한국지반환경공학회 논문집 제4권 제2호 2003년 6월 pp. 93~104

A Review of Porous Media Theory from Woltmans Work to Biots Work

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Abstract

Porous media consist of physically and chemically different materials and have an extremely complicated behavior due to the different material properties of each of its constituents. In addition, the internal structure of porous materials has generally a complex geometry that makes the description of its mechanical behavior quite complex. Thus, classical continuum mechanics cannot explain the behavior of materials with pore spaces, such as concrete, soils and organic materials in waste landfill. For these reasons, porous media theory has been developed in the nineteenth century.

Biot had the greatest influence on the development of porous media theory. Biot's work has been referred by many authors in the literature. Development of numerous fundamental equations in porous media theory were made possible due to Biot's work. His contributions made the greatest influence on porous media theory. Therefore, it is highly advantageous to review Biot's publications. This work presents a review of Biot's work. It shows how porous media theory has been developing so far and provides a chance to discuss the contribution of his work to the modern porous media theory.

Keywords : Porous media theory, constitutive modeling, Biot

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1. Introduction

Solid materials consist of a solid phase as well as closed and open pores, which are termed the internal structure. However, solid materials were generally treated as continuum materials without considering the internal structure in the past. Classical continuum mechanics can be explained successfully these continuum materials that consist of one-component material without an internal structure. However, real solid materials have to be considered as continuum materials with an internal structure. It therefore cannot be explained the behavior of solid materials concerning the different physical phenomena between the solid phase and the pore phase.

The behavior of solid materials that has empty pores may be easily explained in comparison with the behavior of materials that have open pores filled with fluids. The reason of being treated easily is that solid materials with empty pores have the same behavior when materials are loaded and deformed. However, solid materials with open pores filled with fluids usually have a relative velocity to each constituent due to the different material properties. Such solid materials consist of physically and chemically different materials. In addition, the internal structure generally has a complex geometry with a need for a much more complicated mechanical description of the behavior. Porous media theory can explain the mechanical behavior of porous media that are filled with fluids and are considered as an open continuous system. Porous media theory is modeling and predicting how porous media deform when it is subjected to various external agencies and physical phenomena containing deformation of the solid skeleton. Porous media theory incorporates continuum mechanics, thermodynamics, fluid mechanics, etc. In order to describe the complicated behavior of porous media, the corresponding porous media theory has been developed by Coussy (1995), de Boer (1996), and de Boer (2000).

De Boer (1996; 2000) emphasized the historical development and current state on porous media and comprehensively reviewed the vast literature. His investigations of the historical development of porous media theory consist of 'The Early Era', 'The Classical Era', 'The Modern Era' and 'Current State'. An extremely important porous media theory that has not been adequately addressed with yet is the coupling of the macroscopic and microscopic behavior in the porous media concerning volume fraction concept, balance equations, etc. Many authors have contributed in the development of the porous media theory. Among them, it was Maurice A. Biot who had the greatest impact on the development of the porous media theory. He published nearly over two hundred papers on the elasticity theory, soil mechanics and other fields of mechanics, in particular porous media theory. Biot's work has have been referred to by many authors in the literature. The developments concerning numerous fundamental equations in porous media theory were made possible due to Biot's work. Therefore, it is highly advantageous to review Biot's work (1941; 1954; 1955; 1956a; 1956b; 1956c; 1962; 1972; 1977 Biot and Willis, 1957) for understanding the porous media theory and applying it to the behavior of organic materials.

Waste landfill consists of organic materials which are porous media that have physically and chemically complicated behavior. In order to describe the behavior of waste landfill and predict its subsidence, the theory of porous media is essential. Constitutive modeling for large deformation behavior of organic materials will be developed in a forthcoming publication.

This work shows how porous media theory has been developed and in particular it presents a review of Biot's work regarding porous media.

Cartesian tensors are used in this work and the tensorial index notation is employed in all the equations.

2. Fundamental work

First contribution to the theory of porous media was made by Reinhard Woltman in 1794. It addressed the calculation of the earth pressure on retaining walls with the angle of internal friction and the treatment of partially saturated and fully saturated substances in water with the concept of the volume fraction. His primary contributions were given in two ways. Firstly, the introduction to the angle of internal friction that is an extremely important concept for the calculation of failure conditions in soil mechanics was given and the formula for the maximum earth pressure was derived by Woltman (1794). Although his results were checked and consequently believed that the formula was exact, it was later shown that the test values for various granular substances were much smaller than those obtained in his theory. After several years, a second earth pressure theory was developed. The test results showed that they were closer with the second theory than the previous one.

Secondly, the volume fraction concept was first introduced by Woltman to deal with a mixture, such as mud. It was an essential concept of the porous media theory. However, Woltman's concept of volume fractions related the volume element of only a single constituent to the whole volume element and his interesting studies on porous media with the volume fraction concept seemed not to be followed (de Boer, 2000).

Further important contributions were done by Achille Ernest Delesse (1848), Adolf Fick (1855), Henry-Philibert Darcy (1856) and Josef Stefan (1871) on the essential concepts of porous media theory, especially for the theory of mixtures.

Using Woltman's concept of volume fractions, the porous body that was composed of inhomogeneous materials could not be treated with the approach of continuum mechanics. For this reason, another development of volume fraction concept was necessary to evaluate the balance of momentum in the middle of the nineteenth century. Delesse (1848) made an important contribution to evaluate such a concept and arrived at the some conclusion. The conclusion was that the surface fractions were equal to the volume fractions under certain conditions and the concept of surface fractions was equivalent to the concept of volume fractions.

Delesse's important contribution was the basis of the development of a porous media theory in the early twentieth century (de Boer, 1996; 2000).

It was Fick (1855) who made the first attempt to explain the physical phenomenon of porous media. He was interested in diffusion problems in porous media, but there were only a few papers on this subject up to that time. He tried to explain the diffusion phenomenon in porous bodies through observing the simple expansion of a soluble body in a solvent, but did not succeed. However, Fick's differential equation was derived from Fourier equation of heat propagation where the rate of heat conduction in a direction is proportional to the temperature gradient in that direction. His differential equation is called Fick's diffusion law. He obtained

$$\frac{\partial J}{\partial t} = -\bar{k}\left(\frac{\partial^2 J}{\partial h^2} + \frac{1}{A}\frac{dA}{dh}\frac{\partial J}{\partial h}\right) \tag{1}$$

where J is the concentration in a container, t indicates the time, $\bar{k}$ is the proportionality constant, h is the height in a container and A is the cross-section area of a container.
In the case of a constant cross-section area, equation (1) is as follows

$$\frac{\partial J}{\partial t} = -\bar{k}\frac{\partial^2 J}{\partial h^2} \tag{2}$$

Equation (2) is known as Fick's diffusion law where the diffusion rate is proportional to the concentration gradient. This fundamental law for the diffusion process was experimentally proved by Fick (1855). His contribution to the porous media theory was the first attempt to develop a phenomenological mixture theory (Çengel and Boles, 1998; de Boer, 2000; Kaviany, 1995).

In 1856, with experimental observations, the relation between the total water quantity flowing through natural sand and the variation of pressure was obtained by Darcy (1856). Scheidegger (1963) reviewed Darcy's work and later de Boer (2000) quoted Scheidegger's explanation outlined below:

A liquid percolates through a homogeneous filter of height h, and rises to the height h_1 and h_2 respectively above an arbitrary level. With the results of various experiments, Darcy observed the relation that the total volume of percolating water is proportional to the loss of pressure. The total volume of fluid percolating in unit time, Q is given by

$$Q = -kA\frac{(h_2 - h_1)}{h} \tag{3}$$

where k is the permeability coefficient. This equation (3) is known as Darcy's law. The minus sign in the expression for Q indicates that the flow is in the opposite direction of increasing h.

Darcy's law could be restated in terms of the pressure p, the density ρ and the gravity acceleration g of the liquid. At the upper boundary of the filter denoted by z_2, the pressure is $p_2 = \rho g(h_2 - z_2)$, and at the lower boundary denoted by z_1, the pressure is $p_1 = \rho g(h_1 - z_1)$. Making use of the above and $z_2 - z_1 = h$ into equation (3), one obtains

$$Q = -kA\left[\frac{(p_2 - p_1)}{\rho g h} + 1\right] \tag{4}$$

or, upon introduction of a new constant k' (as $k = \rho g k'$), and assuming ρ and g to be constants, one obtains

$$Q = -k'A\frac{(p_2 - p_1 + \rho g h)}{h} \tag{5}$$

Equations (4) and (5) are equivalent statements of Darcy's law expressed by equation (3).

Although only experimental phenomena were treated, his results were essential to explain the motion of a liquid in a porous media. It was the first approach to deal with the interaction of different constituents in continuum materials where a liquid flows through porous solids (de Boer, 2000).

Another great contribution for the mixture theory in continuum mechanics was introduced by Stefan (1871). From his publication, it was clearly stated that a mixture of two gases should be considered not as a uniform body but as individual constituents in the mixture, and the equations of equilibrium and motion for each constituent are necessary. He also formulated the equilibrium equations and equations of motion for a mixture of two gases.

Through his work, the general equations that are valid for the diffusion of two gases were obtained and another treatment for a mixture of gases was accomplished. The treatment was based on the assumption that the particles of one gas were fixed. Different explanations of two gases with the mixture theory were also given. Thus, it was Stefan who applied for the first time the volume fraction concept with the continuum mechanical treatment (de Boer, 2000).

In the twentieth century, the fundamental mechanical approach in saturated porous solids was begun by Paul Fillunger and Karl von Terzaghi at the Technische Hochschule of Vienna, where they taught. The theory of porous media in the early twentieth century was highly influenced by Fillunger and von Terzaghi. The effects of uplift, friction, capillary and the effective stresses in liquid-saturated porous solids were explained. It was Fillunger (1913) who helped in the early development of the porous media theory of saturated porous solids. The uplift force problem in a saturated porous body such as dams was investigated and discussed in Fillunger's several publications (1913; 1929; 1930; 1934; 1935). Further consideration regarding the uplift problem was given by von Terzaghi (1933) and von Terzaghi and Rendulic (1934). It was von Terzaghi who developed the one-dimensional consolidation theory and approached the concept of effective stresses independently of Fillunger. Numerous experiments were made by von Terzaghi and Rendulic (1934) during 1933 and these experiments yielded the results that von Terzaghi expected. The research and description of effective stress in saturated rigid porous solids by Fillunger and von Terzaghi, were widely accepted in engineering (de Boer, 2000).

The first author to deal with the important problem of saturated deformable porous solids was von Terzaghi (1923). In 1923, a very interesting issue was investigated regarding the calculation of the permeability coefficient of clay. It was said that the permeability coefficient of clay was dependent on the water content, and the water content dependent on the pressure in the clay. Furthermore, making use of numerous tests in a thin clay layer, the partial differential equation describing the consolidation process was derived as follows:

$$\frac{k}{a}\frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} \tag{6}$$

In equation (6), "a" is denoted as the compression coefficient, "p" indicates the porewater overpressure and "z" denotes the depth. It was differential equation (6) that made von Terzaghi (1923) known to the world for his contributions in porous media.

Following the differential equation for the consolidation process, a book (von Terzaghi, 1925) containing his basic theories on soil mechanics was published. De Boer (2000) quoted von Terzaghi's statement (1932) that the relation between the strength properties of sands, clays and solids was recognized and the flow pressure played an important role in the failure of a deformable porous body such as barrages (de Boer, 2000).

Contrary to von Terzaghi's (1923) procedure, Fillunger's (1913; 1936) approach was equivalent to the mixture theory but was restricted by the volume fraction concept. His statement (1913) was that the poreliquid pressure did not have any influence on the strength of the porous solid and a constitutive equation had to be only formulated for the effective stress. Moreover, in his paper of 1936, the balance equations of momentum and mass for both porewater and porous soil for the one-dimensional consolidation were formulated. These remarkable equations were fundamental equations for modeling of materials to deal independently, for the liquid and solid phases. It was an excellent approach corresponding to the modern concept of the treatment of porous media.

Despite the fact that Fillunger's mathematical approach to the treatment of porous media was great, Fillunger's work was almost forgotten. The reason was that there was a big controversy between Fillunger's paper and von Terzaghi's paper due to Fillunger's misunderstanding of von Terzaghi's concepts (de Boer, 2000).

Fillunger's work was followed and continued by Gerhard Heinrich. In Heinrich's paper (1938), Fillunger's field equations in linear case were correctly simplified and the behavior of a saturated porous media in one-dimensional case was described by using Fillunger's fundamental equations. His investigations with Desoyer using Fillunger's concept were also extended to the consolidation problems in three-dimension (Heinrich and Desoyer, 1955; 1956; 1961). Subsequently, partial differential equations that described the consolidation problem were formulated and solved.

However, unfortunately, Heinrich's work has been nearly completely ignored and referred to by only a few authors (de Boer, 2000).

Following the controversy over Fillunger's investigations and von Terzaghi's investigations in the 1930s, further developments on the porous media theory were made. In particular, von Terzaghi's work was followed by Maurice A. Biot. Biot's important contributions made the greatest influence on the porous media theory.

3. Further developments: Biot's work

Soil consolidation was first proposed by von Terzaghi. The assumptions were that the particles in the soil constituted a porous material with elastic properties and the voids were filled with water. If a load was applied to this system, the consolidation was depended on the rate of water squeezed out of the voids. After using experiments where a soil column under a constant load was prevented from lateral expansion, the consolidation theory was created by von Terzaghi and it made the prediction possible for the settlement for soils. However, von Terzaghi's treatment was restricted to the one-dimensional problem of soil column which was prevented from lateral expansion. Biot extended the mathematical formulation of the physical soil properties to the three- dimensional case and established equations for any load with any time.

Table. 1 Brief History of Porous Media Theory

Author	Phblication year	Contribution to porous media theory
Reinhard Wlotman	1794	The earth pressure calculation with the angle of internal friction, First introduction of the volume fraction concept to deal with a mixture
Achille Ernest Delesse	1848	The concept of surface fractions, Influence on the beginning of the development of porous media theory
Adolf Fick	1855	First attempt to develop a phenomenological theory of mixture, Fick's law $\frac{\partial J}{\partial t} = -\bar{k}\frac{\partial^2 J}{\partial h^2}$
Henry-Philibert Darcy	1856	First approach to the interaction of different constituents in a multiphase continuum, Darcy's law $Q = -kA\frac{(h_2 - h_1)}{h}$
Josef Stefan	1871	Considering individual constituents in the mixture of gases, First application to the mixture theory restricted by the volume fraction concept with continuum mechanical treatment
Paul Fillunger	1913	Approach corresponding to the modern concept of the treatment of porous media, namely to the mixture theory restricted by the volume fraction concept
Karl von Terazghi	1923	First author to deal with the problem of saturated deformable porous solids, Development of one-dimensional consolidation theory, Greatest impact on the development of soil mechanics
Gerhard Heinrich	1938	Approach to the three-dimensional consolidation problem with Fillunger's fundamental concept
Maurice A.Biot	1941	Greatest influence on the development of porous media theory Approach to the three-dimensional consolidation problem with Terzaghi's fundamental concept

The following soil properties were assumed by Biot (1941)

i) isotropy of the material,

ii) reversibility of stress-strain relations under final equilibrium conditions,

iii) linearity of stress-strain relations,

iv) small strains,

v) the water contained in the pores is incompressible,

vi) the water may contain air bubbles,

vii) the water flows through the porous skeleton, according to Darcy's law.

A small cubic element of the consolidation soil and its sides being parallel with the coordinate axes are considered, while body forces are neglected. Using these assumptions, the equilibrium conditions of stress in the soil can be represented by

$$\sigma_{ij,j} = 0 \tag{7}$$

where σ_{ij} denotes the stress components and ',' denotes the partial derivative with respect to space.

The strain in the soil is assumed small, so that the strain components can be represented by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (8)$$

where ε_{ij} indicates the strain components and u_i denotes the displacement components of the soil.

The increment of pore water pressure p and the variation in water content W must be considered. First, suppose that increment of pore water pressure p is equal to zero. Hooke's law for an isotropic elastic body is given in this case as follows

$$\varepsilon_{ij} = \frac{1}{2G}\sigma_{ij} - \frac{\nu}{E}\sigma_{ij}\delta_{ij} \quad (9)$$

where G represents the shear modulus, E denotes Young's modulus, ν is Poisson's ratio and δ_{ij} is the Kronecker delta.

Suppose that the effect of the pore water pressure p exists. The pore water pressure p must have the same effect on ε_{11}, ε_{22} and ε_{33} because of isotropy. Hooke's law given by Equation (9) can now be written as follows

$$\varepsilon_{ij} = \frac{1}{2G}\sigma_{ij} - \frac{\nu}{E}\sigma_{ij}\delta_{ij} + \frac{p}{3H}\delta_{ij} \quad (10)$$

where H is a physical constant associated to the compressibility of the soil.

Equation (10)is a function of the stresses in the soil particle and the water pressure in the pores.

In order to express the stresses as functions of the strain and the pore water pressure p, equation (10) can be expressed as follows

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} - \alpha\delta_{ij} \quad (11)$$

where the constant α is defined by Biot (1941) as follows

$$\alpha = \frac{2(1+\nu)}{3(1-2\nu)}\frac{G}{H} \quad (12)$$

The constants μ and λ are termed the Lamé constants.

The constant α measures the ratio of the squeezed water volume to the volume of the soil if the soil is compressed while the increment of pore water pressure p is maintained zero.

Next, the increment of the water content θ is introduced. θ is generally denoted by

$$\theta = c_{ij}\sigma_{ij} + Rp \quad (13)$$

where c_{ij} and R denote physical constants associated to the change in water content. Due to the assumption of isotropy, the effect of shear stress does not exist and three normal stresses are equivalent to hydrostatic stresses. Therefore, the increment of water content θ can be represented by

$$\theta = \frac{1}{3H}\sigma_{ii} + Rp \quad (14)$$

Substituting equation (11) into equation (14), one obtains

$$\theta = \alpha\varepsilon_{ii} + \beta p \quad (15)$$

where β is defined by

$$\beta = R - \frac{\alpha}{H} \quad (16)$$

$1/H$ can be interpreted a measure of the compressibility of the soil for a change in water pressure and β measures the amount of water such that the volume of the soil under pressure is kept constant (Biot, 1941).

In order to obtain the general equations governing consolidation, equations (8) and (11) are substituted into the equilibrium condition (7), such that

$$\mu\nabla^2 u_i + (\lambda + \mu)\,\bar{\varepsilon}_{,i} - \alpha\, p_{,i} = 0 \quad (17)$$

where ∇^2 is denoted by

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \quad (18)$$

and $\bar{\varepsilon}$ is equal to be ε_{ii}.

In order to complete the general equation for consolidation, Darcy's law for the flow of water in soil is introduced by

$$V_i = -k\, p_{,i} \quad (19)$$

where V_i denotes the volume of water flowing per second and per unit area through the faces of a soil cube element. Moreover, it is assumed that the water is incompressible, so that the rate of water content of a soil cube element is equal to the volume of water entering per second through the surface of the soil cube element. Hence, one obtains

$$\dot{\theta} = -V_{i,i} \quad (20)$$

where a dot over symbols denotes the partial time derivative. Combining equations (15), (19) and (20), the differential equation can be obtained as follows

$$k\nabla^2 p = \alpha\dot{\varepsilon}_{ii} + \beta\dot{p} \quad (21)$$

Equation (21) is an extended formulation of von Terzaghi's equation (6) to the three-dimensional case.

Equations (17) and (21) are the differential equations for the transient phenomenon of consolidation obtained by Biot (1941).

4. Current work on porous media

Great effort in elasticity theory, plasticity theory, continuum mechanics and mixture theory has been put for the development of the porous media theory.

In principle, the assumption of the theory of mixtures is not valid for the real saturated solid skeleton because the solid and the liquid phases are immiscible in reality.

For this reason, theories of immiscible mixtures have been developed, in particular in the 1970s and 1980s. Incompressible porous media models were formulated by Bowen (1980) with the use of the thermodynamics of mixtures. They were considered as mixtures where the solid and the fluid constituents were each incompressible (Bowen, 1980).

Bowen (1982) extended his porous media theory to compressible porous solids with compressible immiscible fluids. The formulation of the mixture theory for compressible porous media models allowed immiscibility and also variable volume fractions to be considered. In addition, the linear poroelasticity model that resulted from linearization of the constitutive equations was derived. The constitutive equations for constant porosity models proposed by Biot (1955; 1956a; 1956b; 1956c) and Biot and Willis (1957) were shown to be formally equivalent to Bowen's work (Bowen, 1982).

Within modern porous media theory, porous media can be treated as immiscible mixtures of the solid and the fluid constituents. It also can be treated with continuum mechanics and mixture theory. For each of its constituents, the kinematics and the balance equations for compressible and incompressible porous media were described by de Boer (1996, 2000).

In addition, constitutive equations for compressible and incompressible elastic and plastic porous solids with compressible and incompressible fluids were derived by de Boer (2000). These equations will be introduced and reformulated in a forthcoming paper.

Currently, several problems for transport phenomena in porous media and the application to practical problems are main interesting point and these problems are under study (de Boer, 2000).

5. Conclusion

Many works by many authors have influenced on the development of porous media theory. Woltman (1794) and Delesse (1848) had the influence on the beginning of the development of porous media theory. Fick (1855) made the first attempt to develop a phenomenological theory of porous media. In addition, Darcy (1856) dealt the first approach to the interaction of different constituents in a multiphase continuum. Stefan (1871) applied the mixture theory restricted by the concept of volume fraction with continuum mechanical treatment. Following them, Fillunger (1913) and Heinrich (1938) made an excellent approach corresponding to the modern concept of the treatment of porous media.

Moreover, Terzaghi (1923) developed the one-dimensional consolidation theory and made the greatest impact on the development of soil mechanics. Biot extended the consolidation problem to the three-dimensional case with Terzaghi's concept and his work has been referred by many authors in the literature. Biot had the greatest influence on the development of porous media theory. Therefore, it is advantageous to review Biot's works for the establishment of framework of modern porous media theory. More details in modern porous media theory will be reviewed in a forthcoming paper.

Porous media theory is suitable to deal with the problems of nonhomogeneous and anisotropic porous solids with fluids. The representative problem of nonhomogeneous and anisotropic porous solids with fluids is to research on the behavior of porous media containing organic materials. Since porous media containing organic materials behave bio-chemo-thermo-mechanically, the analysis which is considered coupling effects is essentially needed. Thus, porous media theory may be a powerful method to describe and clarify the problems of porous solids with fluids.

ACKNOWLEDGEMENTS

This work was supported by grant No. R01-2002-000-00063-0(2002) from the Basic Research Program of the Korea Science & Engineering Foundation.

(접수일자 : 2003년 2월 11일)

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