noded boundary elements. Quarter-points singular elements are placed near the crack front. With the use of the proposed procedure the stress intensity factors for some of three dimensional cracks in infinite media are calculated. A comparison of results with the published solutions shows that the SGBEM is very efficient and highly accurate for analysis of three dimensional cracks in infinite bodies.

2. Symmetric Galerkin boundary element method

2.1 Governing equation

Consider an infinite three-dimensional body containing arbitrarily three dimensional cracks of arbitrary geometry. A distributed load is applied at the crack surface. The crack can be described by a distribution of displacement discontinuity with components [5-8].

$$-\int_S\int_S D_\alpha u_i^*(z)C_{\alpha i\beta j}(\xi - z)D_\beta u_j(\xi)dS(\xi)dS(z)$$
$$= \int_S u_i^*(z)t_k dS(z) \qquad (1)$$

Here $S = S_+$ is one of crack surfaces; u_i are displacement discontinuities for the crack surface; u_i^* are the components of a continuous test function; and t_k are crack face tractions.

The two-point weakly singular kernel is given by the following expression:

$$C_{\alpha i\beta j}(\xi) = \frac{\mu}{4\pi(1-\nu)r}[(1-\nu)\delta_{i\beta}\delta_{j\beta} + 2\nu\delta_{i\beta}\delta_j$$
$$-\delta_{ij}\delta_{\alpha\beta} - \frac{\zeta_i\zeta_j}{r^2}\delta_{\alpha\beta} \qquad (2)$$

where ν is Poisson's ratio and μ is the shear modulus. A tangential operator D_α is defined as follows:

$$D_\alpha = \frac{1}{J}\left(\frac{\partial}{\partial\eta_1}\frac{\partial x_\alpha}{\partial\eta_2} - \frac{\partial}{\partial\eta_2}\frac{\partial x_\alpha}{\partial\eta_1}\right) \qquad (3)$$

where, η_1, η_2 are the surface coordinates on the crack surface, $J = |s \times t|$, and s, t are vectors in the plane that is tangent to the crack surface.

2.2 Discretization of the integral equation

Assume the crack that is partitioned into boundary elements. Displacement discontinuities and tractions are defined at element nodes, and are interpolated inside the elements with the use of shape functions N_a:

$$u_i = N_a(\eta_1,\eta_2)\mu_{ia}$$

$$t_i = N_a(\eta_1,\eta_2)t_{ia} \qquad (4)$$

where $i = 1, 2, 3$ is the global coordinate subscript; a is the node number; η_1, η_2 are element local coordinates. With the use of a parametric representation of displacement discontinuities and tractions, we can rewrite the integral equation (1) in the following discretized form:

$$-\int_S\int_S C_{\alpha i\beta j}D_\alpha N_a(z)D_\beta N_b(\xi)dS(\xi)dS(z)u_{jb}$$
$$= \int_S N_a N_q(z)dS(z)t_{iq} \qquad (5)$$

Using the integral equation (5), displacement discontinuities at element nodes of the crack are defined, and then the stress intensity factors can be calculated from their values.

2.3 Modeling of fatigue crack growth.

The fatigue crack growth models of materials(such as Paris, Forman or NASGRO models) express the functional relationship for crack growth rate through the range of the effective stress intensity factor K_{eff}:

$$\frac{da}{dN} = f(\Delta K_{eff}) \qquad (6)$$

where da/dN is the crack growth per cycle and $\Delta K = K_{max} - K_{min}$.

Modeling of fatigue crack growth is performed by finite increments. At each increment, the maximum crack advance is specified as Δa_{max}. The crack advance for a particular point at the crack front is calculated as follows:

$$\Delta a = \Delta a_{max}\frac{(da/dN)}{(da/dN)_{max}} \qquad (7)$$

We use the following procedure for the advancement of the front of the crack:

1) Using SGBEM, solve the problem for the current crack configuration and determine ranges for the stress intensity factors at the crack front.

2) Determine the crack front coordinate system for each corner node at the corner point of two neighboring boundary elements.

3) Calculate the crack advance Δa and the angle, move each corner node in the local coordinate system, and then transform the movement to the global coordinate system.

4) Find the locations of crack front midside nodes, using cubic spline interpolations for corner nodes from

several neighboring elements.

5) Shift the quarter-point nodes on element sides normal to the crack front.

After terminating the crack growth procedure, the total number of cycles N is calculated as a sum ΔN_i of at crack growth increments.

3. Alternating method

In fracture mechanics problems, Combining the symmetric Galerkin boundary element method for modeling an arbitrary non-planar crack in an infinite body, and the finite element method for an uncracked finite body, allows us to employ advantages of both methods. The finite element method is a robust method for elastic and elastic-plastic problems. It can easily incorporate various types of boundary conditions. The finite element method is widely used in industry. There are commercial preprocessor programs, which are capable of transforming any CAD model into a finite element model.

The BEM is most suitable for modeling cracks in infinite bodies. The displacement discontinuity approach provides for a simple modeling of the crack. Only one surface of the crack should be discretized. The independence of the crack model and the finite element model of the body allows to easily change the crack model in order to simulate crack growth under monotonic or cyclic loading. The solution for a finite body with a crack is obtained as a superposition of two models:

1) finite element model for a finite body under external loading, without a crack;

2) an infinite body with a crack modeled by the symmetric Galerkin boundary element method.

Illustration of the superposition principle is presented in Fig. 1. For a correct superposition corresponding to the solution for a finite body with a crack, fictitious forces on the boundary of the finite element model should be found in order to compensate for the stresses caused by the presence of a crack in an infinite body. While this can be done with a direct procedure, the alternating method provides for a more efficient solution, without assembling the joint SGBEM-FEM matrix [1].

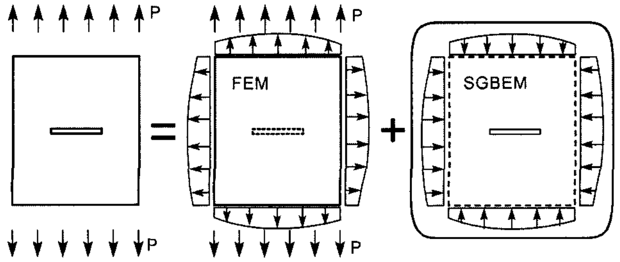

Fig. 1 Superposition principle [10]

The SGBEM-FEM alternating method alternates between the finite element solution for an uncracked body and the displacement discontinuity method solution for a crack in an infinite body. Using an iteration procedure, artificial tractions at the boundary of the finite element-modeled body and at the crack surface, are found.

The steps of the SGBEM-FEM alternating iteration procedure are as follows [9]:

1) Using FEM, obtain the stresses at the location of the crack in a finite uncracked body subjected to given boundary conditions.

2) Using SGBEM, solve the problem of a crack, the faces of which subjected to tractions, as found from FEM analysis of the uncracked body.

3) Determine the residual forces at the outer boundaries of the finite body, from displacement discontinuities at the crack surface.

4) Using FEM, solve a problem for a finiteuncracked body under residual forces from SGBEM analysis.

5) Obtain the stresses at the location of the crack corresponding to FEM solution.

6) Repeat (2)-(5) steps until the residual load is small enough.

7) Compute the solution for a finite body by summing alll the appropriate contributions,

4. Numerical results

In order to demonstrate the accuracy of the SGBEM procedure, the solutions for three dimensional cracks in infinite bodies are presented. And as an example problem of the fatigue crack growth, a rectangular precrack is given. In all examples, 8-noded quadrilateral boundary elements are used for crack surface discretization. Gaussian integration rule, with three points in each of the four directions is employed for computing boundary element matrices for regular and singular cases. Quarter-point singular elements are places at the crack front [10].

4.1 Penny-shaped crack under tension.

The mesh for a penny-shaped circular crack under tensile loading along is shown in Fig. 2. Exact solution for the problem is given by Sneddon [11] and Kassir and Sih [12]:

$$K_I = \frac{2}{\pi}\sigma\sqrt{\pi a} \tag{8}$$

where a is the crack radius (length), σ is the applied remote stress. Three meshes consisting of 12, 20 and 40 quadratic elements are used. Results for the stress intensity factors normalized $K_I/[(2/\pi)\sigma\sqrt{\pi a}]$ as are presented in Fig. 3. Three kinds of meshes all provide accurate results. Especially, the meshes of 40 elements gives values of the stress intensity factors with an error about 0.2%.

4.2 Elliptical crack in an infinite solid under tension.

The mesh of an elliptical crack under tension in an infinite body is shown in Fig. 4, and it is composed of 72 boundary elements and 205 nodes in such a way that it is produced by scaling the circular mesh in one direction. After scaling, the element edges are not normal to the crack front line. The crack is mainly characterized by axis-ratio that varies from 0.5 to 2.0 in 3 steps.

Results for the elliptical crack are given in Fig. 5.

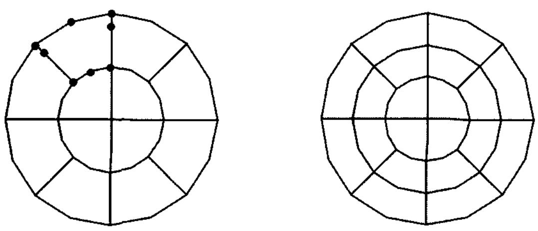

Fig. 2 Meshes for a penny-shaped crack.

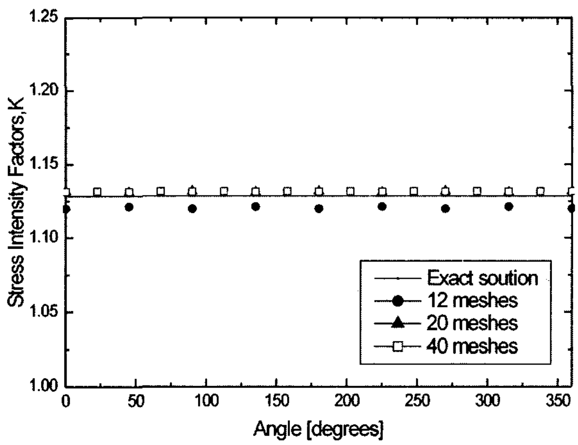

Fig. 3 Stress intensity factors for a penny-shaped crack.

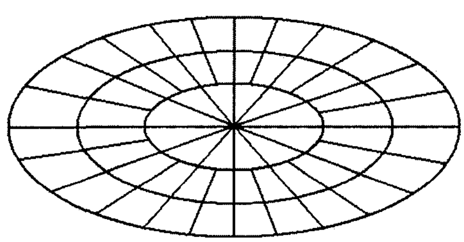

Fig. 4 Mesh for an elliptical crack.

The stress intensity factor values are normalized as $K_I/(\sigma\sqrt{\pi a})$. As shown in Fig. 5, a satisfactory agreement of our results with theoretical solution [13] is observed except points that their singular nodes are not normal to the crack front line.

4.3 Rectangular crack in an infinite solid under tension.

The crack bounded by a rectangular front in an infinite body is shown in Fig. 6. The crack boundary element meshes consisting of 40, 56, 72 and 96 elements. The crack is mainly characterized by axis-ratio *a*/*b* that varies from 0.25 to 1.0 in 4 steps.

Results for the elliptical crack are also given in Fig. 6. The stress intensity factor values obtained at point A are normalized as $K_I/(\sigma\sqrt{\pi a})$. As shown in Fig. 6, a good agreement of our results with those of Murakami and Nemat-Nasser [14] is observed.

4.4 Fatigue crack growth

In order to analyze the fatigue growth of three dimen-

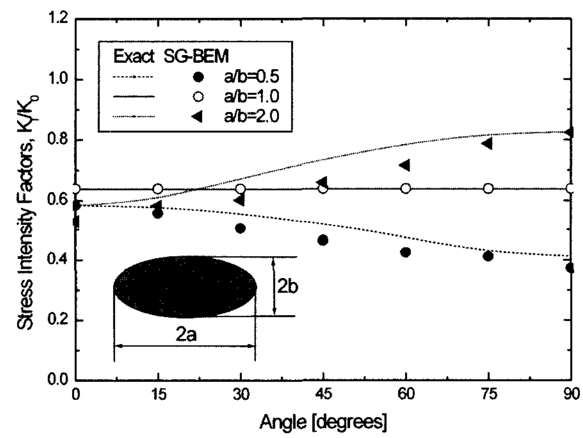

Fig. 5 Stress intensity factors for an elliptical crack.

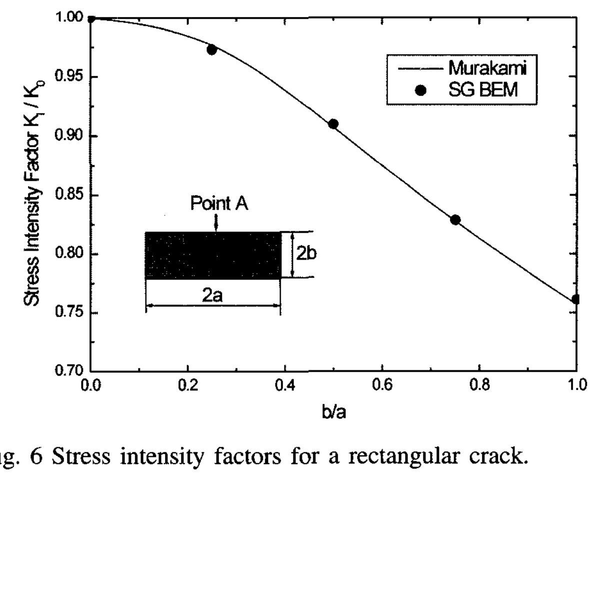

Fig. 6 Stress intensity factors for a rectangular crack.

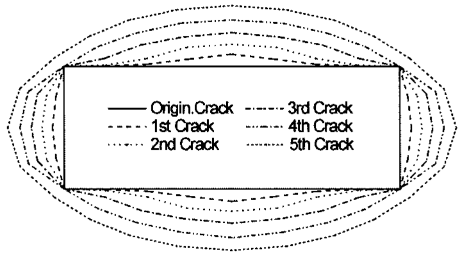

Fig. 7 Growth of a rectangular crack.

sional crack under a uniform tensile loading conditions, a rectangular crack in an infinite body is considered as a precrack. The axis ratio() of a rectangle is 2.0 and is discretized by 56 quadratic boundary elements. The Paris material fatigue model was chosen to simulate fatigue crack growth:

$$\frac{da}{dN} = C(\Delta K)^m \tag{9}$$

where da/dN is the crack growth rate with respect to the loading cycles, $C = 3.0 \times 10^{-13}$ and $m = 3.0$ are material parameters as recommended by Maddox [15] for a wide range of structural steels. Units of da/dN and ΔK are m/cycle and MPa$\sqrt{m}$ respectively.

To analyze the rectangular precrack, the first thing to be done is to calculate the stress intensity factors K_I for the crack front nodes. According to the stress intensity factors, points at the crack front are advanced to new positions with scaling to the specified maximum crack advance da_{max}. A new layer of elements is newly defined by the relationship of old and new crack front lines. Then the new crack model is analyzed and *etc*. Five crack advancements were performed and the view of cracks after crack increments are given in Fig. 7.

5. Conclusion

The symmetric Galerkin boundary element method has been used for the analysis of three dimensional cracks in infinite bodies. Especially, since the finite element mesh for the uncracked body and the boundary mesh for the crack are completely independent, the SGBEM is particularly efficient for modeling of fatigue crack growth.

By using the method, some of problems for the three dimensional cracks, such as penny-shaped, elliptical and rectangular cracks, under tensile loading conditions, is analyzed including an example of fatigue crack growth.

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