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A Study on the Threshold of Avoidance Time in the New Evaluation of Collision Risk

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Abstract : *Evaluating the risk of collision quantitatively plays a key role in developing the expert system of navigation and collision avoidance. This study analysed thoroughly how to determine the threshold function related to the avoidance time as described in the new evaluation of collision risk using sech function, and developed the appropriate equation as applicable.*

Key words : *collision risk, closest distance, approach time, threshold of avoidance time, minimum approach range, collision equi-risk line*

1. Introduction

Evaluating the risk of collision quantitatively plays a key role in developing the expert system of navigation and collision avoidance. There have been several researches into the quantitative assessment of collision risk, which still have a few problems when they are applied to the expert system(A. G., 1982 ; R. F., 1979 ; A. G., 1992 ; H. Imazu, 1978 ; H. Imazu, 1979 ; T. Degre, 1981 ; W. Burger, 1998). For the purpose of solving such problems a new approach to collision risk using sech function was introduced(Jeong, 2003) and the proper method of determining coefficients shown in this approach was developed(Jeong, 2003).

In this paper, of the two thresholds in the new evaluation of collision risk, the threshold function of avoidance time is first analysed and obtained. This threshold function is applied to several practical situations.

2. The method of obtaining the threshold function of avoidance time

The new evaluation of collision risk using sech function is given by(Jeong, 2003, No.2)

$$CR = p \cdot \operatorname{sec}h(a \cdot dcpa) + q \cdot \operatorname{sec}h(b \cdot ta) + r \cdot \Phi(\theta, \alpha) \tag{1}$$

where CR is the collision risk, $dcpa$ is the closest distance and t_a is the approach time. The five coefficients p, q, r, a and b are to determine the change rate of collision risk properly. The amplitude coefficients p, q and r are to determine the amplitude of sech function and the gradient coefficients a and b are to determine the change of sech function. $\Phi(\theta, \alpha)$ is a function of determining whether own ship maintains her course and speed or alters her course and/or speed according to the Collision Regulations. It is called the function of own ship's state and expressed by the bearing θ and the aspect α of a target, the magnitude of which is the naught if own ship maintains and the unity if she alters.

Using the closest distance $dcpa$, the approach time ta can be written as

$$ta = \frac{R^2}{vr\sqrt{R^2 - dcpa^2}} \tag{2}$$

where R , which is equal to or greater than $\sqrt{2}\, dcpa$, is the distance between a target and own ship, and vr is the relative speed of the target.

Substituting Eq. (2) into Eq. (1) yields Eq. (3).

$$CR = \operatorname{sec}h(a \cdot dcpa) + \operatorname{sec}h\left(b \cdot \frac{R^2}{vr\sqrt{R^2 - dcpa^2}}\right) \tag{3}$$

As shown in Eq. (3), the collision risk CR is expressed by the three variables of $dcpa$, R and vr. Representing the threshold of collision risk as the above three variables is not suitable because the distance R is related to avoidance time (or the time of avoiding action). Therefore in this paper the threshold will be expressed by the two variables of the relative speed and the closest distance of a target.

2.1 Things to be considered when determining the threshold of avoidance time

There are two types of thresholds in the new evaluation of collision risk. One is to determine when the avoiding

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action has to be taken if the risk of collision exists. It is called the threshold of avoidance time. The other is to determine which sector will be safe for own ship, which is obtained by the range of own ship's choice, that is, alteration of own ship's course and/or speed. It is called the threshold of avoidance sector.

This paper deals with how to determine the threshold of avoidance time. The followings are assumed or taken into consideration.

① The gradient and amplitude coefficients in the new evaluation of collision risk depend on danger zone (or safe minimum distance), closest distance and approach time. When we assume here that the closest distance ranges from 0.09 to 1.5 mile, that the approach time ranges between 4.6 minutes and 6.9 minutes, and that the danger zone is 2.12(=$1.5\sqrt{2}$) miles, we can obtain the gradient coefficients $a=0.818$, $b=0.180$(Jeong, 2003). Meanwhile the amplitude coefficients are assumed $p=1$, $q=1$ and $r=0$.

② The maximum threshold of the new evaluation of collision risk can be obtained on the assumption that the closest distance of a target is 0 mile and the approach time is 11 minutes. If a target approaches own ship at a relative speed of 1.0 mile per minute, ie 60 knots, own ship can afford to take avoiding action 11 minutes or 11 miles off before the collision takes place. The maximum threshold, $CR_{ah_{max}}$, is obtained as below.

$$\begin{aligned} CR_{ah_{max}} &= \sec h(a \cdot dcpa) + \sec h(b \cdot ta) \\ &= \sec h(0) + \sec h(0.180 \times 9) \\ &= 1.271 \end{aligned} \tag{3a}$$

③ Let the range of relative speed be from 0.1 to 1.0 mile per minute. Because the relative speed governs the time elapsed after avoiding action is taken and the collision risk, the size of danger zone should be governed by the relative speed. As shown in Table 1, we here consider that for the target with her relative speed of 0.39 mile per minute and upwards, avoiding action has to be taken to pass outside the danger zone of 2.12 miles, and that for the target with her relative speed of 0.28 mile per minute and more, but less than 0.39 mile per minute, avoiding action has to be taken to pass outside the danger zone of 1.59 miles. We can also assume that for the target with her relative speed of 0.17 mile per minute and more, but less than 0.28 mile per minute, avoiding action has to be taken to pass outside the danger zone of 1.06 miles, and that for the target with her relative speed of less than 0.17 mile per minute, avoiding action has to be taken to pass outside the danger zone of 0.71 mile.

Table 1 Relative Speed and Danger Zone

Relative Speed (mile/min)	$vr \geq$ 0.39	0.39 > $vr \geq$ 0.28	0.28 > $vr \geq$ 0.17	$vr <$ 0.17
Danger Zone (mile)	2.12	1.59	1.06	0.71

④ As mentioned above in order to have a target pass outside the danger zone the minimum approach range is required, which is obtained as follows. First the distance at which avoiding action has to be taken when the closest distance is 0 mile, is determined. Also, the distance when the closest distance equals the danger zone, is determined. Next they are curve-fitted respectively by the closest distance *dcpa*.

Meanwhile the range at which avoiding action has to be taken if $dcpa=0$, R_i and the range if $dcpa=$ danger zone, R_f are given respectively by

$$R_i = \frac{dz}{\sin \eta}, \qquad R_f = \sqrt{2} \cdot dz$$

where *dz* denotes the danger zone. η is the difference between the relative courses before and after avoiding action when $dcpa=0$. It is considered as 25°here.

The minimum approach range R_{mn} is expressed as below.

if $vr \geq 0.39$

$$\begin{aligned} R_{mn} = {} & 0.0078\,dcpa^4 + 0.0312\,dcpa^3 + 0.0925\,dcpa^2 \\ & - 1.3632\,dcpa + 5.0192 \end{aligned} \tag{4a}$$

if $0.39 > vr \geq 0.28$

$$\begin{aligned} R_{mn} = {} & 0.0188\,dcpa^4 + 0.0548\,dcpa^3 + 0.1240\,dcpa^2 \\ & - 1.3634\,dcpa + 3.7644 \end{aligned} \tag{4b}$$

if $0.28 > vr \geq 0.17$

$$\begin{aligned} R_{mn} = {} & 0.0627\,dcpa^4 + 0.1249\,dcpa^3 + 0.1850\,dcpa^2 \\ & - 1.3632\,dcpa + 2.5096 \end{aligned} \tag{4c}$$

if $vr < 0.17$

$$\begin{aligned} R_{mn} = {} & 0.2157\,dcpa^4 + 0.2760\,dcpa^3 + 0.2796\,dcpa^2 \\ & - 1.3635\,dcpa + 1.6731 \end{aligned} \tag{4d}$$

⑤ In general, the threshold of avoidance time cannot be represented as the only value. It takes a very long time for a target and own ship to keep clear each other in case the

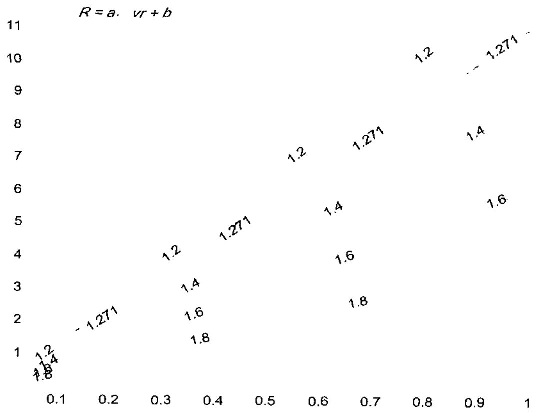

Fig. 1 Collision Equi-Risk Lines at DCPA=0.0(mile)

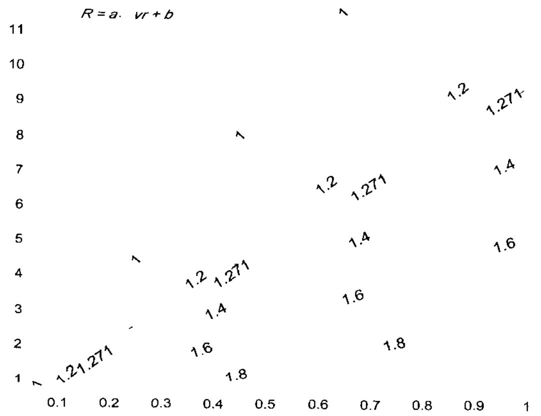

Fig. 2 Collision Equi-Risk Lines at DCPA=0.5(mile)

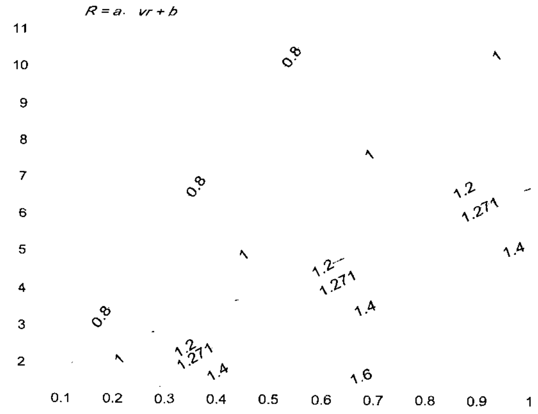

Fig. 3 Collision Equi-Risk Lines at DCPA=1.0(mile)

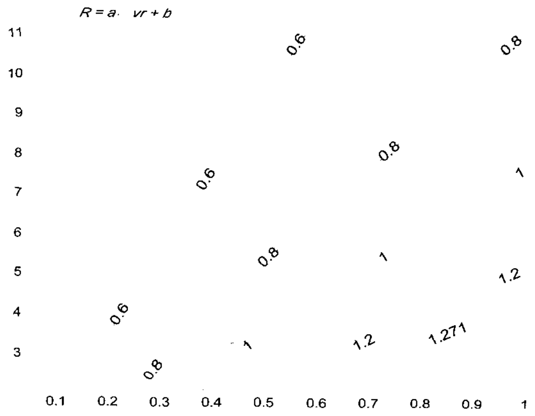

Fig. 4 Collision Equi-Risk Lines at DCPA=1.5(mile)

relative speed is very slow. Therefore the threshold should be used together with the minimum approach range of Eq. (4).

2.2 The method of determining the threshold of avoidance time

In the new evaluation of collision risk the threshold of avoidance time cannot be expressed by the only value as mentioned above. Even so, expressing all kinds of cases as thresholds one by one is highly complicated. Therefore the threshold of avoidance time is here represented as the two variables of relative speed and closest distance.

From the collision risk of Eq. (3) we can get the collision equi-lines represented by the relative speed and the range of a target if the closest distance is given. <Fig. 1>, <Fig. 2>, <Fig. 3> and <Fig. 4> show the examples of collision equi-lines in case $dcpa=0$, $dcpa=0.5$, $dcpa=1.0$ and $dcpa=1.5$ respectively.

In one of these figures (that is, at a given closest distance) we can draw an appropriate straight line and obtain the intersections between this straight line and the collision equ-lines. From the intersections we can get the collision risk and the relative speed and then the collision risk can be curve-fitted by each relative speed. Next because each coefficient in the polynomial of relative speed can be expressed by the closest distance it is also curve-fitted by the closest distance.

The above-mentioned straight line can be obtained as follows. Using the collision equi-line of the maximum threshold of Eq. (3a), we can read the range corresponding to the maximum threshold at a relative speed of 1.0 mile per minute and then can obtain a position expressed by relative speed and range. Next using Eq. (4a) we can get the minimum approach range of about 5.02 miles. Dividing it by the approach time of 11 minutes, we can obtain the relative speed of 0.456 mile per minute. However this relative speed is critical. That is, when the minimum approach range is applied at a bigger relative speed than this, and if the relative speed becomes smaller the threshold is apt to increase. Meanwhile when the minimum approach range is applied at a smaller relative speed than this, and if the relative speed becomes smaller the threshold is apt to decrease. Because the latter is thought to be reasonable, the relative speed of 0.44 mile per minute is used here. So we get another position given by the relative speed of 0.44 mile per minute and the minimum approach range of Eq. (4a). Therefore the straight line that we want can be obtained by connecting the above two positions.

Using the method as mentioned earlier we can obtain the threshold function of avoidance time in the new evaluation of collision risk, CR_{ah} as follows.

$$CR_{ah} = \lambda_1 \cdot vr^2 + \lambda_2 \cdot vr + \lambda_3 \tag{5}$$

$$\lambda_1 = -0.1187\,dcpa^4 + 0.5460\,dcpa^3 - 1.0934\,dcpa^2 - 0.3857\,dcpa - 0.0466$$

$$\lambda_2 = 0.2021\,dcpa^4 - 1.0164\,dcpa^3 + 2.2483\,dcpa^2 - 0.8134\,dcpa + 0.0997$$

$$\lambda_3 = -0.0844\,dcpa^4 + 0.4759\,dcpa^3 - 1.1653\,dcpa^2 + 0.4312\,dcpa + 1.2175$$

Eq. (5) represents a function of obtaining the threshold of avoidance time in case a target approaches at a relative speed between 0.1 and 1.0 mile per minute and at a closest distance between 0 and 1.5 mile. However if the relative speed is small, the range determined by the threshold function becomes smaller than the minimum approach range given by Eqs. (4a), (4b), (4c) and (4d). Therefore they should be used in addition.

3. The application of the threshold function of avoidance time to actual avoiding action

The application of the threshold function of avoidance time to actual avoiding action is as follows. When each target approaches within the danger zone of 2.12 miles, 1.59 mile, 1.06 mile and 0.71 mile respectively and avoiding action is taken to pass outside each danger zone, it would be examined whether the avoidance time is determined by the threshold function of Eq. (5) or the minimum approach range of Eqs. (4a), (4b), (4c) and (4d) as the relative speed increases.

Because the threshold function is governed only by the relative speed and the closest distance of a target, there is not any difference in the relevant rules of Part B, Section Ⅱ(conduct of vessels in sight of one another) of the Collision Regulations. Therefore we here assume that both own ship and a target meet head-on each other or own ship overtakes her and avoiding action is to make an alteration to starboard.

In <Table 2> own ship is steering a course of 000°at a speed of 0.4 mile per minute. First of all, let's consider that a target, which is at a distance of 8.1 miles, bearing 349.4°, approaches within a closest distance of 1.5 mile at a relative speed of 0.39 and more mile per minute. The range corresponding to the avoidance time, at which avoiding action has to be taken so as to make the target pass outside a danger zone of 2.12 miles, is 3.328 miles. It is bigger than the minimum approach range of 3.327 miles. The collision risk at the avoidance time is 0.993, which is bigger than 0.989 generated by the threshold function. At that time the relative speed becomes 0.47 mile per minute. If the relative speed is equal to or bigger than this, avoiding action is taken by the threshold and otherwise avoiding action taken by the minimum approach range. <Fig. 5> shows the result of action taken to avoid the target approaching at a relative speed of 0.47 mile per minute. The collision risk gradually increases as time goes by. When it reaches the threshold of 0.989 avoiding action is taken. As a result the collision risk rapidly drops.

Likewise, we consider that another target, which is at a distance of 7.1 miles, bearing 350.2°, approaches within a closest distance of 1.2 mile at a relative speed of 0.28 mile per minute and more, but less than 0.39 mile per minute. The range corresponding to the avoidance time, at which avoiding action has to be taken so as to make the target pass outside a danger zone of 1.59 mile, is 2.591 miles. It is bigger than the minimum approach range of 2.441 miles. The collision risk at the avoidance time is 0.956, which is bigger than 0.955 generated by the threshold function. At that time the relative speed becomes 0.28 mile per minute. Similarly as before, if the relative speed is equal to or bigger than this, avoiding action is taken by the threshold and otherwise avoiding action taken by the minimum approach range. <Fig. 6> shows the result of action taken to avoid the target approaching at a relative speed of 0.28 mile per minute. The collision risk gradually increases as time goes by. When it reaches the threshold of 0.955 avoiding action is taken. As a result the collision risk rapidly drops.

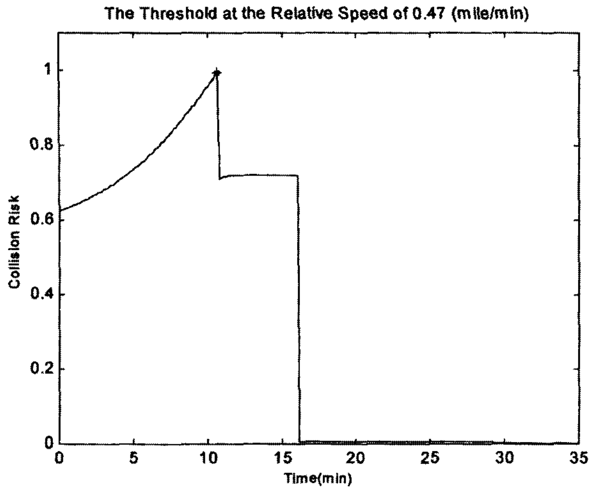

Fig. 5 Action to Avoid a Target at a Relative Speed of 0.47(mile/min), taken by Collision Risk Theshold

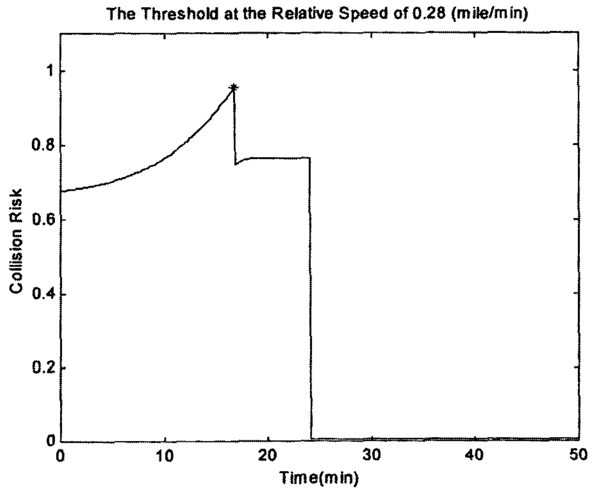

Fig. 6 Action to Avoid a Target at a Relative Speed of 0.28(mile/min), taken by Collision Risk Theshold

Table 2 Data of Ownship and Target

Ownship	Course(°)	000	000	000	000
	Speed(mile/min)	0.4	0.4	0.4	0.4
Tatget	Course(°)	180	000	000	000
	Speed(mile/min)	0.07	0.12	0.21	0.27
Initial Position	Bearing(°)	349.4	350.2	353.3	354.3
	Range(mile)	8.1	7.1	6.0	4.0
DCPA	before Action	1.5	1.2	0.7	0.4
	after Action	2.12	1.59	1.06	0.7
Collision Risk	Threshold	0.989	0.955	1.149	1.240
	Action	0.993	0.956	1.152	1.241
Range	Minimum	3.327	2.441	1.704	1.196
	Action	3.328	2.591	1.844	1.307
Relative Speed(mile/min)		v_r=0.47	v_r=0.28	v_r=0.19	v_r=0.13
Time of Action		Threshold	Threshold	Threshold	Threshold

where, gradient coefficients a=0.818 and b=0.180.

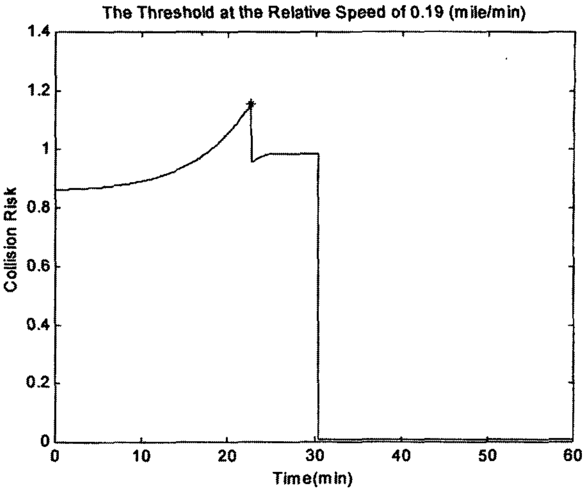

Fig. 7 Action to Avoid a Target at a Relative Speed of 0.19(mile/min), taken by Collision Risk Theshold

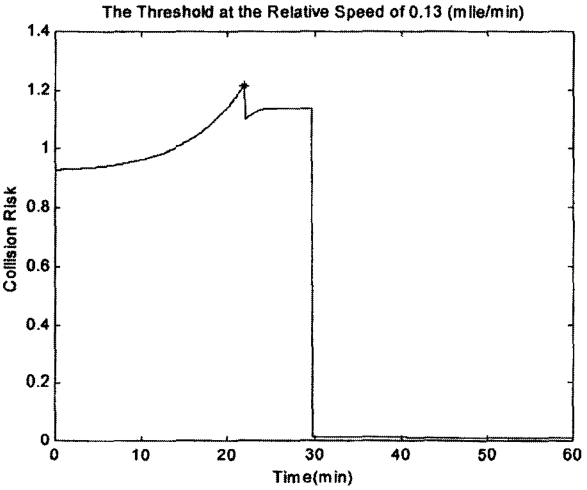

Fig. 8 Action to Avoid a Target at a Relative Speed of 0.13(mile/min), taken by Collision Risk Theshold

Meanwhile, we assume that a target which is at a distance of 6.0 miles, bearing 353.3°, at a relative speed of 0.17 mile per minute and more, but less than 0.28 mile per minute and another target which is at a distance of 4.0 miles, bearing 354.3°, at a relative speed of less than 0.17 mile per minute approach within a closest distance of 0.7 mile and 0.4 mile respectively. The distances corresponding to the avoidance time, at which avoiding action has to be taken so as to make the targets pass outside the danger zones of 1.06 mile and 0.7 mile, are 1.844 mile and 1.307 mile respectively. They are bigger than the minimum approach range of 1.704 mile and 1.196 mile respectively. The collision risks at the avoidance time are 1.152 and 1.241, which are bigger than 1.149 and 1.240 generated by the threshold function respectively. At that time the relative speeds become 0.19 and 0.14 mile per minute respectively. Likewise as before, if the relative speeds are equal to or bigger than these, avoiding action is taken by each minimum approach range. <Fig. 7> and <Fig. 8> show threshold and otherwise avoiding action taken by each the result of each action taken to avoid the targets approaching at a relative speed of 0.19 and 0.13 mile per minute respectively. The collision risks gradually increase as time goes by. When they reach the thresholds of 1.149 and 1.240 respectively each avoiding action is taken. As a result the collision risks rapidly drop.

As mentioned above when the targets approach within danger zones of 2.12 miles, 1.59 mile, 1.06 mile and 0.7 mile, if their relative speeds equal to or greater than 0.47, 0.28, 0.19 and 0.13 mile per minute respectively each avoidance time is given by the threshold function and otherwise it is determined by each minimum approach range.

4. Conclusion

In this paper as the first stage of determining the thresholds in the new evaluation of collision risk using sech function, the method of determining the threshold function of avoidance time was analyzed and was applied to actual avoiding action in the head-on and overtaking situations. As a result, it was concluded as follows.

① The new evaluation of collision risk can be represented as the three variables of approach time, closest distance and relative speed so as to investigate conveniently the threshold function of avoidance time.

② Using gradient coefficients $a = 0.818$ and $b = 0.180$, we can obtain the threshold function of avoidance time, ie. Eq. (5). If the relative speed and closest distance are given, the threshold is determined at once. In case the relative speed is large enough at each danger zone the range corresponding to the avoidance time, which is thought to be safe, can be obtained by the threshold function only.

③ If the relative speed is small, avoiding action would be taken at a distance less than the minimum approach range. Therefore the minimum approach range of Eq. (4) should be used in addition.

However, the maximum threshold of Eq. (3a) and the minimum approach range of Eq. (4) should be investigated again through various experiments aboard ships. The threshold of avoidance sector should be investigated. Finally, when the amplitude coefficients p and q and the function of own ship's state $\Phi(\theta, \alpha)$ are applied to the collision risk, the thresholds should be corrected. All of these will be dealt with in the future study.

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