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Intelligent Digital Controller Using Digital Redesign

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Abstract

In this paper, a systematic design method of the intelligent PAM fuzzy controller for nonlinear systems using the efficient tools-Linear Matrix Inequality and the intelligent digital redesign is proposed. In order to digitally control the nonlinear systems, the TS fuzzy model is used for fuzzy modeling of the given nonlinear system. The convex representation technique also can be utilized for obtaining TS fuzzy models. First, the analog fuzzy-model-based controller is designed such that the closed-loop system is globally asymptotically stable in the sense of Lyapunov stability criterion. The simulation results strongly convince us that the proposed method has great potential in the application to the industry.

Key words : Fuzzy control, Fuzzy modeling, Digital redesign, Extend parallel distributed Compensation (EPDC), PAM Control, Linear matrix inequality(LMI)

1. Introduction

Fuzzy control shows robust performance, especially when the controlled system can hardly modeled mathematically, or the controlled system have nonlinearity and uncertainty [1-7]. There exist three digital design approaches for digital control systems[9-10]. The first approach, called the direct design approach, is to discretize the analog plant and then determine a digital controller for the discretized plant. However this approach has shown the degraded control performance because it ignores the inter-sample behavior of the control system. The second approach, called the digital redesign approach, is to pre-design an analog controller for the analog plant and then carry out the digital redesign for the pre-designed analog controller. The third approach, called the direct sampled-data approach, is directly design a digital controller for the analog plant, which is still under development. Among them, this thesis utilizes on the second approach. called the digital redesign approach.

In general, there exist two types of digital controller [9, 10]. The first is PAM(Pulse-Amplitude Modulated) controller and the second is PWM(Pulse-Width Modulated) controller. The PAM controller, which produces a series of piecewise-constant continuous pulses having variable amplitude and variable or fixed width, is commonly utilized in digital control of all types. The PWM controller, which produces a series of discontinuous pulses with a fixed amplitude and variable width, has become popular in industry for on-off control of DC power converters and stepper motors, etc. [10]. In this paper, we propose that the intelligent digital redesigned PAM fuzzy controller for the digital control of continuous-time nonlinear systems that is represented by the TS fuzzy model. First, a suitable continuous-time TS fuzzy-model based controller is designed such that the controlled TS fuzzy model is globally asymptotically stable in the sense of Lyapunov. The controller design condition is formulated in terms of LMIs. which is quite promising since the extremely efficient numerical algorithms can be used. For the intelligent digital redesign of the pre-designed fuzzy model-based controller, the continuous-time TS fuzzy model is discretized with a sufficiently small sampling period. Using some approximation technique, the state of the discretized version of the digitally controlled system match that of the analogously controlled system as closely as possible. In order to verify the effectiveness of the proposed intelligent digital redesign technique, the flexible joint robot arm are simulated.

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2. Preriminary

2.1 Fuzzy model and Controller

The continuous-time fuzzy model, proposed by Takagi and Sugeno, is described by fuzzy **IF-THEN** rules which locally represent linear input-output relations of nonlinear systems. The i-th rule of T-S fuzzy model is defined by

Plant Rule i :

$$\begin{aligned}&\text{IF } x_1(t) \text{ is } M_1^i \text{ and} \ldots \text{and } x_n(t) \text{ is } M_n^i \\ &\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t)\end{aligned} \tag{1}$$

The final defuzzified output of fuzzy system is inferred by

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{q} \omega_i(t)\,(A_i x(t) + B_i u(t))}{\sum_{i=1}^{q} \omega_i(t)} \\ \omega_i(t) &= \prod_{j=1}^{n} M_j^i(x_j(t))\end{aligned} \tag{2}$$

where, M_j^i $(j=1,2,\ldots,n)$ is i-th fuzzy set, q is the number of rules of this TS fuzzy model, $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the control input vector, $A_i \in R^{n\times n}$

and $B_i \in R^{n\times m}$ are system matrix and input matrix, $x_1(t), \ldots, x_n(t)$ are premise variables.

Controller Rule i :

IF $x_1(t)$ is M_1^i and ... and $x_n(t)$ is M_n^i

$$\text{THEN } u(t) = -K_i x(t) + E_i r(t) \tag{3}$$

where, $K_i = [k_1^i, \ldots, k_n^i]$ is state feedback gain vector, $E_i = [E_1^i \cdots E_n^i]$ is the feedforward gain vector in i-th subspace. $r(t)$ is the reference input.

The final deffuzified output of fuzzy controller for Equ. (3) is as follow:

$$u(t) = \frac{\sum_{i=1}^{q} \omega_i(t)\,(-K_i x(t) + E_i r(t))}{\sum_{i=1}^{q} \omega_i(t)} \tag{4}$$

The fuzzy controller shares the same fuzzy sets with the fuzzy system (2). For each rule, we can use linear control design techniques. The overall closed-loop fuzzy system obtained by combining (1) and (4) becomes

$$\dot{x}(t) = \frac{\sum_{i=1}^{q}\sum_{j=1}^{q} \omega_i(t)\omega_j(t)\,(\,(A_i - B_i K_j)\,x(t) + B_i E_i r(t)\,)}{\sum_{i=1}^{q}\sum_{j=1}^{q} \omega_i(t)\omega_j(t)} \tag{5}$$

2.2 Discretization of the continuous-time TS fuzzy models

In this section, we propose the discrete-time TS fuzzy model against the continious-time TS fuzzy model. Let us consider the observable continuous-time plant by:

$$\begin{aligned} \dot{x}_c(t) &= A\,x_c(t) + B\,u_c(t), \qquad x_c = x_0 \\ y_c(t) &= C x_c(t) \end{aligned} \tag{6}$$

where, $x_c(t)$ is the state vector, $u_c(t)$ is the input vector, $y_c(t)$ is the output vector and (A, B, C) are constant matrices of appropriate dimensions. The continuous-time state feedback control law for the system is as follow:

$$u_c(t) = -K_c x_c(t) + E_c r(t) \tag{7}$$

where, the analog feedback gain K_c and the feedforward gain E_c have been given, and $r(t)$ is the reference input, assumed to be a piecewise- constant signal, $r(t) = r(KT)$ for $kT \le t < k(T+1)$ with T as the sampling period.

Substitute (6) into (7), we can obtain

$$\dot{x}_c(t) = A_c x_c(t) + B E_c r(t), \qquad x_c(0) = x_0 \tag{8}$$

where, A_c is $A - BK_c$.

The corresponding discrete-time model for $r(t) = r(kT)$ with $kT \le t < k(T+1)$ is in (9).

$$x_c(kT+T) = G_c x_c(kT) + H_c u_c(kT) \tag{9}$$

where, $G_c = e^{A_c T}$, $H_c = [G_c - I_n]A_c^{-1}B$.

The fast rate sampled discrete-time model in (8) for $T_N = T/N$, where N is an integer and $r(t) = r(kT)$ for $kT \le t < k(T+1)$, can be written by

$$x_c(kT + iT_N) = G_c^{(i)} x_c(kT) + H_c^{(i)} E_c r(kT) \quad \text{for } i = 1, 2, \ldots, N, \tag{10}$$

Where, $G_c^{(i)} = (e^{A_c T_N})^{(i)}$, $H_c^{(i)} = [G_c^{(i)} - I_n]A_c^{-1}B$.

By applying the piecewise-constant input function $u_d(t)$, the continuous- time state-space equation (6) is as follows:

$$\begin{aligned} \dot{x}_d(t) &= A x_d(t) + B u_d(t) \qquad x_d(0) = x_0 \\ y_d(t) &= C x_d(t) \end{aligned} \tag{11}$$

where, $u_d(t) = u_d(kT)$ for $kT \le t < kT + T$.

The digital control law for (11) with $r(t) = r(kT)$ for $kT \le t < k(T+1)$ is

$$u_d(kT) = -K_d x_d(kT) + E_d r(kT) \tag{12}$$

where, K_d is the feedback digital gain and the E_d is the feedforward digital gain.

By substituting (11) into (12), we can obtain

$$\dot{x}_d(t) = (A - BK_d) x_d(kT) + BE_d r(kT) \tag{13}$$

The corresponding discrete-time model of the sampled-data system in (13) is

$$\dot{x}_d(kT+T) = (G - HK_d) x_d(kT) + HE_d r(kT) \tag{14}$$

where, $G = e^{AT}$, $H = [G - I_n]A^{-1}B$.

The process of finding digital gains (K_d, E_d) in (12) from the analog gains (K_c, E_c) in (7) so that the closed-loop state $x_d(t)$ in (13) closely matches the closed-loop state $x_c(t)$ in (8), is called the state-matching digital redesign.

3. Intelligent PAM Fuzzy Controller Design and Stability Analysis

In this Chapter, we propose the design method of the intelligent PAM fuzzy controller for nonlinear systems. To do this, we propose the stability conditions of both the fuzzy model and the fuzzy control system. The purpose in this Chapter is to design the stable fuzzy controller.

3.1 Intelligent Controller Design Using Digital Redesign

In this Section, we propose the PAM fuzzy controller design using the intelligent digital redesign at i-th subspace.

At i-th subspace, consider a controllable and observable analog nonlinear system represented by

$$\begin{aligned} \dot{x}_c(t) &= A_i x_c(t) + B_i u_c(t) \\ y_c(t) &= C_i x_c(t) \end{aligned} \tag{15}$$

where, $x_c(t) \in R^{n\times 1}$ is the state vector, $u_c(t) \in R^{m\times 1}$ is the input vector.
Control input $u_c(t)$ in (15) at i-th subspace is

$$u_c(t) = -K_c^i x_c(t) + E_c^i r_c(t) \quad (16)$$

Where, $K_c \in R^{m\times n}$, $E_c \in R^{m\times m}$ are the feedback gain and the feed-forward gain and $r(t)$ is the reference input.
In digital control of continuous-time systems, the continuous-time state-space equations need to be converted into discrete-time state-space equations. In the TS fuzzy-model-based controller, the sampling period for the fuzzy modeling and the controller design is assumed to be same.
The state $x_d(t)$ in (16) equals to the state $x_c(t)$ in equation (8) at each sampling instant, $t = KT_f$. By applying the piecewise-constant input function $u_d(t)$, the continuous- time state-space equation (6) is described by

$$\begin{aligned} \dot{x}_d(t) &= A_i x_d(t) + B_i u_d(t) \\ y_d(t) &= C_i x_d(t) \end{aligned} \quad (17)$$

where, $u_d(t) = u_d(kT)$ for $kT \le t < (k+1)T$.
Also, let the digital control law for the system in (17) with $r(t) = r(kT)$ for $kT \le t < (k+1)T$ be

$$u_d(t) = -K_d^i x_d(t) + E_d^i r_d(t) \quad (kT_f \le t < KT_f + T_f) \quad (18)$$

The designed closed-loop sampled-data system using (17) and (18) becomes

$$\dot{x}_d(t) = A_i x_d(t) - BK_d x_d(kT) + B_i E_d r(kT) \quad (19)$$

By utilizing a zero-order in (19), the corresponding discrete-time model of the continuous-time system in (17) is

$$\begin{aligned} x_d(kT+T) &= G_i x_d(kT) + H_i u_d(kT) \\ y_d(kT) &= C_i x_d(kT) \end{aligned} \quad (20)$$

where, $G_i = e^{A_i T_f}$, $H_i = \int_0^{T_f} e^{A_i T_f} B_c dt = (G_i - I_n) A_i^{-1} B_i$.
If $u_c(t) = u_d(t)$, then the analog control input $u_c(t)$ and the digital control input $u_d(t)$ are the same value as follows:

$$u_c(t) = \sum_{T_f=1}^{N} W_{kf}\, \Phi_{kf} = \sum_{T_f=1}^{N} u_d(KT_f) \quad (21)$$

where, Φ_{kf} is orthonormal series.
Using (21), W_{kf} is

$$W_{kf} = \frac{1}{T_f} \int_{KT_f}^{(K+1)T_f} u_c(t)\, dt \quad (22)$$

By applying the Chebyshev quadrature formula to (22), we obtain

$$W_{kf} = \frac{1}{N+1} \sum_{i=0}^{N} x_c\left(KT_f + i\frac{T_f}{N}\right) + E_c r(KT_f) \quad (23)$$

where,

$$x_c\left(KT_f + i\frac{T_f}{N}\right) = G_{cr} x_c(KT_f) + H_{cr} E_c r(KT_f) \quad (24)$$

where,

$$G_{ir} = e^{A_{ir} T_f}, \quad A_{ir} = A_i - B_i K_c^i,$$
$$H_{ir} = \int_0^{T_f} e^{A_{ir} T_f} B_i dt = (G_{ir} - I_n) A_{ir}^{-1} B_i.$$

By substituting (24) into (22), we obtain

$$W_{kf} = \frac{1}{N+1} \sum_{i=0}^{N} \left(G_{cr} x_c(KT_f) + H_{cr} E_c r(KT_f) \right) + E_c r(KT_f) \quad (25)$$

Whrer, W_{kf} is the same control input as the analog control one $u_c(t)$. The control input W_{kf} stabilize the system (24).

$$x_d(kT_f + T_f) = G_i x_d(KT_f) + H_i u(KT_f) \quad (26)$$

The closed-loop system is shown by

$$\begin{aligned} x_d(KT_f + T_f) = {} & \\ & G_c x_d(KT_f) + H_c\Big(\frac{1}{N+1} \sum_{i=0}^{N} (G_{cr} x_c(KT_f) \\ & + H_{cr} E_c r(KT_f)) + E_c r(KT_f)) + E_c r(KT_f) \Big) \end{aligned} \quad (27)$$

In (27), overall closed-loop system is as follows:

$$x_d(KT_f + T_f) = \hat{G}_{CN}\, x_d(KT_f) + \hat{H}_{CN}\, r(KT_f) \quad (28)$$

where, $\hat{G}_{CN} = G_i - H_i K_{dr}^i$, $G_i = e^{A_i T_f}$,
$\hat{H}_{CN} = H_c E_{dr}$, $H_i = (G_i - I_n) A_i^{-1} B_i$.

Because (27) and (28) have same result, then we have

$$\begin{aligned} G_c - H_c K_{dr} &= G_c - H_c K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} \\ H_c E_{dr} &= -H_c \left(K_c \frac{1}{N+1} \sum_{i=0}^{N} H_{cr} - I_n \right) E_c \end{aligned} \quad (29)$$

Solving (29) yields the desired digital control gains as

$$\begin{aligned} K_{dr} &= K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} \\ E_{dr} &= \left(I_n - K_c \frac{1}{N+1} \sum_{i=0}^{N} H_{cr} \right) E_c \end{aligned} \quad (30)$$

Representing (30) yields the digital control gains as

$$\begin{aligned} K_{dr} &= K_c \frac{1}{N+1} (G_{cr} - I_n)^{-1} (G_{cr} - I_n) + G_{cr} \\ E_{dr} &= E_c - K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} - K_c A_{cr}^{-1} B_r E_c \\ &= E_c + (K_c - K_{dr}) A_{cr}^{-1} B_r E_c \end{aligned} \quad (31)$$

Assuming $N \rightarrow \infty$, the digital gains in (31) becomes

$$K_{dr} = \lim_{N\to\infty} K_c[(N+1) (G_{cr} - I_n)^{-1}(G_{cr} - I_n)] + \lim_{N\to\infty} K_c[\frac{1}{N+1} G_{cr}] = \frac{1}{T_f} K_c A_{cr}^{-1}(G_{cr} - I_n)$$

$$E_{dr} = (I_m + K_c A_{cr}^{-1} (B_r - \frac{1}{T_f} H_{cr}))E_c \quad (32)$$

Assuming $N \to 1$, the gains in (33) becomes

$$K_{dr}^i = \frac{1}{2} K_c^i (I_n + G_{ir}) \qquad E_{dr}^i = (I_m - \frac{1}{2} K_c^i H_{ir}) E_c^i \quad (33)$$

We apply the bilinear transform method to (32), then we have

$$K_{dr}^i = \frac{1}{2} (I + \frac{1}{2} K_c^i H_r)^{-1} K_c^i (G_r + I) \qquad E_{dr}^i = (I + \frac{1}{2} K_c H_r)^{-1} E_c \quad (34)$$

where, K_{dr}^i, E_{dr}^i are the digital feedback gain and the digital feed-forward gain in i-th subspace.

3.2 Stability Analysis and Controller Design Using LMI

Theorem 1 [11]: *The equilibrium of a fuzzy system is asymptotically stable in the large if there exists a common positive definite matrix P such that the following two conditions are satisfied.*

$$\{A_i - B_i K_i\}^T P + P\{A_i - B_i K_i\} < 0, \quad i = 1, \ldots, q \quad (35)$$

$$G_{ij}^T P + P G_{ij} < 0, \quad i < j \le q \quad (36)$$

where, $G_{ij} = \frac{\{A_i - B_i K_j\} + \{A_j - B_j K_i\}}{2}$

Theorem 1 is not the stability analysis of nonlinear system but the stability analysis of fuzzy system. The design problem for the fuzzy controller have to satisfy conditions of theorem 1. Where, K_i is the feedback gain. If we obtain the common positive definite matrix P, the stability of closed-loop system enable to decision. But the obtaining of common positive matrix P is difficult, therefore the guaranteed stability of fuzzy system is difficult. In other words, the overall closed-loop system is unstable though the local systems are stable. Also, existed PDC method is not the stability analysis of nonlinear system but the stability analysis of TS fuzzy system. Hence, the tracking problem is not referred to this expression. In this paper, to solve this shortcoming, we proposes extension parallel distributed compensation (EPDC) [1]. In order to solve these problems, we modify the controller rule of PDC with the same premise in (.1) as follows, which is called an EPDC:

Controller Rule i :

IF $x(t)$ is M_1^i and...and $x^{(n-1)}(t)$ is M_n^i

THEN $u(t) = -K_i x(t) + E_i r(t) \quad (i = 1, 2, \ldots, q)$ (37)

where, K_i and E_i are feedback gain and feedforward gain in i-th subspace, respectively, and $r(t)$ is the reference input. The local gains are obtained by LMI, In (37), that are stabilized local systems.

$$u(t) = -\sum_{i=1}^{q} \mu_i(x(t)) K_i x(t) + \sum_{i=1}^{q} \mu_i(x(t)) E_i r(t) = -K(\mu) x(t) + E(\mu) r(t) \quad (38)$$

The control input $u(t)$ in (38) stabilize the overall closed-loop system. And, we obtain the feedback gain and the feedforward gain using LMI.
Consider a continuous-time TS fuzzy model, described by the following state space equation.

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t)) (A_i x(t) + B_i u(t)) \quad (39)$$

$$u(t) = -\sum_{i=1}^{q} \mu_i(x(t)) K_i x(t) + \sum_{i=1}^{q} \mu_i(x(t)) E_i r(t) = -K(\mu) x(t) + E(\mu) r(t) \quad (40)$$

Substituting (39) into (40) gives

$$\dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_j(x(t)) \{(A_i - B_i K_j)x(t) + B_i E_i r(t) \} \quad (41)$$

The main result on the global asymptotic stability of continuous-time TS fuzzy model is summarized in the following theorem.

Theorem 2. *If there exist a symmetric and common positive definite matrix P, some matrices K_i such that the following LMI are satisfied, then the continuous-time TS fuzzy system is asymptotically stabilization via the TS fuzzy model based state feedback controller.*

$$\Gamma > 0$$

$$\begin{pmatrix} A\Gamma + \Gamma^T - B\Phi - \Phi^T B^T & BE \\ BE^T & I \end{pmatrix} < 0 \quad (42)$$

If there exist Φ , E *and the symmetric positive definite matrix* Γ, *Conclusionally, the system stabilized asymptotically. Where* $P^{-1} = \Gamma, KP^{-1} = \Phi$.

Proof)

$$\dot{x}(t) = Ax(t) + B u(t) \qquad u(t) = -Kx(t) + Er(t) \quad (43)$$

Consider the Lyapunov function candidate about the system (43) as follows:

$$V = x^T P x > 0$$
$$\dot{V} = \dot{x}^T P x + x^T P \dot{x} < 0 \quad (44)$$

If the condition is satisfied, overall closed-loop system can be globally asymptotically stabilized.

$$\dot{x}(t) = (A - BK)\, x(t) + BEr(t) \quad (45)$$

Consider the Lyapunov function candidate about the system (45) as follows:

$$\begin{aligned}\dot{V}(t) &= ((A - BK)x(t) + BEr(t))^T Px + x^T P((A - BK)x(t) + BEr(t)) \\ &= \begin{pmatrix} x^T & r^T \end{pmatrix} \begin{pmatrix} (A-BK)^T P + P(A-BK) & PBE \\ (PBE)^T & I \end{pmatrix} \begin{pmatrix} x \\ r \end{pmatrix} < 0\end{aligned} \quad (46)$$

If there exist symmetric positive definite matrices P, K , the closed-loop system is stable in the Lyapunov sense. But the above matrix is QMI, so we need to make the LMI form by changing variables. Pre- and post-multiplying the following matrix both side of the QMI.

Let $P^{-1} = \Gamma, KP^{-1} = \Phi$.

$$\begin{pmatrix} P^{-1} & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} (A-BK)^T P + P(A-BK) & PBE \\ (PBE)^T & I \end{pmatrix} \begin{pmatrix} P^{-1} & 0 \\ 0 & I \end{pmatrix}$$

$$= \begin{pmatrix} A\Gamma + \Gamma^T - B\Phi - \Phi^T B^T & BE \\ BE^T & I \end{pmatrix} < 0 \quad (47)$$

If there exist Φ_i, E_i and the symmetric positive definite matrix Γ satisfying the following LMIs.

$$\Gamma > 0$$

$$\begin{pmatrix} A_i\Gamma + A_i\Gamma^T - B_i\Phi_i - \Phi_i^T B_i^T & B_i E_i \\ (B_i E_i)^T & I \end{pmatrix} < 0 \quad (48)$$

Therefore the stability of the TS fuzzy model can be cast as follows:

$$\begin{aligned} u(t) &= -\sum_{i=1}^{q} \mu_i(x(t))K_i\, x(t) + \sum_{i=1}^{q} \mu_i(x(t))E_i\, r(t) \\ &= -K(\mu)\, x(t) + E(\mu)\, r(t) \end{aligned} \quad (49)$$

4. Single Link Flexible-Joint Robot Arm

Figure 1 shows the mechanism of the single link flexible-joint robot arm. In this figure, M is the total mass of arm, I is the inertia of link, L is length of link, k is the spring of the inertia coefficient, J is the rotor inertia of the actuator, and g is the gravity constant. The mechanism of this robot is derived by

$$\begin{aligned} I\ddot{q}_1 + MgL\sin(q_1) + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= u \end{aligned} \quad (50)$$

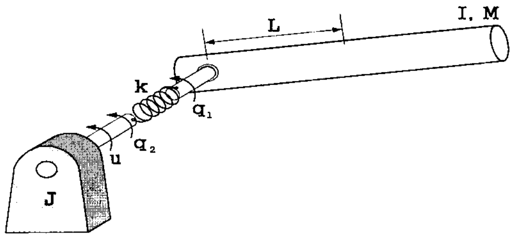

Fig. 1. Single link flexible joint robot arm

Let, $[x_1\ x_2\ x_3\ x_4]^T = [q_1\ \dot{q}_1\ q_2\ \dot{q}_2]^T$. Then, the TS fuzzy model of the system (50) can be obtain by

Plant Rules:

Rule 1 : IF x_1 is *about* 0 THEN $\dot{x} = A_1 x + B_1 u$

Rule 2 : IF x_1 is *about* π THEN $\dot{x} = A_2 x + B_2 u$ (51)

where,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{MgL}{I} - \frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{MgL}{I} - \frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

The parameters of system to do simulation are as follows.

$M = 1kg,\ I = 1kg - m^2,\ L = 1m,\ k = 1N/m,\ J = 1kg - m^2,\ g = 9.8m/s^2$

The membership function for Rule 1 and Rule 2 are shown in Fig. 4.8.

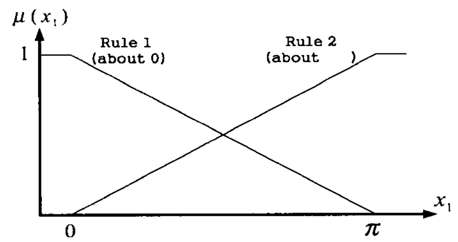

Fig. 4.8. Membership function of the fuzzy model

A common positive definite matrix P that satisfies stability condition defined by Theorem 2 is found to be

$$P = \begin{bmatrix} 812.9896 & 358.0774 & 70.0615 & 6.8172 \\ 358.0774 & 165.2965 & 32.5805 & 3.2415 \\ 70.0615 & 32.5805 & 6.5453 & 0.6460 \\ 6.8172 & 3.2415 & 0.6460 & 0.0700 \end{bmatrix}$$

Then, stability condition of theorem 2 is satisfied. Therefore, overall fuzzy system is stable in Lyapunov sense.

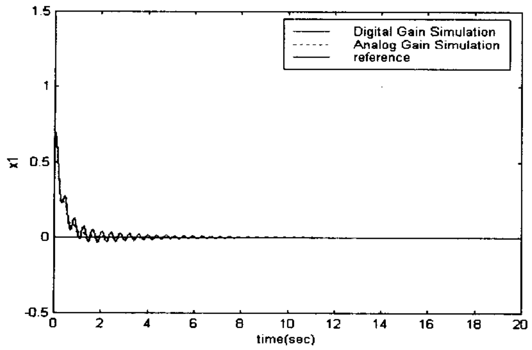

Fig. 2. Response X1

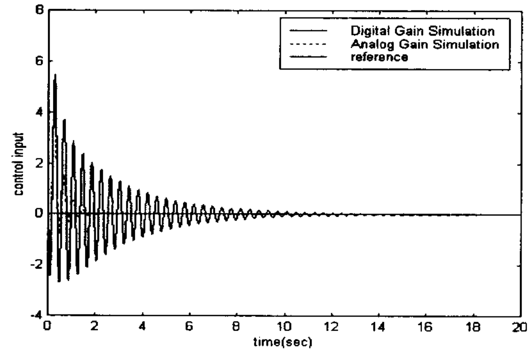

Fig. 3. Control input u(t)

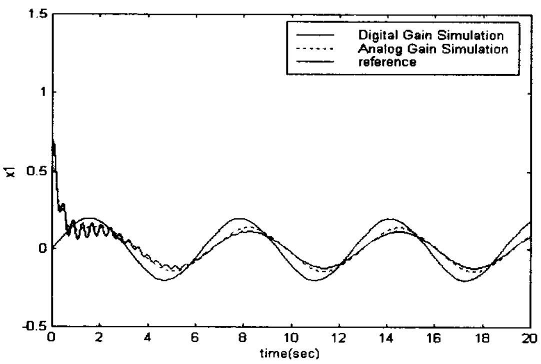

Fig. 4. Response x1

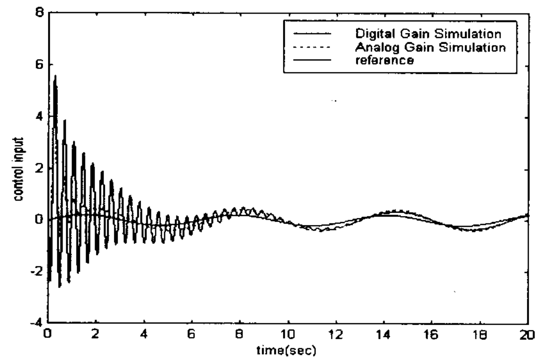

Fig. 5. Control input u(t)

Fig 2. and Fig 3. shows response feature of single link flexible joint robot arm, control input when $r(t)=0$. The initial conditions is $x_0=[\pi/6\ 0\ 0\ 0]^T$. Figure 4 and 5 show response feature of single link flexible joint robot arm, control input when $r(t)=0.2\sin(t)$. The delay phenomenon of Fig 4 and 5 are because of the differences between practical model and computer simulation.

5. Conclusion

In this paper, we have proposed the design method of intelligent PAM fuzzy controller for nonlinear systems represented by the TS fuzzy model. The basic approach are the Lyapunov stability theory using LMI and the intelligent digital redesign using state-mating. In the step of design of analog fuzzy-model-based controller, the design condition was formulated in terms of LMIs. The digital fuzzy-model- based controller has been successfully constructed via the intelligent digital redesign. Finally, simulation results of the single link flexible joint robot arm have convincingly shown the feasibility and effectiveness.

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