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Ultrasonic Measurement of Interfacial Layer Thickness of Sub-Quarter-Wavelength

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Abstract This paper describes a new technique for thickness measurement of a very thin layer less than one-quarter of the wavelength of ultrasonic wave used in the ultrasonic pulse-echo measurements. The technique determines the thickness of a thin layer in a layered medium from constructive interference of multiple reflection waves. The interference characteristics are derived and investigated in theoretical and experimental approaches. Modified total reflection wave g(t) defined as difference between total and first reflection waves increases in amplitude as the interfacial layer thickness decreases down to zero. A layer thickness less than one-tenth of the ultrasonic wavelength is measured using the maximum amplitude of g(t) with a good accuracy and sensitivity. The method also requires no inversion process to extract the thickness information from the waveforms of reflected waves, so that it makes possible to have the on-line thickness measurement of a thin layer such as a lubricating oil film in thrust bearings and journal bearings during manufacturing process.

Keywords: thickness measurement, ultrasonic pulse echo method, thin layer, thrust bearing

1. Introduction

Two factors affecting mechanical energy efficiency of rotary compressors are leakage and friction loss during the process of compression. These losses take place at small clearance gaps of sliding parts in trust bearings and journal bearings of a compression chamber. In order to maximize the efficiency of compressor, the gap clearances of bearings should be managed properly. Mathematical model for trust bearing in rotary compressors can be simply explained by one-dimensional layered structure as shown in Fig. 1, where a thin oil film seals the gap between the first and third solid layers made of the almost same material. Many studies have been done and reported on the measurement of layer thickness including a relatively thin elastic film. Kinra (1988) and Zhu (1992) have measured successfully the layer thickness of a sub-half-wavelength layer based on a time-domain multiple reflection model using the error minimization technique. Iyer (1991) proposed a thickness measurement technique for a sub-wavelength adhesive layer down to 0.1 λ (wavelength) using frequency-domain analysis. A practical novel approach for the thickness measurement was introduced and demonstrated in experiment by Rose (1974) for a thin fluid film enclosed in two plexi-glass plates. He showed that the total amplitude of the reflection waves from the film had a strong dependency on the film thickness especially below the half-wavelength of ultrasound. Although the theoretical proof was not provided in full, the concept was very simple and fast without any needs for time-consuming inversion process because it provided a possibility of measuring the thin film layer directly from the

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amplitude of the reflection wave. However, the SNR (signal-to-noise) ratio decrease very much as the layer gets thin, which leads to the loss of the sensitivity of the reflection waves to the layer thickness. It is mainly because of the destructive interference of multiply superposed reflection waves from the interfaces on both sides of layers.

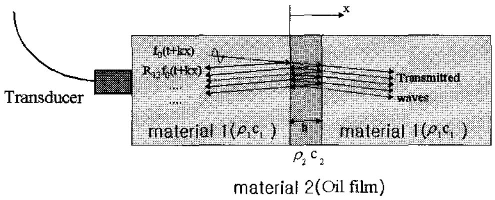

Fig. 1 Multiple reflection waves in a thin fluid film of layered structure

The methods listed above are not applicable to thrust bearings of compressors due to three practical reasons. The most significant problem is that the thickness of oil film is very thin ranging from 10-20μm down to near zero (metal contact). The oil film is also located relatively deep inside the compressor cylinder and contains a micro bubbles (refrigerant gas), so that a low frequency ultrasound should be used for the long penetration depth with a low scattering. One more additional problem for in-situ application of ultrasonic inspection is that the measurement should be completed very fastly, almost in real time, to inspect compressors during the assembling process for the quality control in factory. Otherwise, techniques cannot be adopted in industry. This paper analyzes the interference characteristics of multiple reflection waves occurring in a thin film and introduces a new technique to measure a very thin layer less than 0.1 λ using a relatively low frequency ultrasonic waves without inversion process.

2. Theory

From the mathematical model described in Fig. 1, a total reflection wave f(t+kx) generated from the flat interfaces is expressed by multiple reflections of a normal incident wave $F_0(t+kx)$ as (Kinra and Dayal, 1988)

$$f(t+kx) = R_{12}f_0(t+kx) + \frac{T_{12}T_{21}}{R_{21}}\sum_{m=1}^{\infty} R_{21}^{2m} f_0(t+kx-2msh) \tag{1}$$

where s is the slowness of the wave in the oil film, t and k are time and wave number, R_{12} and T_{12} are reflection and transmission coefficients from the material 1 (compressor cylinder and piston, $\rho_1 c_1$) to the material 2 (oil film, $\rho_2 c_2$), respectively. Defining g(t+kx) as a difference between the total reflected signal f(t+kx) and the reflected signal $R_{12}f_0(t+kx)$ from the first interface between the material 1 and 2, it is expressed from eqn. (1) as

$$\begin{aligned} g(t+kx) &= -[f(t+kx) - R_{12}f_0(t+kx)] \\ &= \frac{T_{12}T_{21}}{R_{12}}\sum_{m=1}^{\infty} R_{12}^{2m} f_0(t+kx-2msh) \end{aligned} \tag{2}$$

At the first boundary surface (x=0), g(t) can be represented using the Fourier series of by

$$\begin{aligned} g(t) &= \frac{T_{12}T_{21}}{R_{12}}\sum_{m=1}^{\infty} R_{12}^{2m} f_0(t-2msh) \\ &= \frac{T_{12}T_{21}}{R_{12}}\sum_{m=1}^{\infty} R_{12}^{2m} \cdot \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos[n\omega(t-2msh)] \right. \\ &\quad \left. + \sum_{n=1}^{\infty} b_n \sin[n\omega(t-2msh)]\right] \end{aligned} \tag{3}$$

where

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f_0(t)\cos(n\omega t)dt, \quad b_n = \frac{1}{\pi}\int_0^{2\pi} f_0(t)\sin(n\omega t)dt,$$

and ω is the fundamental angular velocity. Changing the order of summation in eqn. (3), it becomes

$$\begin{aligned} g(t) &= \frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\sum_{m=1}^{\infty} R_{12}^{2m} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} R_{12}^{2m} a_n \cos[n\omega(t-2msh)] \right. \\ &\quad \left. + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} R_{12}^{2m} b_n \sin[n\omega(t-2msh)]\right] \\ &= \frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\sum_{m=1}^{\infty} R_{12}^{2m} + \sum_{n=1}^{\infty} a_n \sum_{m=1}^{\infty} R_{12}^{2m}\cos[n\omega(t-2msh)] \right. \\ &\quad \left. + \sum_{n=1}^{\infty} b_n \sum_{m=1}^{\infty} R_{12}^{2m}\sin[n\omega(t-2msh)]\right] \end{aligned} \tag{4}$$

For the convenience of calculation, eqn. (4) can be rewritten by the complex notation of the trigonometric functions and rearranged as

$$g(t)=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\sum_{m=1}^{\infty}R_{12}^{2m}+\sum_{n=1}^{\infty}\frac{a_n}{2}\sum_{m=1}^{\infty}R_{12}^{2m}[e^{in\omega(t-2msh)}+e^{-in\omega(t-2msh)}]+\sum_{n=1}^{\infty}\frac{b_n}{2i}\sum_{m=1}^{\infty}R_{12}^{2m}[e^{in\omega(t-2msh)}-e^{in\omega(t-2msh)}]\right]$$

$$=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\sum_{m=1}^{\infty}R_{12}^{2m}+\sum_{n=1}^{\infty}\frac{a_n}{2}\{e^{in\omega t}\sum_{m=1}^{\infty}R_{12}^{2m}e^{-i(2n\omega sh)m}+e^{-in\omega t}\sum_{m=1}^{\infty}R_{12}^{2m}e^{i(2n\omega sh)m}\}+\sum_{n=1}^{\infty}\frac{b_n}{2i}\{e^{in\omega t}\sum_{m=1}^{\infty}R_{12}^{2m}e^{-i(2n\omega sh)m}-e^{-in\omega t}\sum_{m=1}^{\infty}R_{12}^{2m}e^{i(2n\omega sh)m}\}\right]$$

$$=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\sum_{m=1}^{\infty}R_{12}^{2m}+\sum_{n=1}^{\infty}\frac{a_n}{2}\{e^{in\omega t}\sum_{m=1}^{\infty}[R_{12}^{2}e^{-i(2n\omega sh)}]^m+e^{-in\omega t}\sum_{m=1}^{\infty}[R_{12}^{2}e^{i(2n\omega sh)}]^m\}+\sum_{n=1}^{\infty}\frac{b_n}{2i}\{e^{in\omega t}\sum_{m=1}^{\infty}[R_{12}^{2}e^{-i(2n\omega sh)}]^m-e^{-in\omega t}\sum_{m=1}^{\infty}[R_{12}^{2}e^{i(2n\omega sh)}]^m\}\right] \quad (5)$$

From the simple mathematical result, $\sum_{k=0}^{\infty}z^k=1+z+z^2+z^3+.....=(1-z)^{-1}$ when $|z|<1$,

$$\sum_{m=1}^{\infty}[R_{12}^{2}e^{-i(2n\omega sh)}]^m=\frac{R_{12}^{2}e^{-i(2n\omega sh)}}{1-R_{12}^{2}e^{-i(2n\omega sh)}}$$

Then eqn. (5) may be rewritten using this expression by

$$g(t)=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\left(\frac{R_{12}^2}{1-R_{12}^2}\right)+\sum_{n=1}^{\infty}\frac{a_n}{2}\left\{\frac{R_{12}^2e^{in\omega(t-2sh)}}{1-R_{12}^2e^{-i(2n\omega sh)}}+\frac{R_{12}^2e^{-in\omega(t-2sh)}}{1-R_{12}^2e^{i(2n\omega sh)}}\right\}+\sum_{n=1}^{\infty}\frac{b_n}{2i}\left\{\frac{R_{12}^2e^{in\omega(t-2sh)}}{1-R_{12}^2e^{-i(2n\omega sh)}}-\frac{R_{12}^2e^{-in\omega(t-2sh)}}{1-R_{12}^2e^{i(2n\omega sh)}}\right\}\right]$$

$$=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\left(\frac{R_{12}^2}{1-R_{12}^2}\right)+\sum_{n=1}^{\infty}\frac{a_n}{2}(w+\bar{w})+\sum_{n=1}^{\infty}\frac{b_n}{2i}(w-\bar{w})\right] \quad (6)$$

where the complex $w=\frac{R_{12}^2e^{in\omega(t-2sh)}}{1-R_{12}^2e^{-i(2n\omega sh)}}$ and $\bar{w}$ is the complex conjugate of w. Further calculation for $\frac{(w+\bar{w})}{2}$ and $\frac{(w-\bar{w})}{2i}$ leads to

$$\frac{w+\bar{w}}{2}=\frac{R_{12}^2\{\cos[n\omega(t-2sh)]-R_{12}^2\cos(n\omega t)\}}{1+R_{12}^4-2R_{12}^2\cos(2n\omega sh)}=\alpha_n R_{12}^2\{\cos[n\omega(t-2sh)]-R_{12}^2\cos(n\omega t)\}$$

and (7)

$$\frac{w-\bar{w}}{2i}=\frac{R_{12}^2\{\cos[n\omega(t-2sh)]-R_{12}^2\cos(n\omega t)\}}{1+R_{12}^4-2R_{12}^2\cos(2n\omega sh)}=\alpha_n R_{12}^2\{\sin[n\omega(t-2sh)]-R_{12}^2\sin(n\omega t)\}$$

where,

$$\alpha_n=\frac{1}{1+R_{12}^4-2R_{12}^2\cos(2n\omega sh)}=\frac{1}{1+R_{12}^4-2R_{12}^2\cos(4\pi f_n sh)}$$

$$\text{or}\quad \frac{1}{1+R_{12}^4-2R_{12}^2\cos(4\pi\frac{h}{\lambda_n})} \quad (8)$$

In eqn. (7) λ_n and f_n are the wavelength and the frequency of a harmonic wave of angular velocity $n\omega$, i.e., $\frac{1}{\lambda_n}=sf_n=2\pi\cdot n\omega\cdot s$ (Achenbach 1984). Substituting these expressions of eqns. (7) and (8) into eqn. (6) gives

$$g(t)=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\left(\frac{R_{12}^2}{1-R_{12}^2}\right)+R_{12}^2\sum_{n=1}^{\infty}\alpha_n a_n\{\cos[n\omega(t-2sh)]-R_{12}^2\cos(n\omega t)\}+R_{12}^2\sum_{n=1}^{\infty}\alpha_n b_n\{\sin[n\omega(t-2sh)]-R_{12}^2\sin(n\omega t)\}\right]$$

$$=\frac{T_{12}T_{21}}{R_{12}}\left[\frac{a_0}{2}\left(\frac{R_{12}^2}{1-R_{12}^2}\right)+R_{12}^2\sum_{n=1}^{\infty}\alpha_n\{a_n\cos[n\omega(t-2sh)]+b_n\sin[n\omega(t-2sh)]\}-R_{12}^4\sum_{n=1}^{\infty}\alpha_n\{a_n\cos(n\omega t)+b_n\sin(n\omega t)\}\right] \quad (9)$$

In case of no DC component in the incident waves which means= 0, eqn. (9) reduces to

$$g(t)=R_{12}T_{12}T_{21}\left[\sum_{n=1}^{\infty}\alpha_n\{a_n\cos[n\omega(t-2sh)]+b_n\sin[n\omega(t-2sh)]\}-R_{12}^2\sum_{n=1}^{\infty}\alpha_n\{a_n\cos(n\omega t)+b_n\sin(n\omega t)\}\right] \quad (10)$$

Using the Fourier series, the above equation can be expressed as a compact form, i.e. ,

$$g(t)=R_{12}T_{12}T_{21}\{p(t-2sh)-R_{12}^2p(t)\} \quad (11)$$

where $p(t)=\sum_{n=1}^{\infty}\alpha_n\{a_n\cos(n\omega t)+b_n\sin(n\omega t)\}$. The expression for g(t) in eqn. (11) gives a simple and compact mathematical description for the characteristics of the constructive interference of the infinite reflection waves produced from a thin layer. As can be seen from eqn. (11), the film thickness h may be determined implicitly from the signals g(t) and $f_0(t)$ which are easily obtained from the experiments. Comparing with the expression for g(t) in eqn. (3), it is clear that eqn. (11) provides a fast and accurate way to find the thickness h. Moreover it can be simplified to more compact expression if the incident wave $f_0(t)$ is a narrow-band ultrasonic signal with a dominant frequency component near the center frequency f_c. Under this assumption, the coefficients a_n and b_n of p(t) in eqn. (11) are all zero except near the center frequency $f_c = n_c\omega$ and the coefficient α_n contributes only around the center frequency accordingly (Whitehouse 1997). Also α_n becomes a constant ac if $\frac{h}{\lambda} << 1$ since $\cos(4\pi \frac{h}{\lambda_n}) \approx 1$ in eqn. (7). Therefore the signal p(t) in eqn. (11) becomes

$$p(t) = \sum_{n=1}^{\infty} \alpha_n \{a_n \cos(n\omega t) + b_n \sin(n\omega t)\} \approx \alpha_c \sum_{n=1}^{\infty} \{a_n \cos(n\omega t) + b_n \sin(n\omega t)\} = \frac{1}{1 + R_{12}^4 - 2R_{12}^2 \cos(4\pi f_c sh)} \cdot f_0(t) \quad (12)$$

Substituting the expression into eqn. (11) yields a simplified form of g(t) as

$$g(t) = \frac{R_{12} T_{12} T_{21}}{1 + R_{12}^4 - 2R_{12}^2 \cos(4\pi f_c \cdot sh)} \cdot [f_0(t - 2sh) - R_{12}^2 f_0(t)] \quad (13)$$

Since the layer thickness h is much smaller than the wavelength λ, eqn. (13) may be approximated more by the Taylor expansion as

$$g(t) \approx \frac{R_{12} T_{12} T_{21}}{1 + R_{12}^4 - 2R_{12}^2 \cos(4\pi f_c \cdot sh)} \cdot [(1 - R_{12}^2) f_0(t) - 2sh \cdot f_0'(t)]$$

$$\text{for } \frac{h}{\lambda} << 1 \quad (14)$$

Eqn. (13) is an implicit mathematical expression for the thickness h of a thin layer shown in Fig. 1 and eqn. (14) gives an approximation for the layer thickness of the layer. The property of g(t) is described in Fig. 2, where four waveforms of g(t) for different film thicknesses are shown for comparison. In Fig. 2, the maximum amplitude (absolute peak value) of g(t) decreases monotonically as the thickness h increases from 0 μm (physical contact) to 40 μm in thickness. A more precise correlation between the maximum amplitude of g(t) and the thickness h represented by eqns. (13) and (14) is shown in Fig. 3, where the notation |g(t)| represents the maximum absolute peak value of g(t). In the computer simulation of Fig. 3, a reflected wave of 5MHz frequency from an interface boundary (steel to refrigerant oil) is used as the incident wave $f_0(t)$ to calculate the waveform of g(t) in the figure. The figure is normalized by the first reflection wave $R_{12}f_0(t)$ and the wavelength of ultrasound in oil is about 250 μm. In Fig. 3, a very good sensitivity of g(t) to the thickness h is clearly observed in the region below 20 μm in film thickness which is about one-tenth of wavelength. It describes that the thickness h is easily calculated from the chart or the mathematical relationship given in eqn. (13) or (14) by simply measuring the peak amplitude of g(t). This technique eliminates a long computation work needed in conventional techniques for error minimization and reduces a data processing time remarkably. It also makes possible on-line precision measurements of thin layers for the inspection of compressor assembly because the measurement of g(t) is performed in real time.

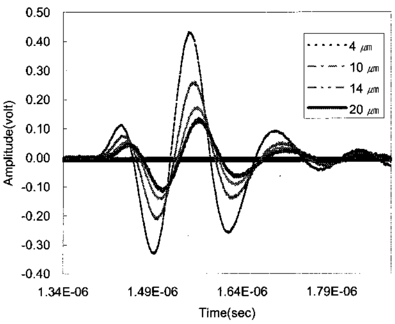

Fig. 2 Amplitude variations of $g(t)$ for thin oil films

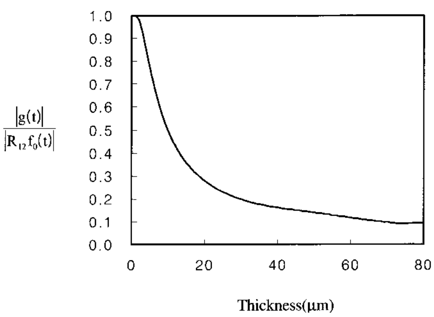

Fig. 3 Relationship between $|g(t)|$ and oil film thickness

3. Experiment and Results

An experimental set-up for the measurement of oil film thickness is shown in Fig.4. A moving plate of one-inch thickness is mounted at the end of the micrometer that controls the thickness of the oil gap between two identical steel plates. On the backside of the moving plate a 5MHz ultrasonic transducer is attached to send and receive ultrasonic signals. The oil film is set 1mm in thickness at first to separate multiple reflection waves from two boundary surfaces and measure the first reflection wave $R_{12}f_0(t)$. Then the film thickness is decreased successively from 80 μm down to zero (contact) while measuring the total reflection wave f(t) from the oil layer at each position. The signal g(t) of our interest is calculated simply by subtracting $R_{12}f_0(t)$ from the total reflection wave f(t) in time domain. Results are shown in Fig.5, where the surface roughness of the plates is less than 1 μm and acoustic impedances of the plate and oil are 36.9×10^6kg/m^{2}s and 1.28×10^6kg/m^{2}s respectively. Once the magnitude of g(t) is measured, the corresponding oil film thickness is calculated directly from eqn. (13) or (14) without any further data processing and calculations. A good agreement is found between theoretical and experimental results in Fig. 5. Especially an excellent sensitivity is observed in the region below 20 μm (about 0.1 λ in wavelength). From the experiments, the oil film thickness up to 1 μm can be determined easily from eqn. (13) or (14) within the measurement error of about 4%.

5. Discussions

The proposed technique in this paper has several advantages in terms of accuracy and speed over the conventional thickness measurement techniques that require an ultra-high frequency ultrasound or a heavy computational work. Simplicity of this technique allows us to apply this technique to automatic on-line inspection of a very thin layer (oil film) because the measurement can be done very fast. Only the reflected wave g(t) from the layer is required to determine the thickness of the thin layer using the expression of eqn. (13). The most important feature of this method is that a very thin layer less than 0.1 λ can be measured easily with high accuracy using a relatively low frequency transducer. This capability is very critical to measure the thickness of a thin layer deep inside the surface of test specimen such as thrust bearings in rotary compressors that are usually located inside the thick cylinder wall made of cast iron. It should be also mentioned that the waviness and the roughness of the boundary surface affect the measurement accuracy in the cases that the layer thickness is as small as the surface roughness. Actually it measures a "mean thickness" of the oil film that is an effective average distance between two interfaces (solid plates) in Fig. 1 and Fig. 4. It is the reason why the measured thickness of oil film is not zero even when two plates are put together completely in the experiment. The insensitive region to the layer thickness appearing for a relatively thick layer over one-tenth of wavelength in Fig. 5 is mainly produced by the narrow-band characteristic of the incident wave $f_0(t)$. Therefore the use of lower frequency or/and wide-band ultrasound can increase the range of high sensitivity.

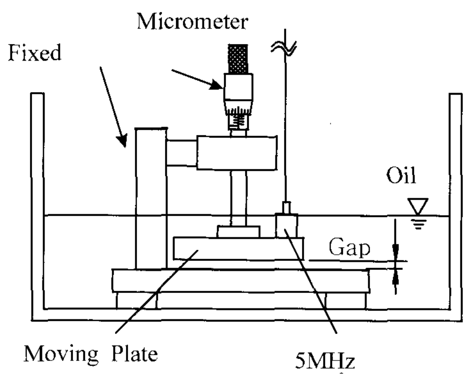

Fig. 4 Experimental set-up for the thickness measurement of oil film

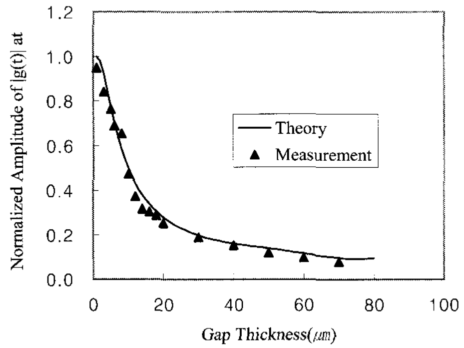

Fig. 5 Experimental results for various film thickness measurements

5. Conclusions

A new ultrasonic method with excellent sensitivity to a very thin layer less than 0.1 λ was suggested and tested to investigate the possibility for on-line measurement of the oil film thickness in trust bearings of rotary compressors. Mathematical derivation for the technique was made with discussion on the characteristics of the constructive interference of multiple reflection waves. The oil films ranging from 1 μm to 80 μm in thickness were measured successfully with the maximum error of 4% using a low frequency transducer (5MHz). A very good agreement between the theoretical and experimental results is found and an excellent sensitivity of the reflection wave g(t) to the layer thickness less than one-tenth of wavelength is also observed. Experimental results showed that the thickness measurement technique described in this paper can be a precise and fast tool for the measurement of a very thin layer in layered structures including trust bearings and journal bearings of rotary compressors.

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