A study on Generalized Synchronization in Hyper-Chaos with SC-CNN

Young-Chul Bae, Ju-Wan Kim, Hag-Hyun Song and **Yoon-Ho Kim**, *Member, KIMICS*

Abstract—In this paper, we introduce a hyper-chaos synchronization method using hyper-chaos circuit consist of State-Controlled Cellular Neural Network (SC-CNN). We make a hyper-chaos circuit using SC-CNN with the n-double scroll. A hyper-chaos circuit is created by applying identical n-double scroll or non-identical n-double scroll and Chua's oscillator with weak coupled method to each cell. Hyper-chaos synchronization was achieved using GS(Generalized Synchronization) method between the transmitter and receiver about each state variable in the SC-CNN.

Index Terms—Chaos, Hyper-chaos, Generalized Synchronization, Nonlinear Dynamics.

I. INTRODUCTION

Recently, there has been interest in studying the behavior of chaotic dynamics. Chaotic systems are characterized by sensitive dependence on initial conditions, making long term prediction impossible, self-similarity, and a continuous broad-band power spectrum, etc. Chaotic systems have a variety of applications, including chaos synchronization and chaos secure communication [1-6].

Chaos synchronization and secure communication has been a topic of intense research in the past decade. However, secure communication or cryptographic using chaos has several problems [7]. First, almost all chaos-based secure communication or cryptographic algorithms use dynamical systems defined on the set of real number, and therefore are difficult for practical realization and circuit implementation. Second, security and performance of almost all proposed chaos-based methods are not analyzed in terms of the techniques developed in cryptography. Moreover, most of the proposed methods generate cryptographically weak and slow algorithms.

To address these problems, we need a hyper-chaos circuit to increase the complexity in secure communication or cryptographic communication. In this paper, we introduce a new hyper-chaos synchronization method called GS (Generalized synchronization) using State-Controlled Cellular Neural Network (SC-CNN) as a hyper-chaos circuit. We make a hyper-chaos circuit using SC-CNN with the n-double scroll [8] and Chua's oscillator.

In order to make a hyper-chaos circuit, we used identical n-double scroll or non-identical n-double scroll and Chua's oscillator with weak coupled method to each cell. Then we accomplished a hyper-chaos synchronization using GS method between the transmitter and receiver about each state in the SC-CNN.

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II. HYPER-CHAOS CIRCUIT

To create a hyper-chaos circuit, we used to the n-double scroll or non-identical n-double scroll and Chua's oscillator using the weak coupling method [8].

A. n-Double scroll circuit

In order to synthesize a hyper-chaos circuit, we first consider Chua's circuit modified to an n-double scroll attractor. The electrical circuit for obtaining n-double scroll, according to the implementation of Arena et al. [12] is given by

$$\begin{aligned} \dot{x} &= \alpha[y - h(x)] \\ \dot{y} &= x - y - z \\ \dot{z} &= -\beta y \end{aligned} \tag{1}$$

with a piecewise linear characteristic

$$h(x) = m_{2n-1}x + \frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|) \tag{2}$$

consisting of 2(2n-1) breakpoints, where n is a natural number. In order to generate n double scrolls one takes $\alpha = 9$ and $\beta = 14.286$. Some special cases are:

1-double scroll

$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, c_1 = 1$

2-double scroll

$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, m_3 = m_1,\ c_1 = 1, c_2 = 2.15, c_3 = 3.6$

3-double scroll

$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7},$

$m_3 = m_1, m_4 = m_2, m_5 = m_3,$

$c_1 = 1, c_2 = 2.15, c_3 = 3.6, c_4 = 8.2, c_5 = 13$

The 2-double scroll attractor and 3-double scroll attractors are shown in Fig. 1 and Fig. 2 respectively.

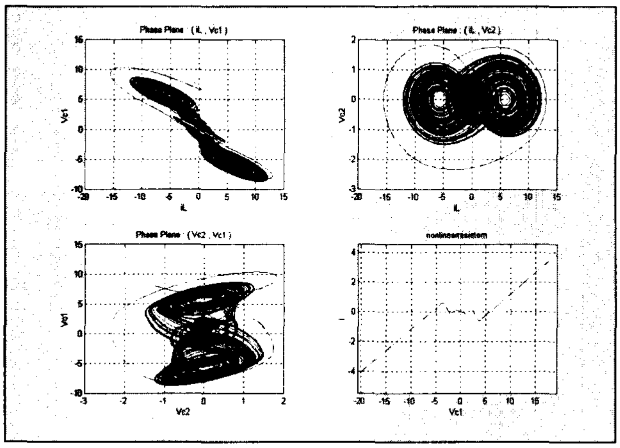


Fig. 1 2-double scroll attractor

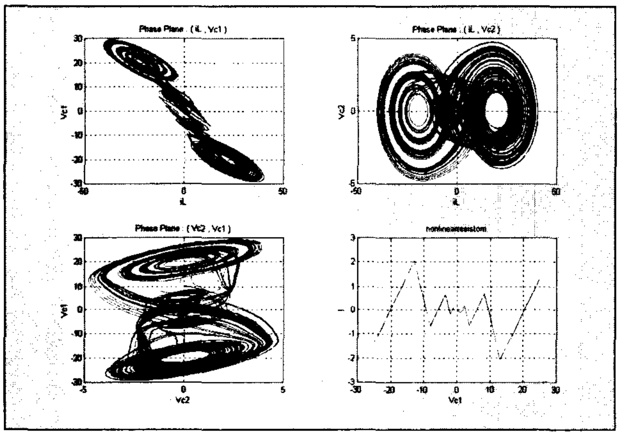


Fig. 2 3-double scroll attractor

B. Hyper-chaos circuit

To synthesize a hyper-chaos circuit, we second consider one-dimension cellular neural network (CNN) with n-double scroll cell [8]. The following equations describe a one-dimensional CNN consisting of identical n-double cell with diffusive coupling as

$$\begin{aligned} \dot{x}^{(j)} &= \alpha\left[y^{(j)} - h(x^{(j)})\right] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} - z^{(j)} \\ \dot{z}^{(j)} &= -\beta\, y^{(j)} \quad j = 1,2,\ldots,L \end{aligned} \tag{3}$$

or

$$\begin{aligned} \dot{x}^{(j)} &= \alpha\left[y^{(j)} - h(x^{(j)})\right] \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} - z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{z}^{(j)} &= -\beta\, y^{(j)} \quad j = 1,2,\ldots,L \end{aligned} \tag{4}$$

where L denotes the number of cells. We impose the condition that $x^{(0)} = x^{(L)}$, $x^{(L+1)} = x^{(1)}$ for equation (3) and (4).

For the coupling constants, $K_0 = 0, K_j = K(j = 1,\ldots,L-1)$ and positive diffusion coefficients D_x, D_y are chosen base on stability theory.

Hardware implementation for hyper-chaos circuit with a CNN using n-double scroll and hyper-chaos attractor are shown in Fig. 3 and 4 respectively.

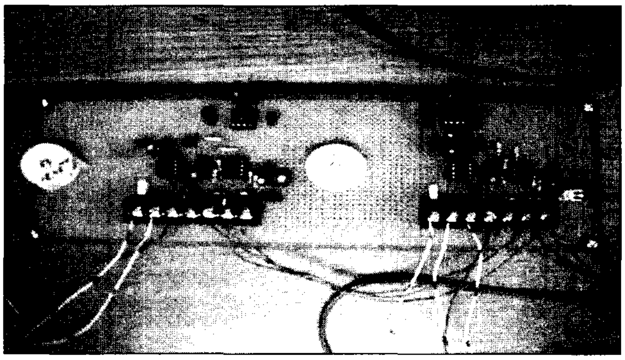


Fig. 3 Hardware implementation for hyper-chaos circuit

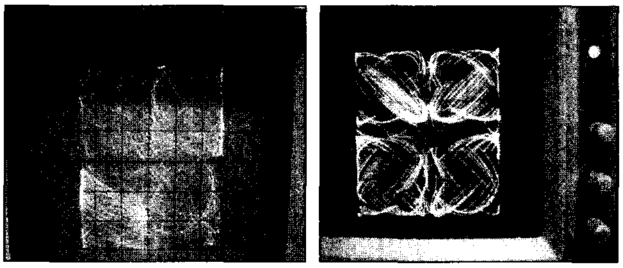


Fig. 4 Hyper-chaos attractor

C. SC-CNN model [12,13]

In [12, 13], the follow generalized cell was introduced:

$$\dot{x}_j = x_j + a_j y_j + G_o + G_s + i_j \tag{5}$$

where j is the cell index, x_j the state variable, y_j the cell output given as

$$y_j = 0.5(|x_j + 1| - |x_j - 1|) \tag{6}$$

where, a_j a constant parameter and i_j a threshold value. In equation (5), G_o is linear combination of the outputs and G_s is state variable of the connected cells.

Generalizing the output nonlinearity (6), the following new output PWL equation is considered

$$y_j = \frac{1}{2}\sum_{k=1}^{2n-1} n_k(|x + b_k| - |x - b_k|) \tag{7}$$

where b_k are the break point and the coefficients n_k are related to the slopes of segments.

SC-CNN cells required to generate the n-double scroll in accordance with the state equation (5) and output equation (7) are given by

$$\begin{aligned}\dot{x}_1 &= -x_1 + a_1 y_1 + a_{12} y_2 + a_{13} y_3 + \sum_{k=1}^{3} s_{1k} x_k + i_1 \\ \dot{x}_2 &= -x_2 + a_{21} y_1 + a_2 y_2 + a_{23} y_3 + \sum_{k=1}^{3} s_{2k} x_k + i_2 \\ \dot{x}_3 &= -x_3 + a_{31} y_1 + a_{32} y_2 + a_3 y_3 + \sum_{k=1}^{3} s_{3k} x_k + i_3\end{aligned} \quad (8)$$

where x_1, x_2, x_3 are state variables and y_1, y_2, y_3 are corresponding outputs. More details about the SC-CNN are given in reference [12, 13]

III. THE SYNCHRONIZATION OF HYPER-CHAOS USING GENERALIZED SYNCHRONIZATION

In order to apply to generalized synchronization theory in the hyper-chaos, we compromised to state equation of dimensionless type of SC-CNN is written as follows:

The state equation of transmitter

$$\begin{aligned}&\dot{x} = Ax + g(x), \\ &g(x) = [g(x_1), 0, 0, g(x_4), 0, 0]^T \\ &\dot{x}' = Ax' + g'(x') + F(x, x')\end{aligned} \quad (9)$$

The state equation of receiver

$$\begin{aligned}&\dot{y} = Ay + g(y), \\ &g(y) = [g(y_1), 0, 0, g(y_7), 0, 0]^T \\ &\dot{y}' = A'x' + g'(y') + F(y, y')\end{aligned} \quad (10)$$

where, $x = [x_1, \ldots x_6]^T$, $y = [y_1, \ldots y_6]^T$ are state variable of 2-double scroll circuit, and $x = [x'_1, \ldots x'_6]^T$, $y = [y'_1, \ldots y'_6]^T$ are Chua's oscillator, $g(x)$ represented as equation (9) is nonlinear element. The Matrix A and A' have the following structures:

$$A = \begin{bmatrix} -\alpha & \alpha & 0 & 0 & 0 & 0 \\ 1 & -1-K & 1 & 0 & K & 0 \\ 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & \alpha & 0 \\ 0 & K & 0 & 1 & -1-K & 1 \\ 0 & 0 & 0 & 0 & -\beta & 0 \end{bmatrix} \quad (11)$$

$$A' = \begin{bmatrix} -\alpha' & \alpha' & 0 & 0 & 0 & 0 \\ 1 & -1-K' & 1 & 0 & K' & 0 \\ 0 & -\beta' & -\gamma' & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha' & \alpha' & 0 \\ 0 & K' & 0 & 1 & -1-K' & 1 \\ 0 & 0 & 0 & 0 & -\beta' & -\gamma' \end{bmatrix} \quad (12)$$

The function vector $F(x, x')$ is used to assure the GS [14] between the 2-double scroll and Chua's oscillator by means of a proper linear feedback action. This action is characterized by the difference between selected state variables of 2-double scroll and Chua's oscillator and by a feedback gain M.

$$\begin{aligned}&F(x, x') = [M(x_1 + x_4 - x'_1), 0, 0, \\ &M(x_1 + x_4 - x'_1 - x'_4), 0, 0]^T\end{aligned} \quad (13)$$

$$\begin{aligned}&F(y, y') = [M(y_1 + y_4 - y'_1), 0, 0, \\ &M(y_1 + y_4 - y'_1 - y'_4), 0, 0]^T\end{aligned} \quad (14)$$

where in order to solve M, we consider equation (9) and (10), have different initial conditions. Following the auxiliary system approach, the time evolution of the differences

$$d_i = x'_i - x''_i, i = 1, 2, \ldots 6 \quad (15)$$

are described by equations.

$$\begin{aligned}\dot{d}_1 &= \alpha'(d_2 - d_1) - \alpha'[g(x'_1) - g(x''_1)] - Md_1 \\ \dot{d}_2 &= d_1 - d_2 + d_3 + K'(d_5 - d_2) \\ \dot{d}_3 &= -\beta d_2 - \gamma d_3 \\ \dot{d}_4 &= \alpha'(d_5 - d_4) - \alpha'[g(x'_4) - g(x''_4)] - Md_1 - Md_4 \\ \dot{d}_5 &= d_4 - d_5 + d_6 + K'(d_2 - d_5) \\ \dot{d}_6 &= -\beta' d_5 - \gamma' d_6\end{aligned} \quad (16)$$

The asymptotical stability of the response system (10) occurs if the dynamical system (16) possess a stable fixed point at the origin d=0, where $d = [d_1, d_2, d_3, d_4, d_5, d_6]^T$

After choosing the following positive definite Lyapunov function as [15]

$$V(d) = d_1^2 + \alpha' d_2^2 + \frac{\alpha'}{\beta'} d_3^2 + d_4^2 + \alpha' d_5^2 + \frac{\alpha'}{\beta'} d_6^2 \quad (17)$$

Thus the derivative of $V(d)$ along the system trajectory can be expressed as:

$$\begin{aligned}\dot{V}(d) \le\ &(-\alpha' - M + \alpha' \max\{|a|, |b|\}) d_1^2 \\ &+ 2\alpha' d_1 d_2 - \alpha' d_2^2 - \gamma' \frac{\alpha'}{\beta'} d_3^2 \\ &+ (-\alpha' - M + \alpha' \max\{|a|, |b|\}) d_2^4 \\ &+ 2\alpha' d_4 d_5 - \alpha' d_5^2 - \gamma' \frac{\alpha'}{\beta'} d_6^2 - Md_1 d_4 \\ &- \alpha' K' d_2^2 - 2\alpha' K' d_2 d_5 + \alpha' K' d_5^2\end{aligned} \quad (18)$$

By considering the worst case, equation (18) can be rewritten as a quadratic form [16]

$$\dot{V}(d) = d^T \Psi d \quad (19)$$

where $\Psi \in \Re^{6\times 6}$ is a symmetric matrix given by:

$$\Psi = \begin{bmatrix} M-p & -\alpha' & 0 & 0 & 0 & 0 \\ -\alpha' & \alpha'(k'+1) & 0 & 0 & -\alpha' K' & 0 \\ 0 & 0 & -\gamma'\frac{\alpha'}{\beta'} & 0 & 0 & 0 \\ M & 0 & 0 & M-p & -\alpha' & 0 \\ 0 & -\alpha' K' & 0 & -\alpha' & \alpha'(K'+1) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma'\frac{\alpha'}{\beta'} \end{bmatrix} \quad (20)$$

with $p = -\alpha' + \alpha' \max\{ |a|, |b| \}$.

Equation (20) can be rewritten as: $\dot{V}(d) = -d_r^T \psi' d_r - \gamma' \frac{\alpha'}{\beta'} d_3^2$

$$-\gamma' \frac{\alpha'}{\beta'} d_6^2 \quad (21)$$

Where $d_r = [d_1, d_2, d_4, d_5]^T$ is a reduced difference vector and $\Psi^1 \in \Re^{4\times 4}$ is the symmetric submatrix obtained from Ψ by deleting the third and the sixth rows and the third and sixth columns, respectively.

By imposing that the matrix Ψ^1 be positive definitive, i.e. that all principal minors be strictly positive [17], the parameter M can be derived. Finally, it can be concluded that $\dot{V}(d)$ is strictly negative for every $d_i (i = 1,2...6)$ when

$$M > \frac{\alpha'}{K'+1} + \alpha' \max\{ |a|, |b| \} \quad (22)$$

The block diagram of the proposed hyper-chaos synchronization system is shown in Fig. 5 and the result of hyper-chaos synchronization is shown in Fig. 6 respectively.

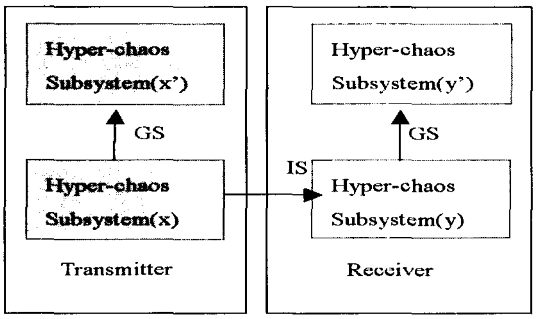
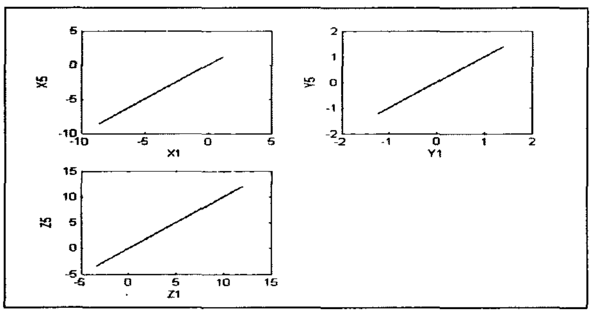
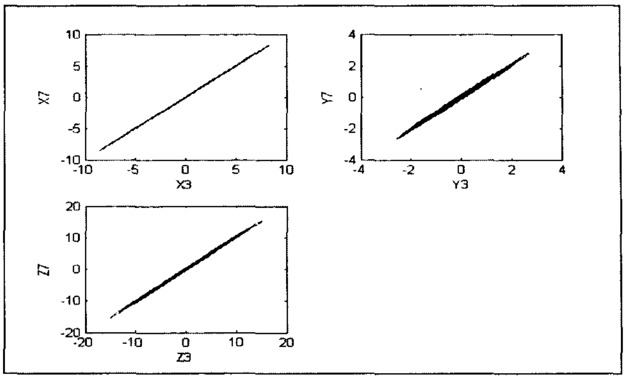


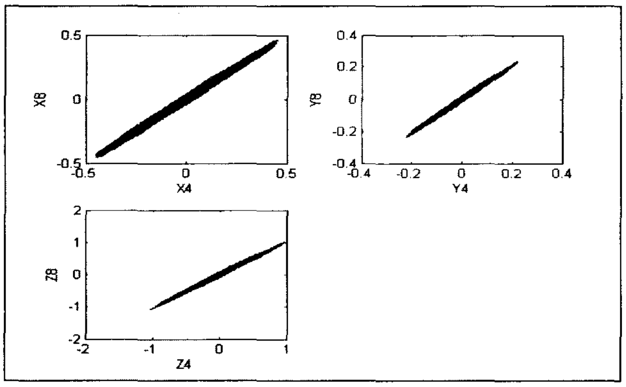
Fig. 5 The Block diagram of hyper-chaos synchronization.



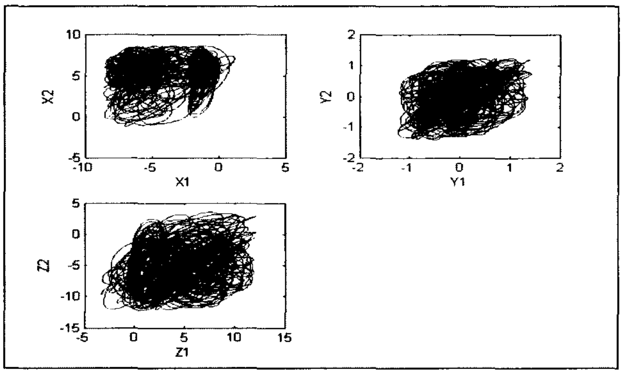
(a) Master subsystem $(x(x_1, x_2, x_3))$ of transmitter vs. master subsystem $(y(y_1, y_2, y_3))$ of receiver



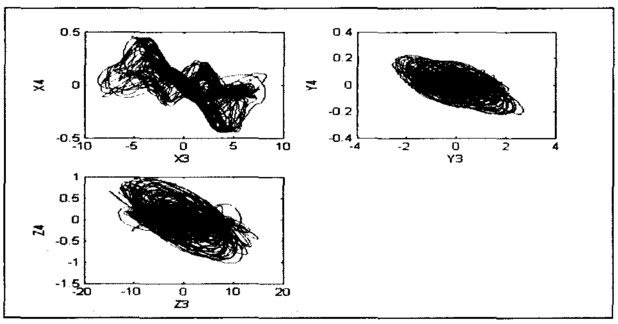
(b) Slave subsystem $(x'(x'_1, x'_2, x'_3))$ of transmitter vs. slave subsystem $(y'(y'_1, y'_2, y'_3))$ of receiver



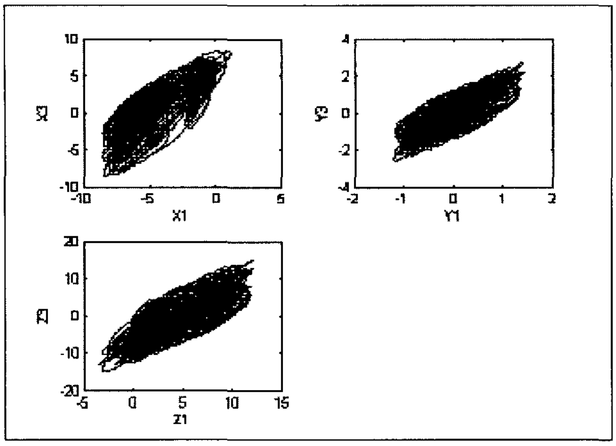
(c) Slave subsystem $(x'(x'_4, x'_5, x'_6))$ of transmitter vs. slave subsystem of $(y'(y'_4, y'_5, y'_6))$ receiver



(d) Phase plane of master subsystems (x_1, x_2, x_3) vs. (y_1, y_2, y_3))



(e) Phase plane of slave subsystem of transmitter (x'_1, x'_2, x'_3) vs. (y'_1, y'_2, y'_3)



(f) Phase plane of master subsystem (x_1, x_2, x_3) vs. slave subsystem (y'_1, y'_2, y'_3)

Fig. 6 The synchronization result

In Fig. 6, we confirmed that effective synchronization result between the transmitter and receiver in the hyper-chaos circuit.

IV. CONCLUSIONS

In this paper, we introduced a new hyper-chaos synchronization method which is called GS(Generalized Synchronization) method using SC-Cellular Neural Network (SC-CNN) as a hyper-chaos. As a computer simulation result, we confirm GS by compare to phase plane in the transmitter and receiver about each other.

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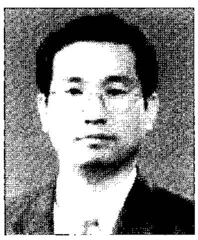

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