Use of the Centroid Method to Estimate Volumes of Japanese Red Cedar Trees in Southern Korea

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Abstract :Cubic-meter volumes estimated from two proxy taper functions were compared to observed volumes of Japanese red cedar trees (*Cryptomeria japonica* D. Don) to evaluate accuracy and precision in the centroid method. Centroid volume estimates were also compared to volume estimates from existing whole-tree volume equations developed for another geographic region. This study found that one proxy function produced unbiased volume estimates while the other was biased. Volume estimates from the whole-tree equations were also biased. However, the volume estimates from the whole-tree equations were more precise than those from the centroid method. These results support previous studies that the centroid method can produce reliable volumes of trees when no other reliable volume equations exist.

Key words : Centroid method, Importance sampling, Max-Burkhart, Taper function, Tree volume

INTRODUCTION

Japanese red cedar tree (*Cryptomeria japonica* D. Don) is one of the most important tree species in terms of high-value wood products in Japan and Southern Korea. The majority of this species were planted for commercial plantations in the 1920s throughout the southern regions of South Korea (Forestry Administration of Korea 1999). Therefore, Japanese red cedar stands are even-aged and intensively managed. However, reliable equations to estimate the volumes of Japanese red cedar trees are currently unavailable in the southern regions of Korea.

Several researchers (Wood *et al.* 1990, Wood and Wiant 1992, Wiant *et al.* 1992) recommended a method to estimate the volumes of tree stems that is useful when other volume equations are unavailable. The centroid method (Wiant *et al.* 1996, Wood *et al.* 1990), derived from importance sampling (Gregoire *et al.* 1986), utilizes proxy taper functions to estimate the volume of a tree stem. Other studies (Coble and Wiant 2000, Wiant *et al.* 1996, Patterson *et al.* 1993, Wood and Wiant 1992, 1990) have shown that the centroid method provides reliable volume estimates of tree stems. In this study, we demonstrate that the centroid method, using two separate proxy taper functions from Coble and Wiant (2000), accurately predicts the stem volumes of Japanese red cedar trees in southern Korea.

MATERIALS AND METHODS

Study areas and data sources

Data were recently collected from Japanese red cedar plantations established during the 1920s throughout the southern regions in South Korea. A total of 32 representative temporary 0.04 hectare sample plots (dimensions: 20m×20m) were installed in even-aged stands throughout the six different southern regions (Gangjin, Jangheung, Jangseong, Namhae, Suncheon and Yangsan) of South Korea (Table 1). The diameter outside bark at breast height (dbh, nearest 0.1 cm) and the total height (nearest 0.1 meter) of all trees were measured in each sample plot. One dominant standard tree was selected and carefully felled for stem analysis from each sample plot. The diameter outside and inside barks (nearest 0.1 cm) were measured on each felled tree.

Table 1. Summary of observed statistics for Japanese red cedar plantations in southern Korea

Regions	No. of plots	Age(years)	DBH(cm)	Height(m)
Gangjin	7	71(24~85)	29.9(21.7~34.5)	16.0(13.4~17.7)
Jangheung	5	36(33~39)	37.1(32.6~41.5)	19.6(17.5~21.5)
Jangseong	7	53(33~84)	29.2(22.7~33.9)	18.9(15.5~21.7)
Namhae	5	37(36~38)	25.8(18.7~30.9)	15.7(13.4~16.9)
Suncheon	5	34(31~39)	27.7(22.9~33.4)	16.5(15.2~18.2)
Yangsan	3	26(24~27)	11.4(8.9~14.1)	10.9(9.1~12.0)

Note: values are means with ranges in parentheses.

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Centroid method

The centroid method (Wiant *et a*l. 1996, Wood *et al.* 1990) utilizes proxy taper functions to estimate the volume of a tree stem. The proxy taper functions are first integrated via calculus to derive proxy volume equations. Next, the centroid height is estimated from the proxy taper functions. The centroid is the height at which half the total volume is above and the other half is below this point (Williams and Wiant 1998, Wood *et al.* 1990). The cross-sectional area (inside bark) is then calculated at the estimated centroid height as well as the actual centroid height. The actual centroid height is approximated by interpolation from closely spaced bole measurements on either side of the centroid position. This value is assumed to be the true centroid height because the true centroid cannot be determined in practice except using water displacement techniques on felled-tree data. The two cross-sectional areas, estimated and actual, are then used in the proxy volume equations to estimate the volume of the stem.

Proxy functions

A simple taper function (Gregoire *et al.* 1986) was used as one of the proxy taper functions in this study:

$$[1] \quad A(h) = A(b)\left(\frac{H-h}{H-b}\right),$$

where: $A(h)$ = cross-sectional area (m^2) at h

$A(b)$ = cross-sectional area (m^2) at b

h = upper-stem height (m), $H_L < h < H_U$

b = 1.2 meters

H = total height (m)

H_L = lower-stem (i.e., stump) height (m)

H_U = upper-stem (i.e., merchantable) height (m).

Equation [1] was integrated between H_U and H_L to derive a volume equation (Patterson *et al.* 1993, Wiant *et al.* 1992, Gregoire *et al.* 1986) that was used as a simple proxy function to estimate stem volume via the centroid method:

$$[2] \quad V_{SIMPLE} = \left(\frac{k}{2}\right) \times \left(\frac{A_{SIMPLE}}{H - H_{SC}}\right)$$

where: $\mathbf{k = 2H(H_U - H_L) + (H_L^2 - H_U^2)}$

$\mathbf{H_{SC} = H - ((H - H_L)^2 - 0.5k)^{0.5}}$ = centroid height (m)

$A_{SIMPLE} = (f \times D_{SC})$=cross-sectional area ($m^2$) at the centroid position

$$f = \frac{\pi}{4 \times 10{,}000}$$

D_{SC} = interpolated diameter (cm) from closely spaced bole measurements on either side of the centroid position.

All other variables are defined as before.

Note that b in equation [1] was set to be the centroid height, Hsc, in equation [2].

The Max-Burkhart taper function (Max and Burkhart 1976) was used as the other proxy taper function in this study:

$$[3] \quad \left(\frac{d}{D}\right)^2 = b_1\left(\frac{h}{H} - 1\right) + b_2\left(\left(\frac{h}{H}\right)^2 - 1\right) + b_3\left(a_1 - \frac{h}{H}\right)^2 I_1 + b_4\left(a_2 - \frac{h}{H}\right)^2 I_2$$

where: D = diameter (cm) at breast height (dbh, 1.2 meters).

I_1 = 1, h $\le$ a_1; = 0, otherwise

I_2 = 1, h $\le$ a_2; = 0, otherwise

b_i = regression coefficients; i = 1, 2, 3, 4 (Table 2)

a_i = inflection points; i = 1, 2 (Table 2).

All other variables are defined as before.

Equation [3] was integrated between H_U and H_L to derive a volume equation that was used as a complex proxy volume equation to estimate the stem volume via the centroid method (Coble and Wiant 2000):

$$[4] \quad P_{COMPLEX} = fD^2H\begin{bmatrix} \frac{b_1}{2}(h_U^2 - h_L^2) + \frac{b_2}{3}(h_U^3 - h_L^3) - (b_1 + b_2)(h_U - h_L) \\ -\frac{b_3}{3}((a_1 - h_U)^3 I_1 - (a_1 - h_L)^3 I_1') \\ -\frac{b_4}{3}((a_2 - h_U)^3 I_2 - (a_2 - h_L)^3 I_2') \end{bmatrix},$$

where: $\mathbf{h_U = \frac{H_U}{H}}$

$\mathbf{h_L = \frac{H_L}{H}}$

I_1 = 1, h_U $\le$ a_1; = 0, otherwise

I_2 = 1, h_U $\le$ a_2; = 0, otherwise

I_1' = 1, h_L $\le$ a_1; = 0, otherwise

I_2' = 1, h_L $\le$ a_2; = 0, otherwise

All other variables are defined as before.

The centroid volume (Wood *et al.* 1990) estimated with equation [4] (Coble and Wiant 2000) is:

$$[5] \quad V_{COMPLEX} = \left(\frac{A_{CC}}{A_E} \times P_{COMPLEX}\right),$$

where: A_E = ($f \times D_E$)= cross-sectional area (m^2) at the centroid based on the Max-Burkhart proxy function

$$D_E = D^2\left[b_1\left(\frac{H_{CC}}{H} - 1\right) + b_2\left(\frac{H_{CC}^2}{H^2} - 1\right) + b_3\left(a_1 - \frac{H_{CC}}{H}\right)^2 J_1 + b_4\left(a_2 - \frac{H_{CC}}{H}\right)^2 J_2\right]$$

J_1 = 1, h_C $\le$ a_1; = 0, otherwise

J_2 = 1, h_C $\le$ a_2; = 0, otherwise

$h_C = H_{CC} / H$

$A_{CC} = (f \times D_{CC})$= cross-sectional area ($m^2$) at the centroid position

D_{CC} = interpolated diameter (cm) from closely spaced bole measurements on either side of the centroid position

H_{CC} = centroid height (m).

All other variables are defined as before.

Data analysis

The volume estimates from both proxy functions were evaluated for accuracy by comparison to actual volumes. They were also compared to volume estimates from whole-tree volume equations for Japanese red cedar in Korea (Forestry Administration of Korea 2000). To provide a fair comparison between the two proxy functions, regression coefficients were estimated for the Max-Burkhart taper function [3] using the red cedar data set (Table 2). The PROC NLIN procedure in SAS (1998) was used to estimate these coefficients.

Reynolds (1984) developed estimation procedures to evaluate the accuracy of models. His procedures test both bias and precision rather than overall prediction accuracy. These procedures were converted to a BASIC program (Rauscher 1986), then later to a SAS program (SASATEST, Gribko and Wiant 1992). SASATEST was used in this study to determine which volume estimator performed best. SASATEST examined both bias and precision on an absolute or percentage basis. In SASATEST, percent bias was calculated as a percentage of the observed volume:

$$\mathrm{BIAS} = 100\left(\frac{(\hat{Y} - Y)}{Y}\right),$$

where $\hat{Y}$ = estimated volume and Y = observed volume. The latter was determined by summation of short bole section volumes using Smalians formula (Avery and Burkhart 1994, page 55). In this study, precision is expressed as the standard deviation of percent bias, which is also calculated by SASATEST. SASATEST then uses the mean percent bias (measure of bias) and the standard deviation (measure of precision) to calculate a 95% prediction confidence interval. If this confidence interval does not contain zero, then the bias is significant at the $\alpha = 0.05$ level. SASATEST also checks the errors between estimated and observed values for departures from normality. If non-normality is detected, a 10% trimmed mean and jackknife standard deviation were used to provide more robust confidence intervals. SASATEST also provides a percent tolerance interval (TI), which indicates with 0.95 probability where 95% of future prediction errors should occur (i.e., BIAS±TI). The mean squared error (MSE = $\mathrm{bias}^2 + \sigma^2$) was also used to evaluate the volume estimators because a biased estimator with a small variance may be preferable to an unbiased estimator with a large variance (Devore 1982, page 513).

Table 2. Parameter estimates, approximate standard errors, and approximate confidence intervals for the Max-Burkhart taper function

Parameter	Estimate	Standard Error	Lower 95% Confidence Level	Upper 95% Confidence Level
B_1	-3.4579	1.4835	-6.3759	-0.5399
B_2	1.7437	0.8586	0.0549	3.4326
B_3	-1.5762	0.7734	-3.0974	-0.0550
B_4	75.4989	16.9759	42.1081	108.9000
A_1	0.6685	0.1600	0.3538	0.9832
A_2	0.0970	0.0130	0.0714	0.1226

RESULTS AND DISCUSSION

The Max-Burkhart proxy function volumes were significantly biased (-4.80%, $p<0.05$), while the simple proxy function volumes were not significantly biased (3.78%, $p>0.05$) (Table 3). The simple proxy function was less precise (SD = 16.80%) than the Max-Burkhart proxy function (SD = 10.37%). The MSE and percent tolerance interval (TI) were also larger for the simple function. The volumes from the whole-tree equations were significantly biased (-3.46%, $p<0.05$), though they were more precise than either of the proxy functions (6.80%).

The mean percent biases of volume predictions of both proxy functions are high, even though the bias for the simple proxy function volume is not significant (Table 3). This non-significant result can likely be attributed to the wide 95% confidence interval; if the 95% confidence interval were not so wide, the bias would most likely be significant. Based on these results, the simple proxy function should be used if less-precise, unbiased volume estimates are desired. The Max-Burkhart proxy function should be used if more precise volume estimates are desired (though they will be biased). As mentioned earlier, a biased estimator with a small variance may be preferable to an unbiased estimator with a large variance (Devore 1982).

We were pleased that, on the average, the simple proxy function provided the unbiased volume estimates. However, we were not surprised that the simple proxy function resulted in imprecise volume estimates. The simple function at best represents a gross approximation of the flared stem bases common in red cedar stems.

Table 3. Percent bias, standard deviations, 95% confidence intervals (CI), 95% tolerance intervals (TI), and mean square errors (MSE) for volume estimates from the simple proxy and Max-Burkhart proxy taper functions, and the whole-tree volume equations

STATISTIC	SIMPLE PROXY	MAX-BURKHART	WHOLE-TREE
MEAN % BIAS	3.78	-4.80*	-3.46*
SD	16.80	10.37	6.80
95% CI	3.78±6.06	-4.80±3.74	-3.46±2.45
95% TI	3.78±42.41	-4.80±26.17	-3.46±17.18
MSE	296.46	130.55	58.30

Note that all values expressed as a percent of observed volume.
* = significant ($p < 0.05$).

The simple proxy function [1] characterizes a stem profile where the cross-sectional area decreases linearly with height (Gray 1956, Gregoire *et al.* 1986). This is not likely a reasonable assumption for red cedar trees. Though Coble and Wiant (2000) found no advantage in using a complex proxy function over a simple one for two species of pine trees, we found that the use of a proxy function (i.e., the Max-Burkhart proxy function [3]) that better accommodates the flared bases of red cedar stems resulted in increased precision in the volume estimates (though bias was significant).

The bias in the volume estimates from the whole-tree equations was not significantly different from bias in the volume estimates from the simple proxy function, though it was different in sign and similar in magnitude (Table 3). This result was unexpected since the simple proxy function does not use species-specific coefficients, like the whole-tree equations or the more complex Max-Burkhart equation (which was fit to the data in this study). This difference was reflected in the much lower precision of the simple proxy function. The bias in the volume estimates from the whole-tree equations was also not significantly different from that of the Max-Burkhart proxy function, though the volume estimates were considerably more precise (Table 3).

The results of this study show that the centroid method can predict volume estimates for Japanese red cedar. These estimates are comparable to those from whole-tree equations designed specifically for Japanese red cedar trees, though they are less precise, on the average. The choice of proxy taper functions that better characterize the stem profile of the red cedar trees leads to increased precision. This study shows that the centroid method can, on the average, provide reliable volume estimates for trees, even without species-specific proxy functions. This could be an important consideration to personnel who need to estimate the volumes of trees for which no volume equations exist.

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