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t-Intuitionistic Fuzzy Subgroups and Subrings

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Abstract

In this paper, we introduce the concepts of t-intuitionistic fuzzy subgroups and t-intuitionistic fuzzy subrings. And we study some properties of t-subgroups and t-subrings.

Key words and phrases : t-intuitionistic fuzzy subgroup, t-intuitionistic fuzzy subring

1. Introduction

In 1965, Zadeh[17] introduced the concept of fuzzy sets. After that time, several researchers[1,7,13,14,16] have applied the notion of fuzzy sets to group theory. Moreover, Anthony and Sherwood[1] introduced the concepts of t-fuzzy subgroups by using the t-norm introduced by Schweizer and Sklar[15].

In 1986, Atanassov[2] introduced the concept of intuitionistic fuzzy sets. Recently, Çoker and his colleagues[5,6,8], Lee and Lee[12] introduced the concept of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. In 1989, Biswas[4] introduced the concept of intuitionistic fuzzy subgroups and investigated some of it's properties. In 2003, Baldev Banerjee and Dhiren Kr.Basnet[3] studied intuitionistic fuzzy subrings and ideals using intuitionistic fuzzy sets. Also Hur and his colleagues[9,10,11] applied the notion of intuitionistic fuzzy sets to group theory.

In this paper, we introduce the concepts of t-intuitionistic fuzzy subgroups and t-intuitionistic fuzzy subrings by using the t-norm. And we investigate some properties of t-subgroups and t-subrings.

2. Preliminaries

We will list some concepts and results needed in the later sections.

For sets X, Y and Z, $f=(f_1, f_2)$: $X \to Y\times Z$ is called a *complex mapping* if $f_1: X\to Y$ and $f_2: X\to Z$ are mappings.

Definition 1.1[2]. Let X be a nonempty set. A complex mapping $A=(\mu_A, \nu_A): X \to I\times I$ is called an *intuitionistic fuzzy set*(in short, IFS) on X if $\mu_A+\nu_A \leq 1$, where the mapping $\mu_A: X\to I$ and $\nu_A: X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership(namely $\nu_A(x)$) of each $x\in X$ to A, respectively.

We will denote the set of all the IFSs in X as IFS(X).

Definitions 1.2[2]. Let X be a nonempty set and let $A=(\mu_A, \nu_A)$ and $B=(\mu_B, \nu_B)$ be IFSs on X. Then

(1) $A\subset B$ iff $\mu_A\leq \mu_B$ and $\nu_A\geq \nu_B$.
(2) $A=B$ iff $A\subset B$ and $B\supset A$.
(3) $A^c=(\mu_A, \nu_A)$
(4) $A\cap B=(\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.
(5) $A\cup B=(\mu_A \vee \mu_B, \nu_A \vee \nu_B)$.
(6) $[\ ]A=(\mu_A, 1-\mu_A)$, $<>A=(1-\nu_A, \nu_A)$.

Definition 1.3[5]. Let $\{A_i\}_{i\in J}$ be an arbitrary family of IFSs in X, where $A_i=(\mu_{A_i}, \nu_{A_i})$ for each $i\in J$. Then

(a) $\cap A_i=(\wedge \mu_{A_i}, \vee \nu_{A_i})$.
(b) $\cup A_i=(\vee \mu_{A_i}, \wedge \nu_{A_i})$.

Definition 1.4[5]. $0_\sim=(0,1)$ and $1_\sim=(1,0)$.

Result 1. A[5, Corollary 2.8]. Let A, B, C, D be IFSs in X. Then

(1) $A\subset B$ and $C\subset D \Rightarrow A\cup C\subset B\cup D$ and $A\cap C\subset B\cap D$.
(2) $A\subset B$ and $A\subset C \Rightarrow A\subset B\cap C$.
(3) $A\subset C$ and $B\subset C \Rightarrow A\cup B\subset C$.
(4) $A\subset B$ and $B\subset C \Rightarrow A\subset C$.
(5) $(A\cup B)^c=A^c\cap B^c$, $(A\cap B)^c=A^c\cup B^c$.

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(6) $A \subset B \Rightarrow B^c \subset A^c$.

(7) $(A^c)^c = A$.

(8) $1^c_\sim = 0_\sim$, $0^c_\sim = 1_\sim$.

Definition 1.5[12]. Let $\lambda, \mu \in (0,1)$ and $\lambda + \mu \leq 1$. An *intuitionistic fuzzy point* (in short IFP) $x_{(\lambda,\mu)}$ of X is the IFS in X defined by

$$x_{(\lambda,\mu)}(y) = \begin{cases} (\lambda, \mu) & \text{if } y = x, \\ (0,1) & \text{if } y \neq x, \end{cases} \quad \text{for each } y \in X.$$

In this case, x is called the *support* of $x_{(\lambda,\mu)}$ and λ and μ are called the *value* and *nonvalue* of $x_{(\lambda,\mu)}$, respectively.

An IFP $x_{(\lambda,\mu)}$ is said to *belong to* an IFS $A = (\mu_A, \nu_A)$ in X, denoted by $x_{(\lambda,\mu)} \in A$ if $\lambda \leq \mu_A(x)$ and $\mu \geq \nu_A(x)$.

Result 1.B[12;Theorem 2.4]. Let $A = (\mu_A, \nu_A)$ be an IFS in X. Then

$$A = \bigcup \{x_{(\lambda,\mu)} : x_{(\lambda,\mu)} \in A\}.$$

Definition 1.6[15]. A *t-norm* is a mapping $t: I \times I \to I$ satisfying the following conditions : for any $x, y, z, u, v \in I$

(i) $t(x, y) = t(y, x)$, i.e., $xty = ytx$.

(ii) $xt(ytz) = (xty)tz$.

(iii) If $x \leq u$ and $y \leq v$ then $xty \leq utv$

In particular, if $y \leq v$, then $xty \leq xtv$.

(iv) $xt1 = x$ and $xt0 = 0$.

t-norms which are frequently encountered are :

(a) $xt_0y = \min\{x, y\}$ for $x, y \in I$.

(b) $xt_1y = \mathrm{Prod}\{x, y\} = xy$ for $x, y \in I$.

(c) $xt_2y = \max\{x + y - 1, 0\}$ for $x, y \in I$.

Definition 1.7[15]. A *t-conorm* or *s-norm* is a mapping $s_t: I \to I$ defined by : for any $u, v \in I$,

$$us_tv = 1 - (1-u)t(1-v).$$

It is clear that s_t satisfies the following conditions : for any $x, y, z, u, v \in I$,

(i) $xs_ty = ys_tx$.

(ii) $xs_t(ys_tz) = (xs_ty)s_tz$.

(iii) If $x \leq u$ and $y \leq v$ then $xs_ty \leq us_tv$.

In particular, if $y \leq v$, then $xs_ty \leq xs_tv$.

(iv) $xs_t0 = x$ and $xs_t1 = 1$.

t-conorms corresponding to the above *t*-norms t_0, t_1, t_2 are as follows :

(a′) $xs_0y = \max\{x, y\}$ for any $x, y \in I$.

(b′) $xs_1y = x + y - xy$ for any $x, y \in I$.

(c′) $xs_2y = \min\{1, x + y\}$ for any $x, y \in I$.

Definition 1.8[11]. Let $(X, \cdot)$ be a groupoid and let $A, B \in IFS(X)$. Then the *intuitionistic fuzzy product of A and B under t-norm t* (in short, *t-intuitionistic fuzzy product of A and B*), $A \circ_t B$, is defined as follows : for any $x \in X$,

$$\mu_{A \circ_t B}(x) = \begin{cases} \bigvee_{yz = x} [\mu_A(y)\, t \mu_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{A \circ_t B}(x) = \begin{cases} \bigwedge_{yz = x} [\nu_A(y)\, t \nu_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 1 & \text{otherwise}. \end{cases}$$

Definition 1.9[11]. Let $(X, \cdot)$ be a groupoid and let $0_\sim \neq A \in IFS(X)$. Then A is called an *intuitionistic fuzzy subgroupoid in X under a t-norm t* (in short, *t-IFGP*) in X if $A \circ_t A \subset A$.

Definition 1.9′[11]. Let $(X, \cdot)$ be a groupoid and let $A \in IFS(X)$. Then A is called *t-intuitionistic fuzzy groupoid* (in short, *t-IFGP*) of X, if for any $x, y \in X$,

$$\mu_A(xy) \geq \mu_A(xy)\, t \mu_A(y)$$

and

$$\nu_A(xy) \leq \nu_A(x)\, s_t \nu_A(y).$$

It is clear that $0_\sim$ and $1_\sim$ are both *t-IFGPs* of X.

Definition 1.10[9]. Let $(X, \cdot)$ be a groupoid and let $A, B \in IFS(X)$ Then the intuitionistic fuzzy product of A and B, $A \circ B$, is defined as follows : for any $x \in X$,

$$\mu_{A \circ B}(x) = \begin{cases} \bigvee_{yz = x} [\mu_A(y) \wedge \mu_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{A \circ B}(x) = \begin{cases} \bigwedge_{yz = x} [\nu_A(y) \vee \nu_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 1 & \text{otherwise}. \end{cases}$$

Definition 1.11[9]. Let $(G, \cdot)$ be a groupoid and let $0_\sim \neq A \in IFS(G)$. Then A is called an *intuitionistic* *fuzzy subgroupoid* in G (in short, *IFGP*) in G if $A \circ A \subset A$.

Definition 1.11′[9]. Let $(G, \cdot)$ be a groupoid and let $A \in IFS(G)$. Then A is called an *intuitionistic fuzzy subgroupoid* (in short, *IFGP*) of G if for any $x, y \in G$,

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$

and

$$\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y).$$

Definition 1.12[10]. Let G be a group and let $A \in IFGP(G)$. Then A is called an *intuitionistic fuzzy subgroup* (in short, *IFG*) of G if $A(x^{-1}) \geq A(x)$, i.e., $\mu_A(x^{-1}) \geq \mu_A(x)$ and $\nu_A(x^{-1}) \leq \nu_A(x)$ for each $x \in G$.

We will denote the set of all IFGs of G as $IFG(G)$.

Definition 1.13[10]. Let $(R, +, \cdot)$ be a ring and let $0_{\sim} \neq A \in IFS(R)$. Then A is called an *intuitionistic fuzzy subring* (in short, *IFR*) in R if it satisfies the following conditions :

(i) A is an IFG with respect to the operation " $+$ " (in the sense of Definition 1.12),

(ii) A is an IFGP with respect to the operation " $\cdot$ " (in the sense of Definition 1.11 or 1.11′).

It is clear that subrings of R are IFRs of R.

2. t-intuitionistic fuzzy subgroups and t-intuitionistic fuzzy normal subgroups

Definition 2.1. Let X be a group and let $0_{\sim} \neq A \in IFS(X)$. Then A is called an *intuitionistic fuzzy subgroup in X under a t-norm t* (in short, *t−IFG*)in X if it satisfies the following conditions:

(i) A is a t-IFGP in X, i.e., $\mu_A(xy) \geq \mu_A(xy)\, t\mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x)\, s_t\nu_A(y)$ for any $x, y \in X$.

(ii) $A(x^{-1}) \geq A(x)$, i.e., $\mu_A(x^{-1}) \geq \mu_A(x)$ and $\nu_A(x^{-1}) \leq \nu_A(x)$ for each $x \in X$.

Proposition 2.2. Let A be a t-IFG in a group X. Then $A(x^{-1}) = A(x)$, i.e., $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ for each $x \in X$.

Proof. Let $x \in X$. Then :

$$\mu_A(x) = \mu_A((x^{-1})^{-1}) \geq \mu_A(x^{-1}) \geq \mu_A(x)$$

and

$$\nu_A(x) = \nu_A((x^{-1})^{-1}) \leq \nu_A(x^{-1}) \leq \nu_A(x).$$

Hence $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ for each $x \in X$.

Proposition 2.3. If A is a t-IFG in a group X, then $H = \{x \in X : A(x) = 1_{\sim}$, i.e., $\mu_A(x) = 1$ and $\nu_A(x) = 0\}$ is a subgroup of X.

Proof. Let $x, y \in H$. Then :

$$\mu_A(xy^{-1}) \geq \mu_A(x)\, t\mu_A(y^{-1}) = \mu_A(x)\, t\mu_A(y) = 1\, t1 = 1$$

and

$$\nu_A(xy^{-1}) \leq \nu_A(x)\, s_t\nu_A(y^{-1}) = \nu_A(x)\, s_t\nu_A(y) = 0\, s_t0 = 0.$$

Thus $\mu_A(xy^{-1}) = 1$ and $\nu_A(xy^{-1}) = 0$. So $xy^{-1} \in H$. Hence H is a subgroup of X.

Proposition 2.4. If A is a t-IFG in a group X and if there is a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} \mu_A(x_n)\, t\mu_A(x_n) = 1$ and $\lim_{n\to\infty} \nu_A(x_n)\, s_t\nu_A(x_n) = 0$, then $\mu_A(e) = 1$ and $\nu_A(e) = 0$, where e is the unity in X.

Proof. Let $x \in X$. Then :

$$\mu_A(e) = \mu_A(xx^{-1}) \geq \mu_A(x)\, t\mu_A(x^{-1}) = \mu_A(x)\, t\mu_A(x)$$

and

$$\nu_A(e) = \nu_A(xx^{-1}) \leq \nu_A(x)\, s_t\nu_A(x^{-1}) = \nu_A(x)\, s_t\nu_A(x).$$

Thus, for each n, $\mu_A(e) \geq \mu_A(x_n)\, t\mu_A(x_n)$ and $\nu_A(e) \leq \nu_A(x_n)\, s_t\nu_A(x_n)$. On the other hand, $1 \geq \mu_A(e) \geq \lim_{n\to\infty} \mu_A(x_n) t\mu_A(x_n) = 1$ and $0 \leq \nu_A(e) \leq \lim_{n\to\infty} \nu_A(x_n)\, s_t\nu_A(x_n) = 0$. Hence $\mu_A(e) = 1$ and $\nu_A(e) = 0$.

Proposition 2.5. Let A be a t-IFG in a group X. If $A(xy^{-1}) = 1_{\sim}$, i.e., $\mu_A(xy^{-1}) = 1$ and $\nu_A(xy^{-1}) = 0$, then $A(x) = A(y)$, i.e., $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.

Proof. Let $x, y \in X$. Then :

$$\begin{aligned}\mu_A(x) &= \mu_A((xy^{-1})y) \geq \mu_A(xy^{-1}) t\mu_A(y) = 1\, t\mu_A(y)\\ &= \mu_A(y) = \mu_A(y^{-1}) = \mu_A(x^{-1}(xy^{-1}))\\ &\geq \mu_A(x^{-1}) t\mu_A(xy^{-1}) = \mu_A(x)\, t1 = \mu_A(x)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x) &= \nu_A((xy^{-1})y) \leq \nu_A(xy^{-1}) s_t\nu_A(y)\\ &= 0\, s_t\nu_A(y) = \nu_A(y) = \nu_A(y^{-1})\\ &= \nu_A(x^{-1}(xy^{-1})) \leq \nu_A(x^{-1}) s_t\nu_A(xy^{-1})\\ &= \nu_A(x) s_t0 = \nu_A(x).\end{aligned}$$

Hence $\mu_A(x)=\mu_A(y)$ and $\nu_A(x)=\nu_A(y)$.

Proposition 2.6. Let X be a group and let $0_\sim \neq A \in IFS(X)$ with $A(e)=(1,0)$. Then A is a t-IFG in X if and only if $\mu_A(xy^{-1}) \geq \mu_A(x) t \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) s_t \nu_A(y)$ for any $x, y \in X$.

Proof. $(\Rightarrow)$: Suppose A is a t-IFG in X and let $x, y \in X$. Then by Proposition 2.4, $\mu_A(xy^{-1}) \geq \mu_A(x) t \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) s_t \nu_A(y)$.

$(\Leftarrow)$: Suppose the necessary conditions hold and let $x, y \in X$. Then :

$$\begin{aligned}\mu_A(x^{-1}) &= \mu_A(ex^{-1}) \\ &\geq \mu_A(e) t \mu_A(x) \\ &= 1 t \mu_A(x) = \mu_A(x)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x^{-1}) &= \nu_A(ex^{-1}) \\ &\leq \nu_A(e)\, s_t\, \nu_A(x) \\ &= 0\, s_t\, \nu_A(x) = \nu_A(x).\end{aligned}$$

So $\mu_A(x^{-1}) \geq \mu_A(x)$ and $\nu_A(x^{-1}) \leq \nu_A(x)$ for each $x \in X$.

On the other hand :

$$\begin{aligned}\mu_A(xy) &= \mu_A(x(y^{-1})^{-1}) \geq \mu_A(x) t \mu_A(y^{-1}) \\ &\geq \mu_A(x) t \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}\nu_A(xy) &= \nu_A(x(y^{-1})^{-1}) \\ &\leq \nu_A(x) s_t\, \nu_A(y^{-1}) \leq \nu_A(x) s_t\, \nu_A(y).\end{aligned}$$

Hence A is a t-IFG in X.

Proposition 2.7. Let X_p be the cyclic group of prime order p, let t a t-norm and let $A \in IFS(X_p)$ with $A(e)=(1,0)$, where e is the identity in X_p. If $A(x)=A(a) \leq A(e)$, i.e., $\mu_A(x)=\mu_A(a) \leq \mu_A(e)$ and $\nu_A(x)=\nu_A(a) \geq \nu_A(e)$ for each $e \neq x \in X_p$, where $X_p=(a)=\{e=a^0, a^1, a^2, \cdots, a^{p-1}\}$, then A is a t-IFG in X_p.

Proof. Let $x, y \in X_p$.

Case(i) : Suppose $x \neq e$, $y \neq e$ and $xy^{-1} \neq e$. Then, by the hypothesis, $\mu_A(xy^{-1})=\mu_A(x)=\mu_A(y)$ and $\nu_A(xy^{-1})=\nu_A(x)=\nu_A(y)$. Thus $\mu_A(xy^{-1}) \geq \mu_A(x)\, t \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x)\, s_t \nu_A(y)$.

Case(ii) : Suppose $x \neq e$, $y \neq e$ and $xy^{-1}=e$. Then, by the hypothesis, $\mu_A(x)=\mu_A(y) \leq \mu_A(e)=\mu_A(xy^{-1})$ and $\nu_A(x)=\nu_A(y) \geq \nu_A(e)=\nu_A(xy^{-1})$. Thus

$$\mu_A(xy^{-1}) \geq \mu_A(x) t \mu_A(y)$$

and

$$\nu_A(xy^{-1}) \leq \nu_A(x) s_t \nu_A(y).$$

Case(iii) : Suppose $x \neq e$ and $y=e$. Then, by the hypothesis, $\mu_A(x)=\mu_A(xy^{-1}) \leq \mu_A(e)=\mu_A(y)=1$ and $\nu_A(x)=\nu_A(xy^{-1}) \geq \nu_A(e)=\nu_A(y)=0$. Thus

$$\mu_A(xy^{-1}) \geq \mu_A(x) t 1 = \mu_A(x) t \mu_A(y)$$

and

$$\nu_A(xy^{-1}) \leq \nu_A(x) s_t 0 = \nu_A(x) s_t \nu_A(y).$$

Case(iv) : Suppose $x=e$ and $y \neq e$. Then it is the same as case (iii).

In all,

$$\mu_A(xy^{-1}) \geq \mu_A(x) t \mu_A(y)$$

and

$$\nu_A(xy^{-1}) \leq \nu_A(x) s_t \nu_A(y).$$

Hence A is a t-IFG in X_p.

Definition 2.8. Let A be a t-IFG in a group X. Then A is called a *t-intuitionistic fuzzy normal subgroup* (in short, *t-IFNG*) in X if $A(xy)=A(yx)$, i.e., $\mu_A(xy)=\mu_A(yx)$ and $\nu_A(xy)=\nu_A(yx)$ for any $x, y \in X$.

Proposition 2.9. Let A be a t-IFNG in a group X.

(1) For each $B \in IFS(X)$, $A \circ_t B = B \circ_t A$.

(2) If B is a t-IFG in X, then so is $B \circ_t A$.

Proof. (1) Let $z \in X$ with $z=xy$. Then :

$$\begin{aligned}\mu_{A \circ_t B}(z) &= \bigvee_{xy=z} \mu_A(x) t \mu_B(y) \\ &= \bigvee_{x=zy^{-1}} \mu_A(x) t \mu_B(y) \\ &= \bigvee_{x=zy^{-1}} \mu_A(zy^{-1}) t \mu_B(y) \\ &= \bigvee_{x'=y^{-1}z} \mu_A(x') t \mu_B(y) \\ &\quad (A \text{ is a } t-IFNG \text{ in } X) \\ &= \bigvee_{yx'=z} \mu_B(y)\, t \mu_A(x') = \mu_{B \circ_t A}(z)\end{aligned}$$

and

$$\begin{aligned}\nu_{A \circ_t B}(z) &= \bigwedge_{xy=z} \nu_A(x) s_t\, \nu_B(y) \\ &= \bigwedge_{x=zy^{-1}} \nu_A(zy^{-1}) s_t\, \nu_B(y) \\ &= \bigwedge_{x'=y^{-1}z} \nu_A(zy^{-1}) s_t\, \nu_B(y) \\ &= \bigwedge_{yx'=z} \nu_A(x') s_t\, \nu_B(y) \\ &= \bigwedge_{yx'=z} \nu_B(y) s_t\, \nu_A(x') \\ &= \nu_{\circ_t A}(z)\end{aligned}$$

So $\mu_{A \circ_t B} = \mu_{B \circ_t A}$ and $\nu_{A \circ_t B} = \nu_{B \circ_t A}$. Hence $A \circ_t B = B \circ_t A$

(2) By Definition 1.9 and Proposition 2.9 in [11],

$(B\circ_t A)\circ_t(B\circ_t A)=B\circ_t(A\circ_t B)\circ_t A=B\circ_t(B\circ_t A)\circ_t A=(B\circ_t B)\circ_t(A\circ_t A)\subset B\circ_t A$. Thus $B\circ_t A$ is a t-IFG in X. Now let $x\in X$ with $x^{-1}=yz$. Then :

$$\begin{aligned}\mu_{B\cdot_t A}(x^{-1}) &= \bigvee_{yz=x^{-1}} \mu_B(y)t\mu_A(z)\\ &= \bigvee_{x=z^{-1}y^{-1}} \mu_B((y^{-1})^{-1})t\mu_A((z^{-1})^{-1})\\ &\geq \bigvee_{x=z^{-1}y^{-1}} \mu_B(y^{-1})t\mu_A(z^{-1})\\ &= \bigvee_{x=z^{-1}y^{-1}} \mu_A(z^{-1})t\mu_B(y^{-1})\\ &= \mu_{A\cdot_t B}(x)=\mu_{B\cdot_t A}(x)\end{aligned}$$

and

$$\begin{aligned}\nu_{B\cdot_t A}(x^{-1}) &= \bigwedge_{yz=x^{-1}} \nu_B(y)s_t\nu_A(z)\\ &= \bigwedge_{x=z^{-1}y^{-1}} \nu_B((y^{-1})^{-1})s_t\nu_A((z^{-1})^{-1})\\ &\leq \bigwedge_{x=z^{-1}y^{-1}} \nu_B(y^{-1})s_t\nu_A(z^{-1})\\ &= \bigwedge_{x=z^{-1}y^{-1}} \nu_A(z^{-1})s_t\nu_B(y^{-1})\\ &= \nu_{A\cdot_t B}(x)=\nu_{B\cdot_t A}(x).\end{aligned}$$

Thus $\mu_{Bo_tA}(x^{-1})\geq\mu_{Bo_tA}(x)$ and $\nu_{Bo_tA}(x^{-1})\leq\nu_{Bo_tA}(x)$. Hence $B\,o_t A$ is a t-IFG in X.

3. t-intuitionistic fuzzy rings and ideals

Definition 3.1. Let $(X, +, \cdot)$ be a ring and let t a t-norm and let $0_\sim\neq A\in IFS(X)$. Then A is called a *t-intuitionistic fuzzy subring*(in short, *t−IFR*) in X if it satisfies the following conditions:

(i) A is a t-IFG in X with respect to "+" (in the sense of Definition 2.1),

(ii) A is a t-IFGP in X with respect to " $\cdot$ " (in the sense of Definition 1.9 or Definition 1.9′).

Proposition 3.2. Let X be a ring and let $0_\sim\neq A\in IFS(X)$ such that $A(0)=1_\sim$ where 0 is the zero element for "+" in X. Then A is a t-IFR in X if and only if for any $x, y\in X$,

$$\mu_A(x)t\mu_A(y)\leq\mu_A(x-y)\wedge\mu_A(xy)$$

and

$$\nu_A(x)s_t\nu_A(y)\geq\nu_A(x-y)\vee\nu_A(xy).$$

Proof. A is a t-IFR in X if and only if $\mu_A(x-y)\geq\mu_A(x)t\mu_A(y), \nu_A(x-y)\leq\nu_A(x)s_t\nu_A(y)$ (by Proposition 2.6) and $\mu_A(xy)\geq\mu_A(x)t\mu_A(y)$, $\nu_A(xy)\leq\nu_A(x)s_t\nu_A(y)$ for any $x,y\in X$ (by Definition 1.9′) if and only if $\mu_A(x)t\mu_A(y)\leq\mu_A(x-y)\wedge\mu_A(xy)$ and $\nu_A(x)s_t\nu_A(y)\geq\nu_A(x-y)\vee\nu_A(xy)$ for any $x, y\in X$.

Corollary 3.2. Let X be a ring and let $0_\sim\neq A\in IFS(X)$. Then A is an IFR in X if and only if $\mu_A(x)\wedge\mu_A(y)\leq\mu_A(x-y)\wedge\mu_A(xy)$ and $\nu_A(x)\vee\nu_A(y)\geq\nu_A(x-y)\vee\nu_A(xy)$ for any $x, y\in X$.

Definition 3.3. Let X be a ring and let $0_\sim\neq A$ a t-IFR in X. Then A is called a :

(1) *t-intuitionistic fuzzy left ideal* (in short, *t−IFLI*) in X if $A(xy)\geq A(y)$ i.e., $\mu_A(xy)\geq\mu_A(y)$ and $\nu_A(xy)\leq\nu_A(y)$ for any $x, y\in X$.

(2) *t-intuitionistic fuzzy right ideal* (in short, *t−IFRI*) in X if $A(xy)\geq A(x)$ i.e., $\mu_A(xy)\geq\mu_A(x)$ and $\nu_A(xy)\leq\nu_A(x)$ for any $x, y\in X$.

(3) *t-intuitionistic fuzzy ideal* (in short, *t−IFI*) in X if it is both t-IFLI and t-IFRI in X.

Proposition 3.4. Let X be a ring and let $0_\sim\neq A\in IFS(X)$ such that $A(0)=1_\sim$. Then A is a t-IFI [resp. t-IFLI, t-IFRI] in X if and only if $\mu_A(x-y)\geq\mu_A(x)t\mu_A(y)$, $\nu_A(x-y)\leq\nu_A(x)s_t\nu_A(y)$ and $\mu_A(xy)\geq\mu_A(x)s_t\mu_A(y)[\geq\mu_A(y),\geq\mu_A(x)]$, $\nu_A(xy)\leq\nu_A(x)t\nu_A(y)[\leq\nu_A(y),\leq\nu_A(x)]$ for any $x, y\in X$.

Proof. It is obvious from Proposition 3.2 and Definition 3.3.

Corollary 3.4[10, Proposition 4.5]. Let X be a ring and let $0_\sim\neq A\in IFS(X)$. Then A is an IFI[resp. IFLI, IFRI] in X if and only if $\mu_A(x-y)\geq\mu_A(x)\wedge\mu_A(y)$, $\nu_A(x-y)\leq\nu_A(x)\vee\nu_A(y)$ and $\mu_A(xy)\geq\mu_A(x)\vee\mu_A(y)$ $[\geq\mu_A(y),\geq\mu_A(x)]$, $\nu_A(xy)\leq\nu_A(x)\,\nu_A(y)\wedge$ $[\leq\nu_A(y), \leq\nu_A(x)]$for any $x, y\in X$.

Proposition 3.5. Let X be a skew-field and let $0_\sim\neq A\in IFS(X)$. Then A is a t-IFI in X if and only if

(1) $A(x)=A(e)$, i.e., $\mu_A(x)=\mu_A(e)$ and $\nu_A(x)=\nu_A(e)$ for each $x\in X-\{0\}$,

(2) $\mu_A(0)=\mu_A(0)s_t\mu_A(x)\geq\mu_A(e)t\mu_A(e)$ and $\nu_A(0)=\nu_A(0)t\nu_A(x)\leq\nu_A(e)s_t\nu_A(e)$ for each $x\in X$,

(3) $\mu_A(e)=\mu_A(e)s_t\mu_A(e)$ and $\nu_A(e)=\nu_A(e)t\nu_A(e)$.

Proof.(⇒) : Suppose A is a t-IFI in X and let $0\neq x\in X$. Then :

$$\begin{aligned}\mu_A(x) &= \mu_A(xe) \geq \mu_A(x) s_t \mu_A(e)\\ &= \mu_A(e) s_t \mu_A(x) \geq \mu_A(e) s_t 0\\ &= 1-(1-\mu_A(e)) t (1-0)\\ &= 1-(1-\mu_A(e)) = \mu_A(e) = \mu_A(x^{-1}x)\\ &\geq \mu_A(x^{-1}) s_t \mu_A(x) = \mu_A(x) s_t \mu_A(x^{-1})\\ &\geq \mu_A(x) s_t 0 = \mu_A(x)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x) &= \nu_A(xe) \leq \nu_A(x) t \nu_A(x)\\ &= \nu_A(e) t \nu_A(x) \leq \nu_A(e) t 1\\ &= 1-(1-\nu_A(e)) s_t (1-1)\\ &= 1-(1-\nu_A(e)) = \nu_A(e)\\ &= \nu_A(x^{-1}) t 1 \leq \nu_A(x^{-1}) t \nu_A(x)\\ &= \nu_A(x) t \nu_A(x^{-1}) \leq \nu_A(x) t 1\\ &= \nu_A(x).\end{aligned}$$

So $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$ for each $x \in X-\{0\}$, i.e., the condition (1) holds.

Let $x \in X$. Then:

$$\begin{aligned}\mu_A(0) &= \mu_A(x0) \geq \mu_A(x) s_t \mu_A(0)\\ &= \mu_A(0) s_t \mu_A(x) \geq \mu_A(0) s_t 0 = \mu_A(0)\\ &= \mu_A(0) s_t \mu_A(0) = \mu_A(e-e) s_t \mu_A(e-e)\\ &\geq [\mu_A(e) t \mu_A(e)] s_t [\mu_A(e) t \mu_A(e)]\\ &\geq [\mu_A(e) t \mu_A(e)] s_t 0 = \mu_A(e) t \mu_A(e)\end{aligned}$$

and

$$\begin{aligned}\nu_A(0) &= \nu_A(x0) \leq \nu_A(x) t \nu_A(0) = \nu_A(0) t 1\\ &= \nu_A(0) t \nu_A(0) = \nu_A(e-e) t \nu_A(e-e)\\ &\leq [\nu_A(e) s_t \nu_A(e)] t [\nu_A(e) s_t \nu_A(e)]\\ &= [\nu_A(e) s_t \nu_A(e)] t 1 = \nu_A(e) s_t \nu_A(e).\end{aligned}$$

So $\mu_A(0) = \mu_A(0) s_t \mu_A(x) \geq \mu_A(e) t \mu_A(e)$ and $\nu_A(0) = \nu_A(0) t \nu_A(x) \leq \nu_A(e) t \nu_A(e)$, i.e., the condition (2) holds.

Now let $0 \neq x \in X$. Then, by (1),

$$\begin{aligned}\mu_A(e) &= \mu_A(x) = \mu_A(xe) \geq \mu_A(x) s_t \mu_A(e)\\ &= \mu_A(e) s_t \mu_A(e) \geq \mu_A(e) s_t 0 = \mu_A(e)\end{aligned}$$

and

$$\begin{aligned}\nu_A(e) &= \nu_A(x) = \nu_A(xe) \leq \nu_A(x) t \nu_A(e)\\ &= \nu_A(e) t \nu_A(e) \leq \nu_A(e) t 1 = \nu_A(e).\end{aligned}$$

So $\mu_A(e) = \mu_A(e) s_t \mu_A(e)$ and $\nu_A(e) = \nu_A(e) t \nu_A(e)$, i.e., the condition (3) holds.

($\Leftarrow$): Suppose the necessary conditions hold and let $x \in X$. Since $\mu_A(0) = \mu_A(-0)$ and $\nu_A(0) = \nu_A(-0)$, let $x \neq 0$. Then, by (1), $\mu_A(x) = \mu_A(e) = \mu_A(-x)$ and $\nu_A(x) = \nu_A(e) = \nu_A(-x)$. So $\mu_A(-x) = \mu_A(x)$ and $\nu_A(-x) = \nu_A(x)$ for each $x \in X$. (*)

Let $x, y \in X$.

Case (i): Suppose $x+y \neq 0$ with $y \neq 0$. Then ;

$$\begin{aligned}\mu_A(x+y) &= \mu_A(x+y) t 1\\ &\geq \mu_A(x+y) t \mu_A(x)\\ &= \mu_A(e) t \mu_A(x) && (by(1))\\ &= \mu_A(y) t \mu_A(x) && (by(1))\\ &= \mu_A(x) t \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x+y) &= \nu_A(x+y) s_t 0\\ &\leq \nu_A(x+y) t \nu_A(x)\\ &= \nu_A(e) s_t \nu_A(x) && (by(1))\\ &= \nu_A(y) s_t \nu_A(x) && (by(1))\\ &= \nu_A(x) s_t \nu_A(y).\end{aligned}$$

Case(ii) : Suppose $x+y=0$ with $x=0$. Then :

$$\begin{aligned}\mu_A(x+y) &= \mu_A(0) = \mu_A(0) t 1 \geq \mu_A(0) t \mu_A(y)\\ &= \mu_A(x) t \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x+y) &= \nu_A(0) = \nu_A(0) s_t 0 \leq \nu_A(0) s_t \nu_A(y)\\ &= \nu_A(x) s_t \nu_A(y).\end{aligned}$$

Case(iii) : Suppose $x+y=0$ with $0 \neq x = -y$. Then ;

$$\begin{aligned}\mu_A(x+y) &= \mu_A(0) \geq \mu_A(e) t \mu_A(e) && (by(2))\\ &= \mu_A(x) t \mu_A(-y) && (by(1))\\ &= \mu_A(x) t \mu_A(y) && (by(*))\end{aligned}$$

and

$$\begin{aligned}\nu_A(x+y) &= \nu_A(0) \leq \nu_A(e) s_t \nu_A(e) && (by(2))\\ &= \nu_A(x) s_t \nu_A(-y) && (by(1))\\ &= \nu_A(x) s_t \nu_A(y). && (by(*))\end{aligned}$$

In all, for any $x, y \in X$, $\mu_A(x+y) \geq \mu_A(x) t \mu_A(y)$ and $\nu_A(x+y) \leq \nu_A(x) s_t \nu_A(y)$. (**)

Now let $x, y \in X$.

Case(i) : Suppose $xy=0$ with, say, $x=0$.

Then; by (2),

$$\mu_A(xy) = \mu_A(0) = \mu_A(0) s_t \mu_A(y) = \mu_A(x) s_t \mu_A(y)$$

and

$$\nu_A(xy) = \nu_A(0) = \nu_A(0) t \nu_A(y) = \nu_A(x) t \nu_A(y).$$

Case(ii) : Suppose $xy \neq 0$. Then; by (1) and (3),

$$\mu_A(xy) = \mu_A(e) = \mu_A(e) s_t \mu_A(e) = \mu_A(x) s_t \mu_A(y)$$

and

$$\nu_A(xy) = \nu_A(e) = \nu_A(e) t \nu_A(e) = \nu_A(x) t \nu_A(y).$$

In all, for any $x, y \in X$, $\mu_A(xy) \geq \mu_A(x) s_t \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) t \nu_A(y)$. (***)

On the other hand ; by (***),

$$\begin{aligned}\mu_A(xy) &\geq \mu_A(x) s_t \mu_A(y) \geq \mu_A(x) s_t 0\\ &= \mu_A(x) = \mu_A(x) t 1 \geq \mu_A(x) t \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}\nu_A(xy) &\le \nu_A(x) t \nu_A(y) \le \nu_A(x) t 1 \\ &= \nu_A(x) = \nu_A(x) s_t 0 \le \nu_A(x) s_t \nu_A(y)\end{aligned}.$$

So $\mu_A(xy) \ge \mu_A(x) t \mu_A(y)$ and $\nu_A(xy) \le \nu_A(x) s_t \nu_A(y)$ for $x, y \in X$. (****)

Hence, by (*), (**), (***) and (****), A is a t-IFI in X.

Corollary 3.5[10, Proposition 4.7]. Let X be a skew field and let $0_{\sim} \ne A \in IFS(X)$. Then A is an IFI[resp. IFLI, IFRI] in X if and only if $A(x) = A(e) \le A(0)$, i.e., $\mu_A(x) = \mu_A(e) \le \mu_A(0)$ and $\nu_A(x) = \nu_A(e) \ge \nu_A(0)$ for each $0 \ne x \in X$.

Proposition 3.6. Let X be a commutative ring with a unity e. If for any t-IFI A in X; $A(x) = A(e) \le A(0)$, i.e., $\mu_A(x) = \mu_A(e) \le \mu_A(0)$ and $\nu_A(x) = \nu_A(e) \ge \nu_A(0)$ for each $0 \ne x \in X$, then X is a field.

Proof. Let A be an ideal of X such that $A \ne X$. Then clearly $A = (\chi_A, \chi_{A^c})$ is a t-IFI in X such that $A \ne 1_{\sim}$. Then there exists $y \in X$ such that $y \notin A$.

Thus $\chi_A(y) = 0$ and $\chi_{A^c}(y) = 1$. By the hypothesis, $\chi_A(x) = \chi_A(e) \le \chi_A(0)$ and $\chi_{A^c}(x) = \chi_{A^c}(e) \ge \chi_{A^c}(0)$ for each $0 \ne x \in X$. Thus $\chi_A(x) = \chi_A(e) \le \chi_A(0)$ and $\chi_{A^c}(x) = \chi_{A^c}(e) = 1 \ge \chi_{A^c}(0)$ for each $0 \ne x \in X$. So $\chi_A(0) = 1$ and $\chi_{A^c}(0) = 0$, i.e., $A = \{0\}$. Hence X is a field.

Corollary 3.6[10, Proposition 4.9]. Let X be a commutative ring with a unity e. If for any IFI A in X, $A(x) = A(e) \le A(0)$, i.e., $\mu_A(x) = \mu_A(e) \le \mu_A(0)$ and $\nu_A(x) = \nu_A(e) \ge \nu_A(0)$ for each $0 \ne x \in X$. Then X is a field.

References

[1] J.M.Anthony and H. Sherwood, Fuzzy groups redefined, J. Math. Anal. Appl. 69(1979) 124-130.

[2] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986)87-96.

[3] Baldev Banerjee and Dhiren Kr. Basnet, Intuitionistic fuzzy subrings and ideals, J.Fuzzy Mathematics 11(1)(2003)139-155.

[4] R.Biswas, Intuitionistic fuzzy subgroups, Mathematical Forum x(1989)37-46.

[5] D.Çoker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88(1997)81-89.

[6] D.Çoker and A.Haydar Es, On fuzzy compactness in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 3(1995)899-909.

[7] P.S.Das, Fuzzy groups and level subgroups, J. Math. Anal. Appl. 84(1981)264-269.

[8] H.Gürçay, D.Çoker and A.Haydar Es, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5(1997) 365-378.

[9] K.Hur, S.Y.Jang and H.W.Kang, Intuitionistic fuzzy subgroupoids, International J. of Fuzzy Logic and Intelligent Systems 3(1) (2003) 72-77.

[10] K.Hur, H.W.Kang and H.K.Song, Intuitionistic fuzzy subgroups and subrings, Honam Mathematical J. 25(1) (2003) 19-41.

[11] K.Hur, H.W.Kang and J.H.Ryou, t-Intuitionistic fuzzy subgroupoids, To appear.

[12] S.J.Lee and E.P.Lee, The category of intuitionistic fuzzy topological spaces Bull. Korean Math. Soc. 37(1)(2000)63-76.

[13] Wang-jin Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems 8(1982) 133-139.

[14] A.Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35(1971) 512-517.

[15] B.Schweizer and A.Sklar, Statistical metric spaces, Pacific J. Math. 10(1960)313-334.

[16] S.Sessa, On fuzzy subgroups and fuzzy ideals under triangular norms, Fuzzy Sets and Systems 13(1984)95-100.

[17] L.A.Zadeh, Fuzzy sets, Inform. and Centrol8 (1965) 338-353.

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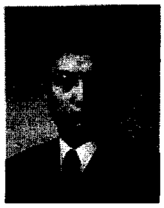

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