Fuzzy Modeling and Control of Differential Driving Wheeled Mobile Robot: To Achieve Performance Objective

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Abstract

The dynamics of the DDWMR depends on the velocity difference of the two driving wheels. And which is known as a type of non-holonomic equation. By this reason, the treatment of DDWMR had become difficult and conservative.

In this paper, the differential-driving wheeled mobile robot is considered. The Takaki-Surgeno fuzzy model and a control method for DDWMR is presented. The suggested controller has three control elements. The first element is fuzzy state feedback designed for eliminating the dependence of time-varying parameter. The second element is weighting controller which is designed for good frequency response. The third controller is PI-controller which is designed for good command following and robustness with un-modeled dynamics. In order for achieving the performance objective, the design of controller is based on the loop-shaping algorithm.

Key words: DDWMR, T-S fuzzy model, pole-placement, loop-shaping, PI controller

1. Introduction

The wheeled mobile robot and its control schemes have been studied by many researchers with various degrees of application and success [1-6]. Most of these studies are concentrated on the development, control and planning the strategy of mobile robot. But, because of the wheeled mobile robot is modeled and controlled by a nonlinear system framework, its treatment is very complicated and conservative.

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In this paper, the differential-driving wheeled mobile robot is considered. The Takaki-Surgeno fuzzy model and a control method for DDWMR is presented. The suggested controller has three control elements. The first element is fuzzy state feedback designed for eliminating the dependence of time-varying parameter. The second element is weighting controller which is designed for good frequency response. The third controller is PI-controller which is designed for good command following and robustness with un-modeled dynamics.

2. Modeling of Wheeled Mobile Robot

In this section, the modeling of DDWMR is presented. Based on the Lagrange dynamics, the dynamic model for DDWMR is derived and T-S fuzzy model of it is presented.

2.1 Modeling of wheeled Mobile Robot

Various types of wheeled mobile robots have been developed, and modeling and its control strategies have been studied. Because of their simple structure and controllability, the differential-drive and car-like WMR are used popularly in the fields of applications. In this paper, some assumptions are made as:

- ① Each wheel dose not allows any kind of slippage.
- ② The dynamic effects of each wheel are neglected.

The structure of the mobile robot, considered in this paper, is shown in Fig. 1. The relation between the forward velocity and the wheel angular velocity is described by

$$\begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$
 (1)

where, v and ϕ are forward and rotation velocities of the

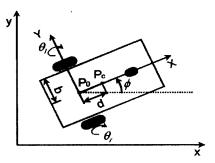


Figure 1. The structure DDWMR.

robot, respectively, and r is the ratio of the wheel. And b is the displacement from center robot to center of wheel. The kinetic equation is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\phi} \end{bmatrix}$$
 (2)

In order to derive the dynamic equations, we now define some variables.

 $I_{\mathcal{C}}$: robot inertia except wheels and rotor

 I_W : motor rotor inertia for wheels and wheel axis

 I_m : motor rotor inertia for wheels and wheel diameter

m: mass of robot except wheels and motor rotor

 m_C : mass of wheels and motor rotor

The dynamic equation of a of robot is described by [4,5]

$$M(q)\ddot{q} + V(q, \ \dot{q}) = E(q)\tau - \hat{A}^{T}(q)\lambda \tag{3}$$

where, λ is Lagrangy multiplier, τ is the torque of each wheels, and d is the displacement from the center of mass to the center of rotation, $q = \begin{bmatrix} x & y & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T$ and

$$M(q) = \begin{bmatrix} m & 0 & -m_c c d \sin \phi & m_c c d \sin \phi \\ 0 & m & m_c c d \cos \phi & -m_c c d \cos \phi \\ -m_c c d \sin \phi & m_c c d \cos \phi & I_c^2 + I_w & -I_c^2 \\ m_c c d \sin \phi & -m_c c d \cos \phi & -I_c^2 & I_c^2 + I_w \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} 2m_c d\dot{\phi}^2 \cos\phi \\ 2m_c d\dot{\phi}^2 \sin\phi \\ 0 \\ 0 \end{bmatrix},$$

$$\hat{A}(q) = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0\\ -\cos\phi & -\sin\phi & cb & cb \end{bmatrix}$$

$$E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

In order to eliminate the Lagrange multiplier, we select the null space of $\hat{A}(q)$ as

$$S(q) = \begin{bmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

then, equation (3) becomes

$$S^{T}(q)M(q)(S(q)\ddot{\theta} + \dot{S}(q)\dot{\theta}) + S^{T}(q)V(q,\dot{q}) = \tau \quad (4)$$

Equation (4) is a type of non-holonomic equation. This type of system cannot be linearized by using the state feedback.

We now present a LPD system model for the mobile robot. Equation (4) becomes

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$
(5)

where.

$$\begin{split} M_{11} &= M_{22} = mc^2b^2 + I_c^2 + I_w \\ M_{12} &= M_{21} = mc^2b^2 - I_c^2 \\ N_{11} &= m_c cbd(c + \dot{\phi}), \qquad N_{12} = m_c cbd(c - \dot{\phi}) \\ N_{21} &= -m_c cbd(c - \dot{\phi}), \qquad N_{22} = -m_c cbd(c + \dot{\phi}) \end{split}$$

In equation (5), the variable $\dot{\phi}$ must be selected as a parameter. Because of the term $\dot{\phi}^2$, the dynamic equation is not linear with respect to the parameter value $\dot{\phi}$. After simple algebraic manipulation, we can obtain the LPD system representation of mobile robot system[7]. Define the state variables, input and the output as

$$x_1 \square \theta_1, x_2 \square \theta_2, x_3 \square \dot{\theta}_1, x_4 \square \dot{\theta}_2 ,$$

$$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad y = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

then, the state space representation of mobile robot is

$$\dot{x}(t) = A_0 x(t) + A_1 (\dot{\phi}(t)) x(t) + B_0 u(t)$$

$$v(t) = Cx(t)$$
(6)

where,

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$$\begin{split} b_{11}(=b_{22}) \approx & \frac{mc^2b^2 + I_c^2 + I_w}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2} \\ b_{12}(=b_{21}) \approx & \frac{I_c^2 - mc^2b^2}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2} \end{split}$$

In the equation (6), controllability matrix $[A_0, B_0]$ is controllable and $[A_1, B_0]$ is controllable except when the variable $\dot{\phi}(t) = 0$.

2.2. Takagi-Sugeno Fuzzy Model of wheeled Mobile Robot

The fuzzy model proposed by Tagaki and Sugeno is described by IF-THEN rules which represent local linear inputoutput relations of a nonlinear system.[8] The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model.

The i-th T-S fuzzy model is [8]

If
$$z_{i}(t) = M_{ii}$$
 and and $z_{p}(t) = M_{ip}$

THEN
$$\begin{cases} \dot{x}(t) = A_{i}x(t) + B_{i}u(t) \\ y(t) = C_{i}x(t), \end{cases} i = 1, 2, \dots, r$$
(7)

The final outputs of the fuzzy systems are inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) \left\{ A_i x(t) + B_i u(t) \right\}}{\sum_{i=1}^{r} w_i(z(t))}$$

$$= \sum_{i=1}^{r} h_i(z(t)) \left\{ A_i x(t) + B_i u(t) \right\}$$

$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))}$$

$$= \sum_{i=1}^{r} h_i(z(t)) C_i x(t)$$

$$= \sum_{i=1}^{r} h_i(z(t)) C_i x(t)$$
(8.b)

Note that the elements of matrices A_0 , B_0 and C are constant and that the only matrix A_1 depend on the value $\dot{\phi}(t)$. The fuzzification of the DDWMR is performed the matrix A_1 . The fuzzy model for the DDWMR, described by the equation (7), becomes

If
$$\dot{\phi}(t) = M_i$$

THEN
$$\begin{cases} \dot{x}(t) = \left[A_0 + A_{1i} \right] x(t) + B_0 u(t) & i = 1, 2, \dots, r \\ y(t) = Cx(t), & \end{cases}$$
 (9)

The final outputs of the fuzzy systems are inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) \left\{ \left[A_0 + A_{1i} \right] x(t) + B_0 u(t) \right\}}{\sum_{i=1}^{r} w_i(z(t))}$$

$$= \sum_{i=1}^{r} h_i(z(t)) \left\{ \left[A_0 + A_{|i|} \right] x(t) + B_0 u(t) \right\}$$

$$= A_0 x(t) + B_0 u(t) + \sum_{i=1}^{r} h_i(z(t)) A_{|i|} x(t)$$

$$= Cx(t)$$
(10)

The equation (10) is a fuzzy state-space representation of DDWMR.

3. Control of Mobile Robot

In this section, the design requirements are described in the view if robust stability and robust performance. And design procedure is presented

3.1. Control Objective: Robust Stability and Performance

The T-S fuzzy model for DDWMR contains some model errors. These errors can make the closed loop be unstable or the response of the system shows bad results. To consider the model error, the following structure which is used in this paper.

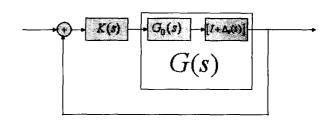


Figure 2. The output multiplicative model error

In figure 2, the used model error is output multiplicative error and the actual plant transfer function is described by

$$G(s) = \left[I + \Delta_0(s)\right] G_0(s) \tag{11}$$

where, G(s) is the actual plant, $G_0(s)$ is nominal plant and $\Delta_0(s)$ is a model error. The control objectives are robust stability and robust performance with respect to the presence of the model error. By well-known small-gain theorem, the closed loop is robustly stable if ① The nominal plant and the perturbed plant have the same number of unstable poles. ② And the following equation must be hold.[]

$$\overline{\sigma}\left\{G_0(s)K(s)\left[I+G_0(s)K(s)\right]^{-1}\right\} < \frac{1}{\overline{\sigma}\left(\Delta_0(s)\right)} \quad (12)$$

where, $\bar{\sigma}(\cdot)$ means the maximum singular value. In order for guarantee the robust stability, the equation (12) shows that the maximum singular value of the closed transfer function must be smaller than $1/\bar{\sigma}\left(\Delta_0(s)\right)$. Also, in order for good

performance properties, the open loop transfer function must be $\underline{\sigma}\left[G_0(s)K(s)\right]>>1$, in the low frequency region and $\overline{\sigma}\left[G_0(s)K(s)\right]<<1$ in the high frequency region. These requirements are shown in figure 3.

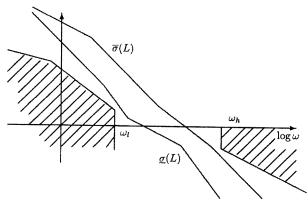


Figure 3. Desired loop shape of open loop transfer function

3.2 Control Structure

The structure of controller presented in this paper is consisted of two loops, the one is state feedback loop and the other is PI control loop. The controller schematic is described by figure 4.

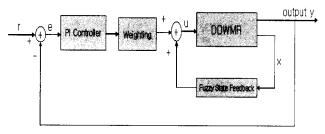


Figure 4. Schematics of Controller

In this paper, the inner fuzzy state feedback loop is designed by pole placement and the outer PI loop is designed for satisfying performance objective. The DDWMR considered in this paper is a MIROSOT soccer robot designed by YuJin Robotics Co. (Parameters of DDRMR are shown by next section.) The membership function is shown by Fig. 5. Fig. 6 shows the singular value plot of DDWMR used. In this figure, note that the singular value plot is changed very largely with respect to the variable, $\dot{\phi}(t)$, which is the velocity difference of two wheels. To compensate this property, the inner statefeedback controller must be designed in order to overcome this change. Also, it is needed the additional weighting controller W(s) which makes the figure of the frequency response of the DDWMR have a good shape. The main objective of this controller scheme is the tracking of reference signals, i.e., good-command following.

3.3. Controller Design

In this paper, the DDWMR considered is MIROSOT and its detailed specifications are summarized in the table 1. The mass of the robot is 0.6Kg and used parameters are

$$b = 35mm, c = r/2b$$

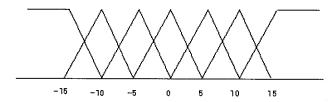


Figure 5. Membership Function

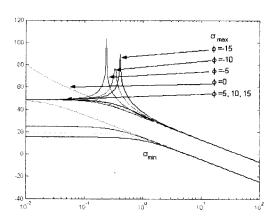


Figure 6. The singular value plot of DDWMR.

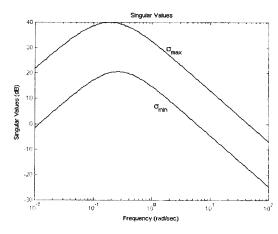


Figure 7. S-V plot of the inner state-feedback loop.

Table 1. The specifications of MIROSOT robot

size	70x70x70 mm	
Wheel diameter	45 mm	
rpm	8000	
Gear ratio	8:1	

Also, we use two random variables, zero mean white Gaussian noise, is added to the velocity of two wheels in order to realize for more real game environment. The maximum power of these random variables is 5% of maximum velocity of wheel.

By using these parameters, the fuzzy state-space representation of the DDWMR is

$$\dot{x}(t) = A_0 x(t) + B_0 u(t) + \sum_{i=1}^{r} h_i(z(t)) A_{1i} x(t)$$

$$y(t) = Cx(t)$$
(11)

where,

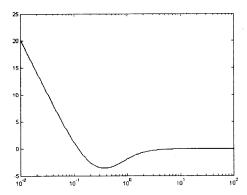


Figure 8. S-V plot of the weighting controller

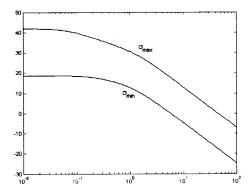


Figure 9. S-V plot of the inner state-feedback loop with weighting controller.

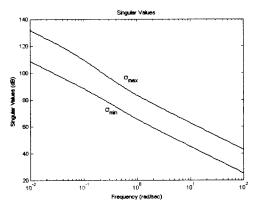


Figure 10. S-V plot of the open-loop transfer function.

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 25.0222 & -19.2256 \\ -19.2256 & 25.0222 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The fuzzy state-feedback controller is designed by the well known pole-placement algorithm. The desired poles of innerloop is selected by

$$P = \begin{bmatrix} -0.1 - 0.2 - 0.3 - 0.4 \end{bmatrix}^T$$

The state feedback gain is shown by Table 2. Fig. 7, Fig. 8 and Fig. 9 are singular value plots of the inner state state-feedback loop, the weighting controller and the inner state state-feedback loop with weighting controller. The weighting controller, which used for compensate the inner state-feedback controller gain in lower frequency region, is selected as

$$W(s) = \begin{bmatrix} s^2 + 0.7s + 0.1 & 0 \\ s^2 + s & 0 \\ 0 & \frac{s^2 + 0.7s + 0.1}{s^2 + s} \end{bmatrix}$$

And the Fig. 10 is the singular value plot of the open-loop transfer function. Because of the PI controller is used, Fig. 10 shows that the controller have good properties i.e., good command-following, good robustness etc. The PI controller gains are

$$K_I = \begin{bmatrix} 310 & 0 \\ 0 & 310 \end{bmatrix}, \quad K_P = \begin{bmatrix} 314 & 0 \\ 0 & 314 \end{bmatrix}.$$

4. Simulation

In simulation, the reference signal, which is reference velocities, is selected as sinusoidal signal which is described by Fig. 11. The magnitude of reference signal is 15 r/s for left wheel and 8 r/s for right wheel. As shown Fig. 11, the system response exactly follows the desired reference signal. Also, the Fig. 12 shows the error between the reference signal and actual output is very small.

Fig. 12 and Fig. 13 shows that the velocity tracking error is very small. Especially, there exist four switching instance, which is occurred to select the appropriate controller. But, these switching is not affect the over-all response of robot. Fig. 15 is a tracking result of test signal. It is shown in this figure that the tracking error is very small. All of these simulation results show that the disturbance effect is negligible and does not affect anymore. This is due to that the presented controller has well disturbance rejection property.

Table	2	State	Feedbe	ch	Gaine
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Table 2. State I cedback Gallis							
h_i	SF Gain						
-15	0.0083 0.0	032 -0.0239	9 0.1217				
	0.0067 0.0	037 -0.0403	0.1264				
-10	0.0083 0.0	032 0.0048	0.0930				
	0.0067 0.0	037 -0.0116	0.0977				
-5	0.0083 0.0	032 0.0335	0.0643				
	0.0067 0.0	037 0.0171	0.0690				
0	0.0083 0.0	032 0.0622	0.0356				
	0.0067 0.0	0.0458	0.0403				
5	0.0083 0.0	0.0909	0.0069				
	0.0067 0.0	037 0.0745	0.0116				
10	0.0083 0.0	032 0.1196	6 -0.0218				
	0.0067 0.0	037 0.1031	-0.0171				
15	0.0083 0.0	032 0.1482	2 -0.0505				
	0.0067 0.0	037 0.1318	3 -0.0458				

Simulation results of other signals, which are not included, show the same result of the test signal adopted in this paper.

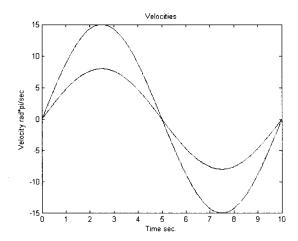


Figure 11. Velocity tracking results sinusoidal signal.

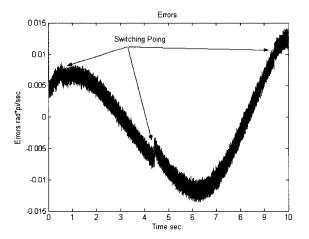


Figure 12. Velocity tracking errors for sinusoidal signal.

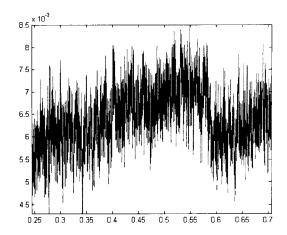


Figure 13. Magnified velocity tracking errors for Fig. 12.

5. Conclusion

In this paper, we studied the modeling and control of DDWMR via T-S fuzzy model. The modeling of DDWMR is based on the well-known T-S fuzzy model and control method developed is based on the classical frequency-domain techniques. It is shown by simulation result that the design requirements are well satisfied. For example, it is shown the commend-following, disturbance rejection, and robustness properties. The presented control algorithm gives some benefits. One of which is very easy and simple method in controller design and implementation. The other is the developed controller can be applicable to other types of robot. Especially, the car like robot can be easily treated via presented controller design algorithm. Also, presented controller can be implemented by simple micro-controller chips.

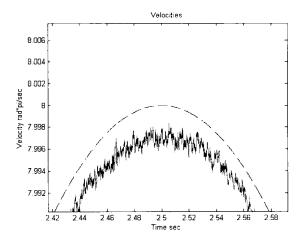


Figure 14. Magnified velocity tracking for Fig. 11.

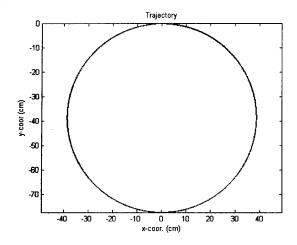


Figure 15. The tracking result for sinusoidal signal.

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