Vol. 2, No. 1, pp. 1-6, June 2003

INTERNATIONAL
JOURNAL OF SAFETY
www.kiis.or.kr

A Study on Mixed Mode Crack Initiation under Static Loading Condition

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(Received January 20, 2003; Accepted May 28, 2003)

Abstract : In this paper, several different fracture criteria using the Eftis and Subramanian's stress solutions [1] are compared with the printed experimental results under different loading conditions. The analytical results of using the solution with non-singular term show better than without non-singular in comparison with the experimental data. And maximum tangential stress criterion (MTS) and maximum tangential strain energy density criterion (MTSE) can get useful results for several loading conditions.

Key words : biaxial load, mixed mode, crack propagation angle, maximum tangential stress criterion, maximum strain energy density criterion, maximum minimum strain energy density criterion

1. Introduction

Traditional applications of fracture mechanics have been concentrated on cracks growing under mode I loading condition. However, many service failure occur from cracks subjected to mixed mode loadings. Also, many uniaxial loading structures and components often contain randomly oriented defects and cracks which develop mixed mode state by virtue of their orientation with respect to the loading axis. The problem involving mixed mode fracture has been extensively investigated not only by several researchers [2, 3] around the globe, but also in Korea [4~6].

A characteristic of mixed mode cracks is that they usually propagate in a non-self similar manner. That is, a crack changes its growth direction when subjected to mixed mode loadings. Consequently, under mixed mode loading conditions, not only the fatigue crack growth rate is of importance, but also the crack growth direction. The crack growth path cannot be predicted without having the knowledge of crack propagation angle and the study of crack propagation angle is important in dealing with arresting the crack.

Recently, Khan and Khraisheh [7] compared the several criteria with the experimental data under uniaxial, pure shear and biaxial loading and stated that none of the criterion can fit the experimental data for all loading conditions. But it is partially caused by using the stress solution without non-singular term. Many researchers [8, 9] stated that non-singular term has an important effect on crack propagation angles.

In mixed mode problem, the representative fracture criteria are as follows:

(1) Maximum tangential stress criterion (MTS)
(2) Maximum energy release rate criterion (MERR)
(3) Minimum strain energy density criterion (SED)

MTS was proposed by Erdogan and Sih [2]. This criterion is based on the assumption that the material behaves ideally brittle.

MERR was, in fact, first mentioned by Erdogan and Sih [2]. The application of MERR is difficult because of the mathematical complexities associated with the elastic stress field analysis of the branched crack, and there are disputes about the analysis method and results.

SED was proposed by Sih [10] and it has been reported that there are an ambiguity in the choice of the relative minimum strain energy density factor [11, 12]. In order to solve this problem, Koo [13] proposed maximum minimum strain energy density criterion(MSED) that the strain energy density factor associated with the sign of a tangential stress.

Also, Koo and Choy [14] proposed the maximum tan-

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gential strain energy density criterion(MSED) and it is assumed that the crack growth takes place in the direction along which the tangential strain energy density factor possesses a maximum value. The authors showed that MTSE agrees well with the experimental data under uniaxial loading.

In this paper, MTS, SED and MTSE will be applied to the slit crack under several loading conditions. But because there is an ambiguity in the choice of the relative minimum strain energy density factor, the maximum minimum strain energy density criterion(MSED) proposed by Koo [13], will be used instead of the minimum strain energy density criterion. And, the analytical results of using the solution with non-singular term will be compared with those without non-singular term.

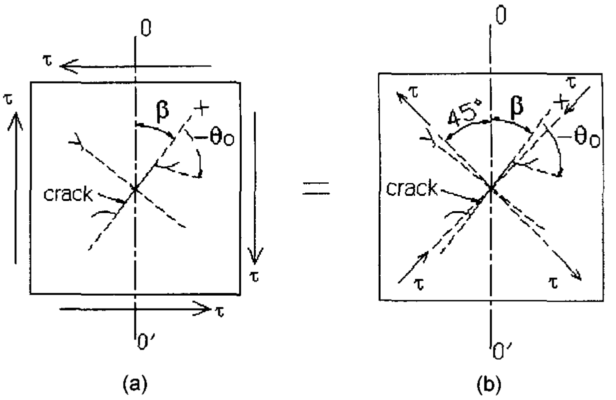

Fig. 2. (a) Loading configuration of the angled crack problem under shear, (b) Equivalence of the pure shear case - the biaxial loading case with $\lambda = -1$.

2. Stress field under biaxial loading

In the case of slit crack under biaxial loading, the stress solution by Eftis and Subramanian [1] has been written as follows (see Fig. 1):

$$\sigma_y = \frac{k_1}{\sqrt{2r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \frac{k_2}{\sqrt{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_x = \frac{k_1}{\sqrt{2r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$-\frac{k_2}{\sqrt{2r}}\sin\frac{\theta}{2}\left[2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right] + \sigma(1-\lambda)\cos(2\beta)$$

$$\tau_{xy} = \frac{k_1}{\sqrt{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{k_2}{\sqrt{2r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right], \tag{1}$$

in which

$$k_1 = \frac{\sigma\sqrt{a}}{2}[(1+\lambda)-(1-\lambda)\cos(2\beta)]$$

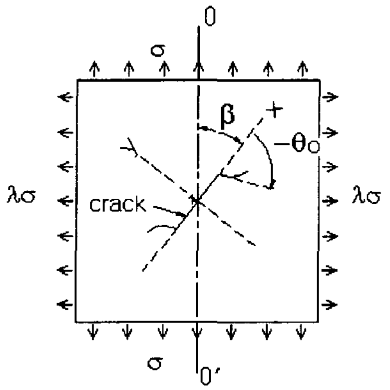

Fig. 1. The configuration of the angled crack.

$$k_2 = \frac{\sigma\sqrt{a}}{2}(1-\lambda)\sin(2\beta), \tag{2}$$

where the ratio of the biaxial loading (λ) may be either positive or negative. When the several criteria of using the stress solution without non-singular term of $\sigma(1-\lambda)\cos(2\beta)$ are compared with the experimental data under several loading conditions, Khan and Khraisheh [7] stated that none of the criterion can fit the experimental data for all loading conditions.

In the uniaxially loaded case, i.e., $\lambda = 0$ in Eqs. (1) and (2), the polar coordinate form of Eq.(1) is equal to the Swedlow's solution [11] with non-singular term. Fig. 2(a) shows the configuration of the angled crack problem in pure shear, in which a uniformly distributed shearing stress is applied at the infinite edge. It can be easily shown that the elastic stress field of the pure shear case is analytically equivalent to that of the biaxial loading with $\lambda = -1$. In fact, the case of $\beta = \beta_0$ in pure shear is equivalent to the case of $\beta = 45° + \beta_0$ in biaxial loading with $\lambda = -1$. This is clearly illustrated in Fig. 2(b).

Therefore, Eqs. (1) and (2) can be used for the mixed mode problems under several loading conditions.

3. Fracture criteria

Maximum tangential stress criterion (MTS) [2] is the simplest of all, and it states that direction of crack propagation coincides with the direction of the maximum tangential stress along a constant radius around the crack tip. It can be stated mathematically as

$$\frac{\partial\sigma_\theta}{\partial\theta} = 0, \quad \frac{\partial^2\sigma_\theta}{\partial\theta^2} < 0. \tag{3}$$

Minimum strain energy density criterion (SED) [10] states that the direction of crack propagation coincides with the direction of minimum strain energy density criterion along a constant radius around the crack tip. In mathematical form, SED can be stated as

$$\frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial^2 S}{\partial \theta^2} < 0 \tag{4}$$

where S is the strain energy density factor, defined as

$$S = r_0 \frac{dW}{dV}, \tag{5}$$

where dW/dV is the strain energy density per unit volume, and r_0, a finite distance from the failure initiation point.

SED has the problem that there exist two or more relative minimum values of S along the arc of the core region for most loading conditions. This implies that one more restrictions are required to determine the direction of crack extension.

In order to solve this problem, Koo [13] presented maximum minimum strain energy density criterion (MSED) as the modified criterion of minimum strain energy density criterion.

It was assumed that the largest of the relative minima of S governs the crack extension. Because, as σ increase, the value of S along the core region increases accordingly, the largest relative minimum S always reach its critical value, S_{cr}, first, which leaves no chance for other relative minima to reach S_{cr}. In addition, the sign of a tangential stress needs to be considered in the crack extension problem because, at $\sigma_\theta > 0$, crack opening occurs and crack extends, but at $\sigma_\theta < 0$, crack closure occurs and crack cannot extend.

The strain energy density factor associated with the sign of a tangential stress is denoted by MS and called the modified strain energy density factor:

$$MS = \mathrm{sign}(\sigma_\theta) \cdot S. \tag{6}$$

The following hypotheses on the crack initiation are made:

(1) The initial crack growth takes place in the direction along which the modified strain energy density factor possesses the largest of the relative minima.

(2) Crack propagation occurs when the modified strain energy density factor reaches a critical value.

In this paper, MSED instead of SED will be applied to the crack propagation problem under several loading conditions.

Maximum tangential strain energy density criterion (MTSE), proposed by Koo and Choy [14], is based on the tangential strain energy density stored in the element $dA = rd\theta dr$, i.e.

$$W_\theta = 1/1\sigma_\theta\varepsilon_\theta. \tag{7}$$

Then, the tangential strain energy density factor, C, can be written as:

$$C = r_0 W_\theta \quad (r_0 : \text{critical distance})$$

$$= b_{11}k_1^2 + b_{12}k_1k_2 + b_{22}k_2^2, \tag{8}$$

in which coefficients $b_{ij}(i, j{=}1, 2)$ stand for

$$b_{11} = 1/(64\mu)\,[(1 + \cos\theta)(x + 2 + \cos\theta)]$$

$$b_{12} = 1/(64\mu)\,[\sin\theta(-3/2 - x - 3\cos\theta)]$$

$$b_{22} = 1/(64\mu)\,[3\sin^2\theta(x + 3\cos\theta)], \tag{9}$$

where μ is shear modulus and $\kappa = 3 - 4\nu$ or $(3-\nu)/(1+\nu)$ for plane strain or plane stress respectively.

This criterion states that:

(1) The initial crack growth takes place in the direction along which the tangential strain energy density factor possesses a maximum value, i.e.,

$$\frac{\partial C}{\partial \theta} = 0, \quad \frac{\partial^2 C}{\partial \theta^2} < 0, \text{ at which } \theta = \theta_0 \tag{10}$$

where θ_0 is the crack initiation angle between $-\pi$ and π.

(2) The crack initiation occurs when the tangential strain energy density factor reaches a critical value, i.e.,

$$b_{11}k^2_1 + b_{12}k_1k_2 + b_{22}k^2_2 = C_{cr} \text{ for } \theta = \theta_0. \tag{11}$$

In the case of mode I crack extension, i.e., $k_2 = 0$, C_{cr} is related to k_{1c} as $C_{cr} = (1 - 2\nu)(8\mu)k_{1c}^2$. And the relations of the critical stress σ_{cr} are as follows:

$$\frac{\sigma_{cr(\beta)}}{\sigma_{cr(90^\circ)}} = \sqrt{\frac{C_{cr(\beta)}}{C_{cr(90^\circ)}}} = \sqrt{\frac{C_{max(90^\circ)}}{C_{max(\beta)}}} \tag{12}$$

where C_{max} is the value in the direction of crack growth.

In the case of a slit crack under uniaxial tension and compression, it was reported that it has good agreements between predictions and experimental data.

4. Results and Discussion

In the uniaxial loaded case, $\lambda = 0$ in Eqs. (1) and (2). Figs. 3 and 4 show the results for uniaxial tension. The shown data are from Williams and Ewing [15] and Palaniswamy and Knauss [16].

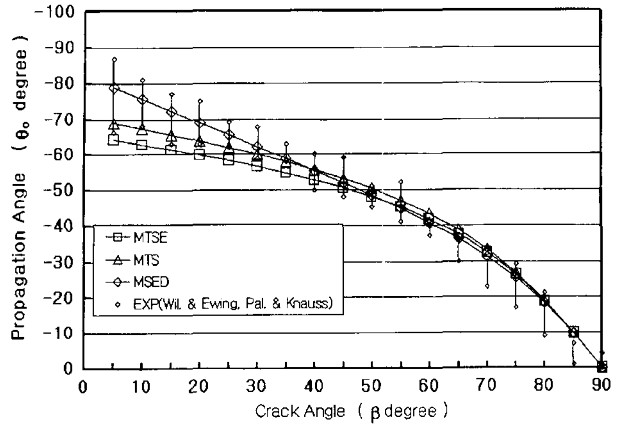

Fig. 3. Crack propagation angle vs. the crack angle in the uniaxial tension without non-singular term.

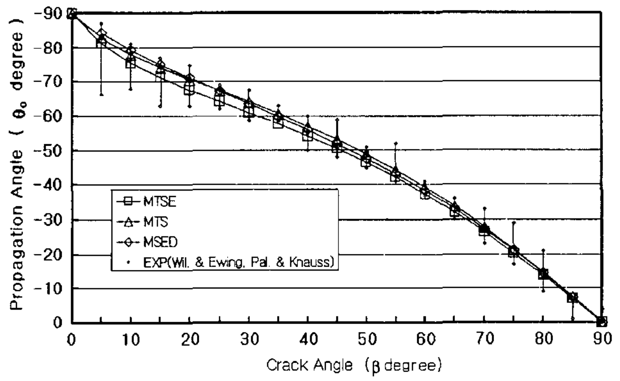

Fig. 4. Crack propagation angle vs. the crack angle in the uniaxial tension with non-singular term.

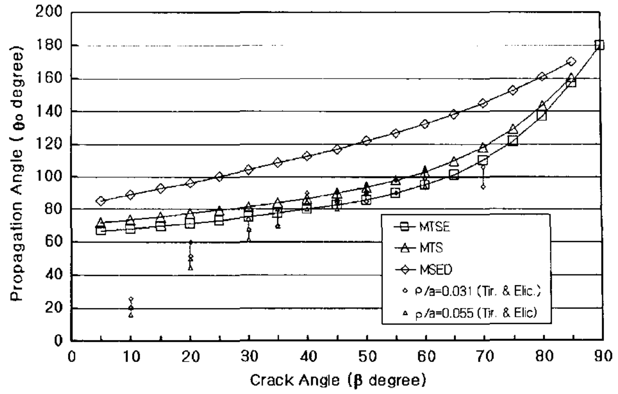

Fig. 5. Crack propagation angle vs. the crack angle in the uniaxial compression without non-singular term.

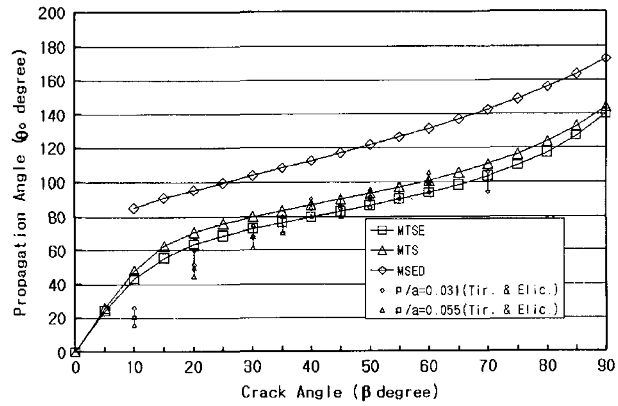

Fig. 6. Crack propagation angle vs. the crack angle in the uniaxial compression with non-singular term.

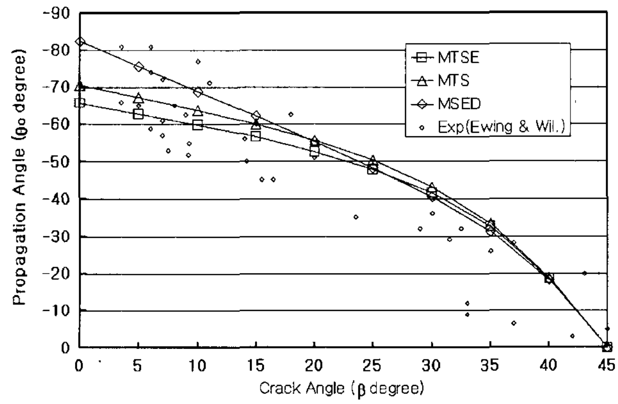

Fig. 7. Crack propagation angle vs. the crack angle in the pure shear without non-singular term.

Fig. 3 shows the results of using the stress solution without non-singular term. It shows that both MTS and MSED agree better with the experimental data than MTSE. All criteria don't have the analytical results at $\beta = 0°$.

Fig. 4 shows the results of using the stress solution with non-singular term. It shows that all criteria agree well with the experimental data. Only MSED doesn't have the analytical results at $\beta = 0°$. When Fig. 3 is compared with Fig. 4, all criteria of using the solution with non-singular term agree better with the experimental data than those of without it for uniaxial tension.

Figs. 5 and 6 show the results for uniaxial compression. The experiment data are from Tirosh and Elicatz [17]. The data are the results for the ratio of curvature radius per half crack length, $\rho/a = 0.031$ and 0.055.

Fig. 5 shows the results of using the solution without non-singular term. MTSE agrees with the experimental data for $30° < \beta < 60°$ and MTS for $40° < \beta < 60°$. But MSED doesn't agree with the data for all crack angle. Also, all criteria don't have the analytical results at $\beta = 0°$.

Fig. 6 shows the results of using the stress solution with non-singular term. It shows that MTS and MTSE agree well with the experimental data but MTSE agrees better with the data than MTS. But MSED doesn't agree with it for all crack angle and have the analytical results at $\beta < 10°$.

Figs. 7 and 8 show the results for pure shear. The

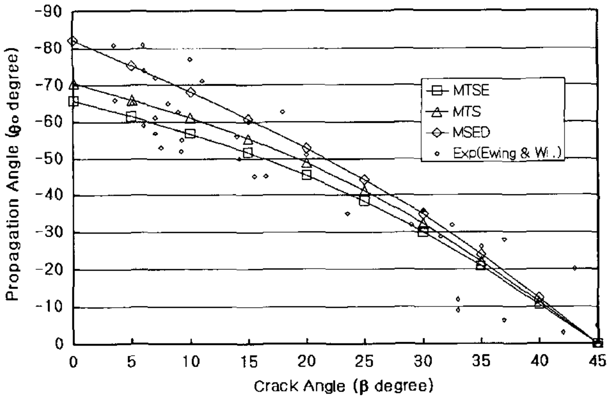

Fig. 8. Crack propagation angle vs. the crack angle in the pure shear with non-singular term.

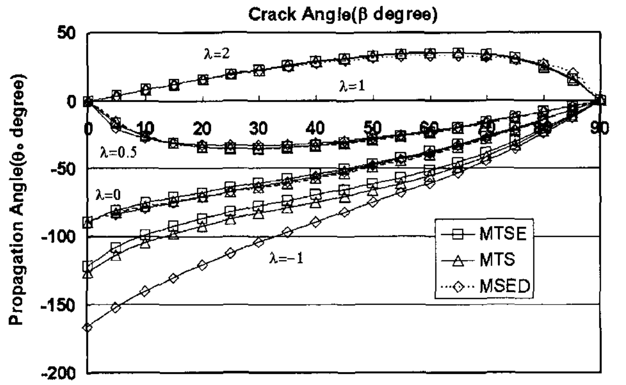

Fig. 10. Crack propagation angle vs. the crack angle in the biaxial load with non-singular term.

experiment data are from Ewing and Williams [18].

Fig. 7 shows the results of using the stress solution without non-singular term. It shows that all criteria agree with the experimental data except for $20° < \beta < 35°$.

Fig. 8 shows the results of using the stress solution with non-singular term. It shows that all criteria agree with the experimental data. When Fig. 8 is compared with Fig. 7, all criteria of using the solution with non-singular term agree better with the experimental data than those of without it for pure shear.

Figs. 9 and 10 show the results for biaxial loading for $-2 \leq \lambda \leq 2$.

Fig. 9 shows the results of using the stress solution without non-singular term. All criteria don't have the analytical results for $\lambda = 2$. Regardless of crack angle, β, all criteria have the same analytical result, i.e., $\theta_o = 0°$ for $\lambda = 1$. The crack propagation angle $\theta_0 \leq 0°$ for $\lambda < 1$. Also, all criteria don't partially have the analytical results for both $\lambda = 0$ and -1.

Fig. 10 shows the results of using the stress solution with non-singular term. All criteria have the analytical results for $\lambda = 2$. In the same way, regardless of crack angle, β, all criteria have the same analytical result, i.e., $\theta_o = 0°$ for $\lambda = 1$. The crack propagation angle $\theta_o \leq 0°$ for $\lambda < 1$. The crack propagation angle $\theta_o \geq 0°$ for $\lambda > 1$. MTS and MTSE except MSED have the analytical result for all biaxial load ratios. The crack propagation angle predictions based on MSED differ significantly from others.

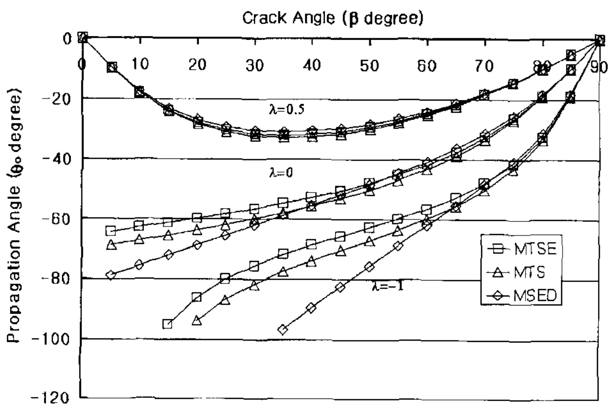

Fig. 9. Crack propagation angle vs. the crack angle in the biaxial load without non-singular term.

5. Conclusion

An analysis of mixed mode crack propagation angles under various loading conditions is presented in this paper. The following can be concluded:

1. All criteria of using the solution with non-singular term agree better with the experimental data than those of without it for mixed mode loading.

2. The crack propagation angle predictions based on MTS and MTSE have similar trends, but the predictions based on MSED differ significantly from others.

3. Both MTS and MSED agree better with the experimental data than MSED.

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