal and minimal.

The *actual* travel cost is defined as the travel cost actually experienced by travelers during their journeys so that the above addressed DUO is also referred as the *ideal* DUO principle. Under the *ideal* DUO state, travelers(Class 3) cannot decrease their actual travel costs by unilaterally changing routes. Therefore, the obtained DUO state can be viewed as an equilibrium. Note that this DUO principle is only suitable for users who have perfect knowledge of real-time traffic conditions through automatic traffic information system(ATIS) equipment. As a result, for those who only have partial information(Class 2) and those who would like to stay on fixed routes(Class 1) for whatever reasons, the route choice criteria should be adjusted, as stated by Dafermos(1972):

The multi-class user model should consider differently the travel demand and travel cost function for different classes of users, and each class of users will select their own shortest path according to their own interpretation of the travel cost.

The route choice criterion of Class 2 travelers is called the *Stochastic Dynamic User Optimal* (SDUO) principle which states:

No traveler can improve his/her perceived actual travel cost by unilaterally changing the route.

3. Notations

Notation used in this paper is shown in <Table 1>.

4. Route Choice Conditions for Class 2 Travelers(SDUO)

As shown in Sheffi(1985), a satisfaction function can be defined as follows:

$$S^{rs}(t) = E[\min_p \eta_p^{rs}(t)] \quad \forall r, s, p \quad (4)$$

(Table 1) Notations

N	Discrete departure time interval n
K	Discrete time interval k
Rs	Origination and destination pair
$f^{rs}(t)$	Departure flow rate from origin r to destination s at time t
$u_{ap}^{rs}(t)$	Inflow rate on link a at time t
$v_{ap}^{rs}(t)$	Exit flow rate on link a at time t
$x_{ap}^{rs}(t)$	Number of vehicles on link a at time t
$E^{rs}(t)$	Cumulative number of vehicles arriving at destination s from origin r by time t
$p_{ap}^{rs}(k)$	Inflow rate on link a at the beginning of time interval k
$q_{ap}^{rs}(k)$	Exit flow rate on link a at the beginning of time interval k
$y_{ap}^{rs}(k)$	Number of vehicles on link a at the beginning of time interval k
$\bar{E}^{rs}(k)$	Cumulative number of vehicles arriving at destination s from origin r by the beginning of time interval k
$e^{rs}(t)$	Arrival flow rate from origin r toward destination s at time t
$\tau_a(t)$	Mean actual travel time over link a for flows entering link a at time t
$\bar{\tau}_a(t)$	Estimated mean actual travel time over link a for flows entering link A at time t
$\eta_p^{rs}(t)$	Mean actual travel time for route p between (r, s) for flows departing origin r at time t
$\pi^{rs}(t)$	Minimal mean actual route travel time between (r, s) for flows departing origin r at time t

The satisfaction function, $S^{rs}(t)$, captures the expected minimum travel time of route p between (r, s) at time t that a traveler derives from a set of route travel times. The partial derivative of the satisfaction function with respect to the mean actual travel time for route p between (r, s) for flows departing from origin r at time t , $\eta_p^{rs}(t)$, equals the proportion of flows between (r, s) that follow route p at time t .

$$\frac{\partial S^{rs}(t)}{\partial \eta_p^{rs}(t)} = P_p^{rs}(t) \quad \forall r, s, p \quad (5)$$

The SDUO route choice condition is defined in Equation (6). It states the departure flows of Class 2 travelers from r to s on route p at time t , $f_{p2}^{rs}(t)$, equals the Class 2 travelers from r to s at time t multiply the proportion of flows between (r, s) using route p at time t .

$$f_{p2}^{rs}(t) - f_2^{rs}(t)P_p^{rs}(t) = 0 \quad \forall r, s, p \quad (6)$$

Note that mean actual route travel time $\eta_p^{rs}(t)$ increases with route departure flows of Class 2 travelers, $f_{p2}^{rs}(t)$, as shown in Equation (7).

$$\frac{\partial \eta_p^{rs}(t)}{\partial f_{p2}^{rs}(t)} > 0 \quad \forall r, s, p \quad (7)$$

For each route p and O-D pair rs , define an auxiliary cost term $F_p^{rs}(t)$ as follows:

$$F_p^{rs}(t) = [f_{p2}^{rs}(t) - f_2^{rs}(t)P_p^{rs}(t)] \frac{\partial \eta_p^{rs}(t)}{\partial f_{p2}^{rs}(t)} = 0 \quad \forall r, s, p \quad (8)$$

The above system of equations is equivalent to the following VE for each time instant $t \in [0, +\infty)$:

$$\int_0^T \sum_{rs} \sum_p \{F_p^{rs*}(t)\} dt = 0 \quad (9)$$

where * denotes the SDUO state $t \in [0, +\infty)$, thus

$$F_p^{rs*}(t) = [f_{p2}^{rs*}(t) - f_2^{rs}(t) P_{p2}^{rs}(t)] \frac{\partial \eta_p^{rs}(t)}{\partial f_{p2}^{rs*}(t)} = 0 \quad (10)$$

5. Route Choice Conditions for Class 3 Travelers(DUO)

The Class 3 travelers' link-time-based *ideal* DUO route choice conditions are as follows(Shin et al, 2002).

$$\pi^{ri*}(t) + \tau_a[t + \pi^{ri*}(t)] - \pi^{rj*}(t) = 0, \forall a = (i, j), r \quad (11)$$

$$u_{3a}^{rs*} [t + \pi^{ri*}(t)] \{ \pi^{ri*}(t) + \tau_a[t + \pi^{ri*}(t)] - \pi^{rj*}(t) \} = 0, \forall a = (i, j), r, s \quad (12)$$

$$u_{3a}^{rs} [t + \pi^{ri*}(t)] > 0, \forall a = (i, j), r, s \quad (13)$$

As defined in the temporal generalization of Wardrop's first principle, when a link a is visited at certain time $t + \pi^{ri*}(t)$ from the flows departing from origin r at time t , the link a is on the shortest route between origin r to destination s at time $t + \pi^{ri*}(t)$.

The equivalent VE formulation of the link-time-based *ideal* DUO route choice conditions defined in Equations (11)-(13) can be written as follows, where * denotes the DUO state.

$$\int_0^T \sum_{rs} \sum_a \{ \tau_a [t + \pi^{ri*}(t)] - \pi^{rj*}(t) \} dt = 0 \quad (14)$$

6. Variational Equality of Multi-Class DTA Problem

Combining Equations (9) and (14), the equivalent

VE formulation of this multi-class DTA problem can be stated as follows(where * denotes the optimal state):

$$\int_0^T \sum_{rs} \left\{ \sum_a \{ \tau_a [t + \pi^{ri*}(t)] - \pi^{rj*}(t) \} + \sum_p F_p^{rs*}(t) \right\} dt = 0 \quad (15)$$

The first term in Equation (15) represent the DUO route choice condition, and the second term describes the SDUO route choice condition. For Class 1 travelers who travel on fixed or predetermined routes, no mathematical expressions are required in Equation (15) to represent their route choice condition because they have no choice to choose other alternative routes.

IV. Physical Network Approach

We propose a diagonalization technique to solve Equation (15). In the proposed approach, two separate solution steps are included: 1) outer step and 2) inner step. In inner step, two solution algorithms are combined Frank-Wolfe(FW) algorithm for Class 3 and method of successive average(MSA) for Class 2. Expanded time-space network approach (ETSNA) is inevitable when the existing FW and MSA algorithms are applied because the two algorithm needs to deal with three variables(inflow, flow, outflow) in link performance function(Ran and Boyce, 1996). The main idea of physical network approach (PSA) can be implemented by expressing two variables(inflow and flow) by inflow. It means link performance function can be solely expressed by inflow, thus ETSNA is no longer required.

1. FW Algorithm Using ETSNA

LeBlanc et al(1975) have shown that the Frank-Wolfe(F-W) algorithm can be directly used to solve

the NLP formulation of the static traffic assignment(route choice) problem. Ran and Boyce(1996) proved that the F-W algorithm is also appropriate to solve the dynamic traffic assignment problem if a time-space network is considered.

Denote the subproblem variables as p , q , y corresponding to the main problem variable, u , v , x . Applying the Frank-Wolfe algorithm to the minimization of the discretized DUO traffic assignment program requires, at each iteration, a solution of the following linear program(LP):

$$\begin{aligned} \min_{p,q,y,E} Z = & \nabla_u Z(u,v,x) p^T + \nabla_v Z(u,v,x) q^T \\ & + \nabla_x Z(u,v,x) y^T \end{aligned} \quad (16)$$

Since there are 3 variables associated with each physical link, the LP subproblem needs to be decomposed to deal with 3 variables together by replacing each link by 3 separate artificial links for each time period and adding artificial nodes to define the new links(Ran and Boyce, 1996).

2. A Physical Network Algorithm for Class 3 Travelers

The earlier approach inevitably introduces significant complexity if large-scale networks are considered. The FW algorithm can be applied to the dynamic traffic assignment problem without performing time-space network expansions.

Without considering link flow dispersion and compression, the discrete flow propagation constraint is expressed as:

$$u_{ap}^{rs}(k) = v_{ap}^{rs}[k + \tau_a(k)] \quad (17)$$

Equation (17) states that the exit flows of link a at time interval $k + \tau_a(k)$ are equal to the inflows of link a at time interval k . Furthermore,

Equations (18) and (19) show that v (exit flow) and x (flow) can be expressed solely by u (inflow).

$$v_a(t) = \sum_k u_a(k) \delta_a^k(t) \quad (18)$$

$$\text{where } \delta_a^k(t) = \begin{cases} 1, & k + \tau_a(k) = t \\ 0, & \text{otherwise} \end{cases}$$

$$x_a(t) = \sum_k u_a(k) \delta_a^k(t) \quad (19)$$

$$\text{where } \delta_a^k(t) = \begin{cases} 1, & k < t, k + \tau_a(k) \geq t \\ 0, & \text{otherwise} \end{cases}$$

Denote t as the entering interval of certain flows to a link. If $\delta_a^k(t)=1$, Equation (18) simply states: at time interval t , the exit flows of link a are equal to the sum of flows entering link a at time interval k (kt) and exiting link a at time interval t ($k + \tau_a(k) = t$). Equation (18) shows theoretically that exit flows are replaceable by inflows. If $\delta_a^k(t)=1$, Equation (19) represents that the flows on link a at time interval t are equal to those flows that enter link a before time interval t and exit link a after time interval t . Similar to Equation (18), Equation (19) shows theoretically that link flows are replaceable by inflows.

Based on Equations (18) and (19), a simplified LP subproblem is obtained in Equation (20).

$$\min Z = \nabla_u Z(u) p^T \quad (20)$$

The minimization of Equation (20) can be accomplished by assigning all flows between rs at each time interval to the time-dependent shortest routes connecting rs at that time interval. Note that the search of time-dependent shortest routes is based on actual travel times. The time-dependent shortest route search over the physical network can thus be directly applied. Accordingly, the time-dependent shortest route search over the expanded time-space network is no longer needed.

3. MSA for Class 2 Travelers

The method of Successive Averages(Sheffi and Powell, 1982) is adopted to solve the relaxed NLP problem for Class 2 travelers. MSA is a descent direction method that uses a predetermined step size. To determine the step size for an objective function like Equation (20) is difficult, if not impossible. It is due to the computational difficulty while evaluating the objective function or lack of analytical expression of the objective function. If the route choice probability distribution is a multinomial Logit, an analytical expression for this choice function may exist, and the objective function for the equivalent optimization and associated partial derivatives can be expressed in closed forms. To evaluate this objective function and its partial derivatives, however, requires explicit enumeration of route which is intractable for large-scale networks. If the route choice probability distribution is a multinomial Probit, the route choice function involves the Normal density function and does not have a closed form. In this case, to evaluate the SDUO objective function can only be approximated by Monte Carlo sampling.

To determine the steepest descent direction, the Monte Carlo simulation is used to sample the link travel time for the stochastic network loading. Travelers' perception errors upon travel times are shown in the Monte Carlo simulation results. The search of time-dependent shortest routes is based on those random link travel times. The stochastic network loading is solved by Equation (21).

$$p_{a2}^r(k) = [(r-1)p_{a2}^{(r-1)}(k) + \hat{p}_{a2}^j(k)] / r \quad \forall a \quad (21)$$

Note that the vector $\{p^r\}$ is used to determine the steepest descent direction. As r equals a pre-specified number, the simulation stops. The step size to combine u and p is determined by

$$\alpha_n = \frac{k_1}{k_2 + n} \quad (22)$$

where k_1 is a positive constant, and k_2 is a non-negative constant. By setting $k_1=1$ and $k_2=0$, the step size is equal to $1/n$. It yields a new solution

$$u_{a2}^{n+1}(k) = u_{a2}^n(k) + \frac{1}{n} [p_{a2}^n(k) - u_{a2}^n(k)] \quad \forall a \quad (23)$$

The variables, $x_{a2}(k)$ and $v_{a2}(k)$, can be obtained through Equations (24) and (25).

$$v_{a2}(t) = \sum_k u_{a2}(k) \delta_a^k(t) \quad (24)$$

$$x_{a2}(t) = \sum_k u_{a2}(k) \delta_a^k(t) \quad (25)$$

4. The Algorithm

In the combined algorithm, we define the travel time approximation procedure(relaxation) as the outer iteration and the FW/MSA procedure as the inner iteration.

This algorithm is summarized as follows.

Step 0 : Initialization

Set $\{x_{am}^{(0)}(k)\}$, $\{u_{am}^{(0)}(k)\}$, and $\{v_{am}^{(0)}(k)\}$, to zero and compute initial travel time estimates $\tau_a^{(1)}(k)$ for all classes of travelers. Set outer iteration counter $l=1$.

Step 1 : Relaxation

Set inner iteration counter $n=1$. Find a new approximation of actual link travel times: $\bar{\tau}_a^{(n)}(k) = \tau(x_a^{(*)}(k))$, where $*$ denotes the solution obtained from the most recent inner iteration. According to $\bar{\tau}_a^{(n)}(k)$, update $\delta_a^k(t)$, $\delta_{a^n}^k(t)$ by Equations (18) and (19). Solve the route choice problems for different traveler classes.

Class 1 Travelers : Predetermined Routes

[Step 1.1] : Fixed Route Assignment. Assign O-D departure flows $f_1^n(k)$ of Class 1

travelers to predetermined routes with prespecified route flow shares. Compute the resulted link flows $\{u_{a1}(k)\}$, $\{v_{a1}(k)\}$, and $\{x_{a1}(k)\}$ by flow conservation and propagation constraints.

Class 2 Travelers : Stochastic DUO Problem

[Step 1.2.1] : Stochastic Dynamic Network Loading. Perform Monte Carlo simulation to obtain random link travel times. Search for shortest routes based on perceived link travel times. Obtain minimal route travel times $\pi^{rs}(k)$ and assign all departure flows $f_2^{rs}(k)$ of Class 2 travelers to these routes in each Monte Carlo iteration. Let \hat{p}^i be the temporary link flow vector resulted from the all-or-nothing network loading at Monte Carlo iteration i . Solve the stochastic network loading by the recursive Equation (21). If i equals a prespecified number, stop. The vector p^i is used as the converged link flows (p_2^n) at inner iteration n . It is determined by the stochastic network loading of the O-D flows based on the specified multinomial Probit route choice density function.

[Step 1.2.2] : Method of Successive Averages(MSA). Using the predetermined step size $1/n$, yields a new MSA main problem solution through Equation (23).

Class 3 Travelers : Deterministic DUO Problem

[Step 1.3.1] : Update. Calculate Equation (20).

[Step 1.3.2] : Direction Finding. Based on $\bar{\Omega}_a^r(k)$, search for shortest routes for all O-D pairs over the physical network without time-space expansions. Perform an all-or-nothing assignment, yielding subproblem solution $p_{a3}^r(k)$. Compute $q_{a3}^r(k)$ and $y_{a3}^r(k)$ by Equ-

ations (24) and (25).

[Step 1.3.3] : Line Search. Find the optimal step size using a line search procedure such as the golden section method that solves the one-dimensional search problem.

[Step 1.3.4] : Move. Find a new solution by combining $u_{a3}^r(k)$, $v_{a3}^r(k)$, $x_{a3}^r(k)$ and $p_{a3}^r(k)$, $q_{a3}^r(k)$, $y_{a3}^r(k)$ using the optimal step size found in Step 1.3.3.

[Step 1.3.5] : Convergence Test for the Inner Iteration. If n equals a prespecified number, go to Step 2; otherwise, set $n=n+1$ and go to Step 1.3.1.

Step 2 : Convergence Test for the Outer Iteration.

If $\tau_a^{(l)}(k) \cong \tau_a^{(l-1)}(k)$, stop. The current solution $\{u_{am}(k)\}$, $\{v_{am}(k)\}$, $\{x_{am}(k)\}$ is in a near optimal state; otherwise, set $l=l+1$ and return to Step 1.

V. Computational Results

1. Link Travel Time Function

A modified Greenshields speed-density function is adopted in the solution algorithm to determine link speed because of the monotone relationship between traffic density and travel time expressed in the function.

$$u = \begin{cases} u_{\min} + (u_{\max} - u_{\min}) \left(1 - \frac{k}{k_j}\right) & \text{if } k \leq k_j \\ u_{\min} & \text{if } k > k_j \end{cases} \quad (26)$$

where

u : speed (ft/sec)

u_{\min} : minimum speed at jam density (ft/sec)

u_{\max} : free flow speed (ft/sec)

k : density (veh/mile)

k_j : jam density (veh/mile)

The link travel time is obtained by Equations

(48) and (49).

$$\begin{aligned}
 & \text{if } k \leq k_j, \\
 \tau_a(t) &= \frac{L_a k_j}{u_{\min} k_j + (u_{\max} - u_{\min})(k_j - k_a(t))} \\
 &= \frac{k_j L_a^2}{u_{\min} k_j L_a + (u_{\max} - u_{\min})(k_j L_a - x_a(t))} \tag{27}
 \end{aligned}$$

$$\begin{aligned}
 & \text{if } k > k_j, \\
 \tau_a(t) &= \frac{L_a}{u_{\min}} \tag{28}
 \end{aligned}$$

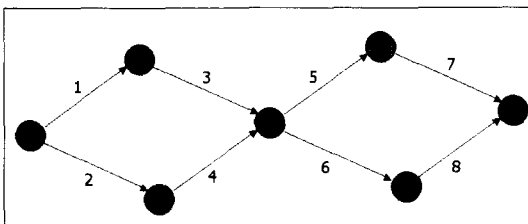
where

- $\tau_a(t)$: travel time on link a in time interval t
- L_a : length of link a
- $x_a(t)$: number of vehicles on link a in time interval t

2. Test of A Hypothetical Network

A hypothetical network that contains 7 nodes and 8 links is constructed to test the assumptions of the proposed model. <Figure 1> shows the topology of this test network. In this network, each link is assumed as a one-lane freeway link with length of one-mile. A single OD pair (1, 2) is considered in the test runs. The assumptions for this network are:

- Free flow speed of 50 mph.
- Link cost function is based on a modified Greenshields speed-density function.



<Figure 1> Test Network

- No incidents and accidents.
- Perceived travel times of Class 2 travelers are modeled by a truncated Normal Distribution derived from the Monte-Carlo simulation method (Sheffi and Powell, 1982).
- 100 time intervals are assumed.
- Each time interval has 20 seconds.
- There is one fixed route for Class 1 travelers:

<Table 2> Fixed Route for Class 1 Travelers

Route	Fixed Route (Node Sequence)
1	1 -> 3 -> 5 -> 6 -> 2

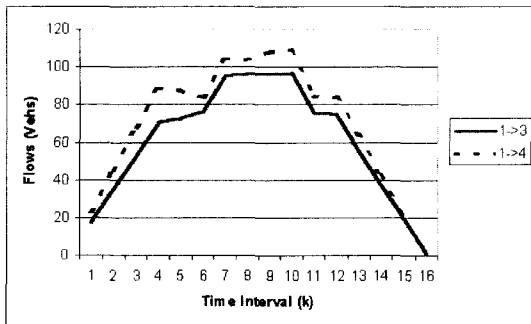
- Ten time intervals are specified for multi-class OD pairs and at each time interval, each class has the constant departure flow rates as shown in <Table 2>:

<Table 3> Departure flow rate for each class

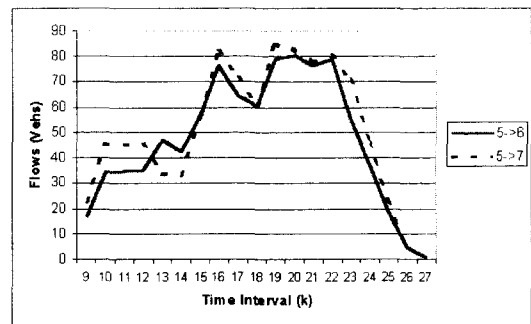
Time Intervals	Total	Class 1 (Guided)	CLASS 2 (Unguided)	CLASS 3 (Guided)
1 10	40	5	20	15

The results are verified from two aspects: first, the variation in flow over time; and second, the tracing of actual travel times on each possible route. Since the network is symmetric, if there is no incident considered, the flow pattern on symmetric parts of two links should have a similar trend. This can be derived from the fact that Class 3 drivers receive correct traffic information, which compensates for the biased link flows generated by Class 1 and Class 2 drivers. As shown in <Figure 2>~<Figure 5>, the flow patterns of the two symmetric links are within a 10% margin of error. These small discrepancies result from temporal discretion and convergence errors.

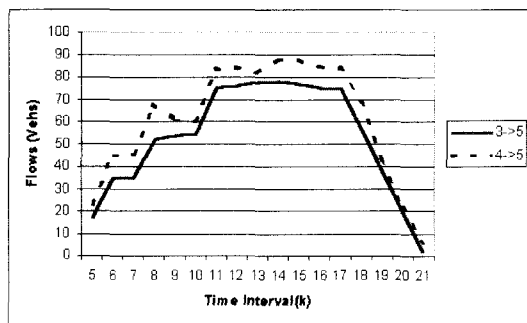
The second indicator is to compare the actual route travel times to determine whether the DUO flow requirements are satisfied. Since Class 3 drivers compensate for the biased travel time caused by Class 1 and 2, the actual travel times



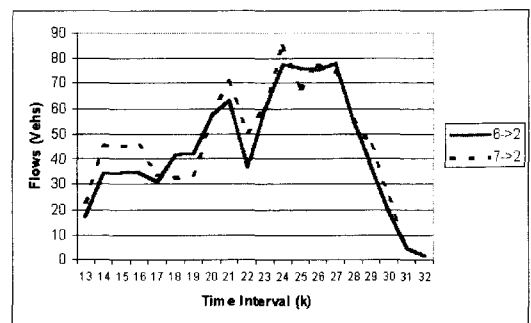
〈Figure 2〉 Flow Pattern over Time for Link (1->3) and (1->5)



〈Figure 4〉 Flow Pattern over Time for Link (5->6) and (5->7)



〈Figure 3〉 Flow Pattern over Time for Link (3->5) and (4->5)



〈Figure 5〉 Flow Pattern over Time for Link (6->2) and (7->2)

of all used routes needs to be as identical as possible. 〈Table 3〉 shows the actual travel times for 10 departure time intervals when first platoon of OD flows departing from the origin. The route travel times can be determined by tracing the link travel times experienced by these vehicles. The results show that the DUO flow patterns are almost satisfied. As can be seen in 〈table 3〉, the experienced actual travel times for departure flows are almost identical with some marginal errors. This fact indicates that the solution algorithm is able to achieve the DUO state in multi-class traffic situation if there are enough guided travelers. Note that these marginal errors will be reduced when more travelers equip the ATIS information system in their cars.

VI. Concluding Remarks

We have presented a dynamic traffic assignment

model and solution algorithm that consider multiple classes of travelers. In this paper, three traveler classes (fixed route, SDUO, and DUO) are classified according to different assumptions upon travelers' route choice behavior. The formulated model is a link-time-based VE. The algorithm's outer iteration is a relaxation procedure to approximate the travel time, and the inner iteration includes F-W and MSA.

A notable contribution is that we have proposed a new efficient algorithm to solve the analytical DTA models. Mathematically, we have shown that link flow (x) and exit-flow (v) can be solely represented by in-flow (u). With this realization, the LP subproblem in the inner iteration is proved as a typical time-dependent shortest route problem on a physical network. Traditionally, solving DTA models exploits an expanded time-space network. Nevertheless, by using the proposed new algorithm in this paper, the time-space network expansion

is no longer needed. The resulting complexity is thus greatly relieved, that increases the applicability of analytical DTA models in solving real world traffic problems.

<Table 4> Experienced Actual Travel Time Between An OD PairTime

Time Interval	Used Path (Node Sequence)	Route Time (Minutes)	Route Speed (MPH)
1	1->3->5->6->2	15.5	46.6
	1->3->5->7->2	15.7	46.0
	1->4->5->6->2	15.7	46.0
	1->4->5->7->2	15.8	45.4
2	1->3->5->6->2	16.7	43.1
	1->3->5->7->2	17.2	42.0
	1->4->5->6->2	17.1	42.0
	1->4->5->7->2	17.6	41.0
3	1->3->5->6->2	17.7	40.8
	1->3->5->7->2	17.4	41.3
	1->4->5->6->2	18.4	39.1
	1->4->5->7->2	18.2	39.5
4	1->3->5->6->2	18.3	39.4
	1->3->5->7->2	17.9	40.2
	1->4->5->6->2	19.7	36.6
	1->4->5->7->2	19.7	36.6
5	1->3->5->6->2	19.0	37.8
	1->3->5->7->2	19.0	37.9
	1->4->5->6->2	21.0	34.4
	1->4->5->7->2	21.3	33.8
6	1->3->5->6->2	20.3	35.4
	1->3->5->7->2	20.7	34.8
	1->4->5->6->2	20.8	34.6
	1->4->5->7->2	21.2	34.0
7	1->3->5->6->2	20.5	35.1
	1->3->5->7->2	20.5	35.1
	1->4->5->6->2	22.0	32.7
	1->4->5->7->2	22.4	32.1
8	1->3->5->6->2	21.6	33.4
	1->3->5->7->2	22.0	32.8
	1->4->5->6->2	22.2	32.5
	1->4->5->7->2	22.6	31.9
9	1->3->5->6->2	21.6	33.4
	1->3->5->7->2	21.4	33.6
	1->4->5->6->2	22.4	32.1
	1->4->5->7->2	22.2	32.4
10	1->3->5->6->2	21.4	33.7
	1->3->5->7->2	21.5	33.6
	1->4->5->6->2	22.3	32.3
	1->4->5->7->2	22.3	32.4

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♣ 주 작 성 자 : 신성일

♣ 논문투고일 : 2002. 11. 4

논문심사일 : 2002. 12. 20 (1차)

2003. 3. 3 (2차)

2003. 11. 17 (3차)

심사판정일 : 2003. 11. 17

♣ 반론접수기한 : 2004. 4. 30