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안 성 제**

Risk Sharing in a Supply Chain*

Seongje Ahn**

■ Abstract ■

This paper suggests that the profit sharing contract can be Pareto optimal for both supplier and the purchaser. It is shown that Pareto optimal risk sharing contract can be obtained even though the decisions are made in a decentralized manner. The effect of risk attitude of the members of the supply chain is discussed. We examined various aspects of the risk sharing contract such as risk attitude, bargaining power, and cost of information system. The different risk attitude changes the optimal parameters and decision variables. Especially, we proved that, when both the supplier and the purchaser are risk averse, the purchaser orders less quantity than when the one is risk neutral and the other is risk averse. If the fixed cost for the information system is big enough to satisfy a certain condition, it is Pareto optimal not to share the profit and the purchaser takes all the risk even though he is risk averse.

Keyword : Supply Chain, Supply Contract, Risk Sharing, Pareto Optimal Contract, Profit Sharing Contract

1. Introduction

In a supply chain, firms are always faced with the make-or-buy decision. There are many factors to be considered in the decisions made in a supply chain. Due to intense competition and decreasing transaction costs, more firms tend to favor outsourcing. Many reasons such as lower

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cost, available capacity, quality, technology, delivery time, etc., drive manufacturers to outsource. Outsourcing of components and subassemblies is emerging increasingly as a key strategy for companies seeking competitive advantages in the global environment, and it represents a sizable amount of their management activities. Drastic cost advantages provide strong motivation for multinational firms to seek sourcing arrangements in environments with significant uncertainties of price and demand, with developing countries serving as prominent examples (Davis, 1992). The difficulties in managing the supply chain arises mainly from the fact that there are multiple decision makers in a supply chain, which may be different firms or different divisions in a single firm. The management of supply chain consisting of multiple decision makers with possibly conflicting objectives require considerations of the relationships among the decision makers. Uncertainties in a supply chain should be considered in managing the supply chain, too. Therefore, the outsourcing contracts have arrangement for the interests of the decision makers involved and the uncertainties facing them.

Uncertainties in a supply chain come from the fluctuating demand and the volatile price. Today's intense competition and rapid technological changes contribute to the demand uncertainties in the supply chain. Shortened life cycle due to the technological advances and diverse customer preferences are key drivers for the demand uncertainties. The risk associated with the demand uncertainty makes the management of the supply chain a difficult task. One example for the difficulties in managing the operation is due to the presence of the bullwhip effect in the supply chain. Many works have focused on finding the reasons of the bullwhip effect and the remedy for reducing the effect (Lee *et al.* 1997a). Most of the remedies stress on the use of information system to reduce the amplification of the demand uncertainty in a supply chain (Lee *et al.* 1997b). Even though the use of information system can reduce the demand uncertainty, the uncertainty that comes from the demand for the final product still remains. The risk associated with the demand uncertainties sometimes require a risk sharing contract for the companies in a supply chain. As we are going to discuss in this paper, the risk sharing could be realized in a form of profit sharing contract.

As for the price uncertainties, many reasons contribute to the perceived price uncertainties in sourcing arrangements within a supply chain. One major reason is the exchange rate movement, where a buyer can end up paying substantially more or less than the original contract price (Carter and Vickery 1988). Hyperinflation conditions in some developing countries also contribute to volatility of sourcing prices from local suppliers (Austin, 1990, Ch. 7, pp.185-230). Tight and difficult-to-forecast capacity conditions in fast growing industries combined with fluctuating prices of substitute components and material/process technologies further contribute to price uncertainties. In many industries (e.g., paper, agriculture, electronics, textiles) the sourcing of commodity inputs involves substantial price uncertainty. The spot prices of many commodities (e.g., commodity fibers, petro-chemicals) fluctuate substantially but there are no futures markets for them. Under such sourcing environments, it is not uncommon that a supplier will offer supply contracts with risk-sharing features to its buyers (see Carter and Vickery, 1988 ; Dornier *et al.*, 1998).

In this paper, we study supply contracts in environments of demand uncertainty. Our emphasis is on 1) developing a risk sharing contract between a purchaser and a supplier within a supply chain under stochastic demand, and 2) demonstrating and discussing various aspects of the Pareto optimal contract that enables both parties to share the risk.

2. Literature Review

Supply contracts formalize supply arrangements with respect to cost of components, quantity purchased, time of delivery, quality of goods, and other variables specific to the sourcing environment. There are lots of models in the inventory literature that can also be thought of as analyzing supply contracts of a specific form (see Lee and Nahmias, 1993 ; Porteus, 1990 ; Tsay *et al.*, 1999 for recent surveys). Much of the past work on the inventory literature involves with the multi-echelon inventory theory. The multi-echelon inventory theory is primarily about the decision of the timing and the flow of the material. However, the problems faced in supply chain management cannot be solved with the multi-echelon inventory theory, since there are multiple decision makers in a supply chain. A decision that is locally efficient can be inefficient in the perspective of the whole supply chain. Therefore, the outsourcing contract in a supply chain should have different arrangements for decision rights, pricing, lead time, quality, etc in order to be more efficient.

The recent operations management literature on supply contracts emphasizes the use of contractual flexibility in terms of purchased quantities in more effectively handling uncertain demand situations. Bassok and Anupindi (1997) analyzed the so-called total minimum quantity commitment contracts. The buyer is contractually committed to purchase a minimum cumulative quantity by the end of a planning horizon. Such contracts are common practices in the electronic industry.

Cachon and Lariviere (2000) analyzed the revenue sharing contracts between a supplier and a retailer. This type of contract was shown to be successful in the video cassette rental industry. It is shown that the revenue sharing contract enhances the channel coordination. But, the revenue sharing contract has no uncertainty in demand and is not a risk sharing contract. More extensive review on the supply chain contracts can be found in Tsay *et al.* (1999). The key element of previous research work on supply contracts is the optimization of contract parameters to better handle uncertain demand environments. Most of the previous works treats the firms in a supply chain to be risk neutral.

It would not be difficult to make the firms in a supply chain understand that a decision that is locally efficient can be inefficient in the perspective of the whole supply chain. Therefore, they can be persuaded to make a supply contract that they should fully cooperate in their decisions as if there were only one decision maker. Even though the contract of full cooperation is possible, their risk attitudes may differ from each other. hence, the contract should be designed to firms with different risk attitude. In our paper, we emphasize selecting a risk sharing contract through profit sharing and optimizing their parameters in handling the risk associated with the demand uncertainty.

3. The Model

The supply chain consists of a supplier and a purchaser. The supplier would be a component manufacturer when the purchaser is an assembler. When the purchaser is a retailer, the supplier would be a vendor. The supplier supplies an item to the purchaser. The purchaser either assembles a product with the supplied item, or sells it to the customer.

The profit sharing contract between the supplier and the purchaser is characterized as (q, w, t). The contract requires that the supplier's selling price for the item is w, the amount to be purchased by the purchaser is q units, and that the supplier gets the portion of the purchaser's profit, which is $(1-t)$ multiplied by the profit of the purchaser. The supplier's unit cost of production is c. The purchaser decides the amount to buy from the supplier. The purchaser acquires q unit of the item at the unit price of w. After the purchaser's activity, which would be either manufacturing or selling, his profit is realized. The profit is redistributed between the supplier and the purchaser and the purchaser keeps a final profit, which is t multiplied by the profit. Therefore, the ratio of the profit sharing between the supplier and the purchaser is $(1-t):t$. The profit that occurs at the purchaser before sharing the profit is called ex-ante profit.

We assume the contract lasts only for one period. After the contract has ended, the item has no value. In many industries, the product life cycle is shortened because of the intense competitions and the rapid technology advances. For example, it is known that personal computer has a life cycle of 3 months. It means that supplying a specific item could only last for 3 months. Hence, assuming one period contract between the supplier and the purchaser is not unrealistic.

The purchaser sells the assembled product to the customers and the ex-ante profit is realized. The ex-ante profit is determined by the demand for the product and the demand is random. Let the demand be d. Then, the ex-ante profit of the purchaser be $P(q,d)$. Note that the ex-ante profit is a function of q and d. Assembler forecasts the demand of the product and decides the purchasing amount. But, the actual demand will be different from the forecast. If the demand is higher than the forecast, the purchaser can only manufacture the amount that can be produced with q units of the item and the demand that exceeds the manufactured amount of the product will be lost. If the demand is lower than the forecast, the purchaser can satisfy the demand, but there will be some items that are left and cannot be used elsewhere. This is similar to the newsboy problem. Let $P(q)$ be the expected ex-ante profit. Since the demand d is random, we can define the variance of $P(q,d)$. Let $v(q)$ be the variance of $P(q,d)$.

The profit of the supplier will be $(w-c)q+(1-t)P(q,d)-F_s$ after the supplier and the purchaser shares the profit. $(w-c)q$ is the profit from producing q units of item at a unit cost of c and supplying them at the price of w. The shared profit after the purchaser's en-ante profit is realized would be $(1-t)P(q,d)$. Note that the shared profit is random. F_s is the fixed cost of the supplier. The fixed cost occurs when the supplier has to make a capital investment to manufacture the item. In a fast-changing industry such as the semiconductor industry, a large capital investment is needed to start manufacturing a new product. The fixed cost also includes the investment in developing the information system that would be used for the communication with the purchaser. The information system could be used for monitoring the activities of the purchaser. After the product is introduced in the market, the need for change of the product design may occur due to changing customer preferences. The changes in customers' preferences requires the rapid change of the design of the item. It makes the communication between the purchaser and the supplier necessary. Furthermore, when the purchaser employs a Just-In-Time production system, the supplier may have to supply the item in a JIT manner. These require the supplier to invest in an information system that enables fast communications between the supplier and the purchaser. By the same reason, the purchaser should make a capital investment, too. The purchaser should invest on facility and information system to implement the contract.

The profit of the purchaser will be $tP(q,d) - qw - F_p$. After the purchaser's ex-ante profit is realized, $(1-t)P(q,d)$ is paid to the supplier and $tP(q,d)$ is the profit of the purchaser after the sharing is done. Since the purchaser pays w for the item, it is accounted in the profit. F_p is the amount of capital invested by the purchaser to fufill the contract with the supplier.

Let's assume that the expected utility is affected by the expected value and the variance of the profit. We can assume that the expected utility of the supplier is as follows.

$$\begin{aligned} EU_s(q,w,t) &= E[U_s(q,w,t,d)] \\ &= (w-c)q + (1-t)P(q) - \\ &\quad F_s - \alpha(1-t)^2 v(q), \end{aligned} \tag{1}$$

where α is a coefficient for the supplier's risk attitude. If he is more risk-averse, he has a larger α. The variance of the profit is $(1-t)^2 v(q)$ and the expected profit is $(w-c)q+(1-t)P(q)-F_s$.

The expected utility of the purchaser is as follows.

$$\begin{aligned} EU_p(q,w,t) &= E[U_p(q,w,t,d)] \\ &= tP(q) - qw - F_p - \beta t^2 v(q), \end{aligned} \tag{2}$$

where β is a coefficient for the purchaser's risk attitude. If he is more risk-averse, he has a larger β. The variance of the profit is $t^2 v(q)$ and the expected profit is $tP(q) - qw - F_p$.

3.1 The Pareto Optimal Contract

We can obtain the Pareto optimal contract (q, w, t) in this simple supply chain. The expected utility of the supply chain is the sum of supplier's expected utility and purchaser's expected utility, if we assume that the utility transfer between the supplier and purchaser is possible. When we maximize the total profit of the supply chain, the total profit will be always better than or equal to the sum of profits that are maximized for each participant in the supply chain.

The expected utility of the supply chain is as follows.

$$\begin{aligned} EU(q,t) &= EU_s(q,w,t) + EU_p(q,w,t) \\ &= P(q) - cq - F_s - F_p \\ &\quad - \{\alpha(1-t)^2 + \beta t^2\} v(q). \end{aligned} \tag{3}$$

Note that w is not a factor in determining the

Pareto optimal contract. Since the purchaser pays qw to the supplier and it is just a transfer of money from the purchaser to the supplier, there is no effect of w in the utility of the whole supply chain. Therefore, the contract should specify only q and t.

The first order conditions are as follows.

$$\frac{\partial EU(q,t)}{\partial q} = P'(q) - c - \{\alpha(1-t)^2 + \beta t^2\} v'(q) = 0. \quad (4)$$

$$\frac{\partial EU(q,t)}{\partial t} = 2\{\alpha(1-t) - \beta t\} v(q) = 0. \quad (5)$$

Let q^* and t^* be the optimal contract obtained from (4) and (5). It is obvious that an optimal t^* is as follows.

$$t^* = \frac{\alpha}{\alpha+\beta} \quad (6)$$

Using the optimal t^* from the equation (6), the equation (4) can be rewritten as follows.

$$P'(q) - c + \frac{\alpha\beta}{\alpha+\beta} v'(q) = 0 \quad (4')$$

The optimal q^* should satisfy the equation (4′).

The maximized expected utility of the supply chain can be obtained as follows.

$$EU(q^*, t^*) = P(q^*) - cq^* - F_s - F_p - \frac{\alpha\beta}{\alpha+\beta} v(q^*) \quad (7)$$

3.2 Real World Contract

The Pareto optimal contract may not be established in the real world. The supplier and purchaser don't make decisions as if they were one company. We will consider a contract that is possible in the real world. They will make decisions separately. At first, the supplier decides (w, t, F_p) and suggest it to the purchaser. Note that F_p is one of decision variables of the supplier. It means that the supplier makes the first move for the contract knowing that a fixed cost of F_s would occur to him. In other words, the supplier decides to make an offer even though the contract would cost him F_s. The offer to the purchaser requires the purchaser to make a capital investment of F_p if he decides to take it. When the purchaser decides whether he will accept the offer or not, F_s does not affect the decision of the purchaser. The offer also specifies that the price of the item is w and the ratio of profit sharing is t.

When we consider the Pareto optimal contract, we don't have to worry about F_p or F_s, since they are independent of other decision variables. Whatever their values are, they are not determinants of the Pareto optimal contract. But, they play a major role in the expected utility of the firms in a supply chain. When the supplier and the purchaser make decisions on their own, they are key determinants of the contract along with w. The purchaser decide on q after the supplier offers him a contract of (w, t, F_p). Let's assume that the reservation utility of the purchaser is z.

For a given (w, t, F_p), the purchaser solves the following problem to decide on q.

$$Max_q\, EU_p(q, w, t) = tP(q) - qw - F_p - \beta t^2 v(q) \quad (8)$$

$$\text{s.t.}\quad tP(q) - qw - F_p - \beta t^2 v(q) \geq z \quad (9)$$

$$q \geq 0$$

Let's call the above problem as the purchaser's problem. The objective is to maximize the expected utility of the purchaser for a given (w, t, F_p) with a constraint of reservation utility (9). Assume that, for a given (w, t, F_p), the purchaser could find q that satisfies the constraint of reservation utility and the following condition, which is the first order condition of the purchaser's problem without the constraint for the reservation utility.

$$tP'(q) - w - \beta t^2 v'(q) = 0 \tag{10}$$

The supplier knows that the purchaser will solve the above problem if he suggest (w, t, F_p) to the purchaser. Note that, if we can maximize the expected utility of the supply chain satisfying the purchaser's reservation utility, we will have a Pareto contract and it will be maximizing purchaser's expected utility, too.

The Pareto optimal q maximizes the expected utility of the supply chain given (w, t, F_p). If q satisfying equation (10) satisfies equations of the first order condition for the Pareto optimal equations (4) and (5), we will have a optimal contract for the purchaser which is also Pareto optimal. Let's define w^* as follows.

$$w^* = tc + t(1-t)\{\alpha(1-t) - \beta t\} v'(q) \tag{11}$$

Then, w^* satisfies both (4) and (10). The supplier suggest a contract (w^*, t, F_p) to the purchaser. Then, the purchaser solves the purchser's problem and obtains the optimal q^* by solving the equation (10) if q^* satisfies the constraint (9). Both w^* and q^* satisfy the first order conditions of (4) and (10). Therefore, the Pareto optimal ratio of profit sharing is $t^* = \dfrac{\alpha}{\alpha+\beta}$.

If the supplier uses the Pareto optimal t^*, w^* becomes $w^* = t^* c = \dfrac{c\alpha}{\alpha+\beta}$.

If the supplier can satisfy the reservation utility of the purchaser and suggest w^* and t^*, it would be Pareto optimal and also be optimal for the supplier and purchaser when they make their own decisions. The supplier should satisfy the reservation utility constraint to make the contract possible. If the fixed cost of the purchaser is set as follows, it always satisfies the reservation utility of the purchaser. The fixed cost of the purchaser should be

$$F_p^* = t^* P(q^*) - q^* w - \beta t^{*2} v(q^*) - z. \tag{12}$$

If the supplier suggests a contract (w^*, t^*, F_p^*) to the purchaser, it satisfies the reservation utility of the purchaser. Under this contract, the purchaser decides on q^*. It will be Pareto optimal and optimal for both the supplier and the purchaser. We have proved the following theorem holds.

Theorem 1 : Pareto optimal contract is possible when the supplier and purchaser make decisions on their own. The Pareto optimal contract (w^*, t^*, F_p^*) is $w^* = \dfrac{c\alpha}{\alpha+\beta}$, $t^* = \dfrac{\alpha}{\alpha+\beta}$, and $F_p^* = t^* P(q^*) - q^* w - \beta t^{*2} v(q^*) - z$, where q^* satisfies $P'(q^*) - c + \dfrac{\alpha\beta}{\alpha+\beta} v'(q^*) = 0$.

4. Charicteristics of the Risk Sharing Contract

We will discuss the characteristics of the risk sharing contract in the previous section in terms of risk attitude, bargaining power, and cost of the information system.

4.1 Risk Attitude

When the variance associated with the demand is zero, we don't need to consider the risk. It means that we have $v(q)=0$. If we let $w^*=tc$ and find q^* satisfies the first order condition of the Pareto optimal, which is $P'(q)=c$, they will be optimal when the supplier and purchaser make their own decisions. The utility of the supply chain will be $P(q)-cq^*-F_s-F_p$. If t and F_p satisfy the reservation utility constraint $tP(q^*)-q^*w^*-F_p \geq z$, it will be sufficient.

Even though there is a risk associated with the uncertain demand (i.e. $v(q)>0$), if the supplier and the purchaser are risk neutral, we don't have to consider the risk in the contract. If they are risk neutral, we have $\alpha=\beta=0$. In this case we have the same result as in the case when there is no variance of the demand. If we let $w^*=tc$ and find q^* satisfies the first order condition of the Pareto optimal $P'(q)=c$, they will be optimal when the supplier and purchaser make their own decisions. The utility of the supply chain will be $P(q^*)-cq^*-F_s-F_p$. If t and F_p satisfy the reservation utility constraint $tP(q^*)-q^*w^*-F_p \geq z$, it will be sufficient.

When both parties are not risk neutral and the variance of the demand is not negligible, further discussion is needed. Let us examine the changes of the optimal contract with respect to the risk attitude. As we discussed earlier, the profit sharing ratio is determined by $t^*=\frac{\alpha}{\alpha+\beta}$. If the purchaser is more risk averse, β becomes larger and the purchaser have less proportion of the ex-ante profit. In contrast, if the supplier is more risk averse, the supplier has less proportion of the ex-ante profit and the purchaser has larger portion of the ex-ante profit. It is obvious that the uncertainty comes from the ex-ante profit and that, if one is more risk averse, he has less risk and less proportion of the uncertain ex-ante profit.

If the supplier is risk averse ($\alpha>0$) and the purchaser is risk neutral ($\beta=0$), $t^*=1$. All the risk is taken by the purchaser. It would be the case that the purchaser is large enough to take the risk and the supplier is relatively small. For the opposite case ($\alpha=0, \beta>0$), the supplier should be large enough to takes the risk.

The supplier sells the item at the price of $w^*=t^*c$ to the purchaser. If the purchaser has more proportion of the ex-ante profit, the price gets higher. It is interesting to note that w^* is always smaller than or equal to c, since $t^* \leq 1$. If w^* is larger than c, the supplier would always be better off and the purchaser would always be worse off with the profit sharing contract. So, the contract cannot be established. The profit sharing contract requires both parties to share the risk when they are risk averse. Therefore, even though the supplier sells the item at the price lower than the cost, he will get positive total profit after receiving the portion of the ex-ante profit from the purchaser.

The optimal order quantity q^* should satisfy $P'(q)-c-\frac{\alpha\beta}{\alpha+\beta}v'(q)=0$. If either the supplier or the purchaser is risk neutral, the equation becomes $P'(q)=c$. This is the same as the case when the profit maximizing and risk neutral firms decide their production quantity. In other words, they produce the quantity satisfying that the marginal profit equals to the marginal cost. If the supplier is risk neutral and the purchaser is risk averse, the supplier takes all the risk and the purchaser don't have to take the risk into consideration with the optimal contract. In this case, the purchaser acts as if he is risk neutral. If both parties are risk averse, the optimal order quantity is determined by $P'(q) = c + \frac{\alpha\beta}{\alpha+\beta} v'(q)$. It means that they produce the quantity satisfying that the marginal profit equals to the marginal cost plus the cost associated with the risk.

The optimal order quantity when both parties are risk averse will be determined by q satisfying $P'(q) = c + \frac{\alpha\beta}{\alpha+\beta} v'(q)$. The optimal order quantity when the one is risk neutral and the other is risk averse, will be determined by q satisfying $P'(q) = c$. It is the same for the case when both parties are risk neutral as we discussed earlier. It would be interesting to compare the optimal order quantities between the case of risk averse supplier and purchaser and the case of risk neutral supplier or purchaser. The following theorem examines the relationship.

Theorem 2 : When both the supplier and the purchaser are risk averse, the purchaser orders less quantity than when the one is risk neutral and the other is risk averse. If both are risk neutral, the order quantity is the same as the case when the one is risk neutral and the other is risk averse.

Proof : Assume that only one item supplied by the supplier is need by the purchaser to assemble one unit of the product[1]. Let p be the contribution margin of the product to the purchaser. The demand d follows a probability distribution function $g(x)$. $G(x)$ is a cumulative distribution function of $g(x)$. Let d' be the quantity that were sold. The expected value of the ex-ante profit is as follows.

$$\begin{aligned} P(q) &= E[P(q,d)] = E[pd'] = pE[d'] \\ &= p\{\int_0^q x dG(x) + q(1-G(q))\} \\ &= p(q - \int_0^q G(x)dx) \end{aligned}$$

The variance of the ex-ante profit is as follows.

$$\begin{aligned} v(q) &= Var[P(q,d)] = Var[pd'] \\ &= p^2 Var[d'] \\ &= p^2\{\int_0^q (x-E[d'])^2 dG(x) \\ &\quad + (q-E[d'])^2(1-G(q))\} \\ &= p^2\{(q-E[d'])^2 G(q) \\ &\quad -2\int_0^q (x-E[d'])G(x)dx \\ &\quad + (q-E[d'])^2(1-G(q))\} \end{aligned}$$

If we differentiate $P(q)$ with respect to q,

$$\frac{dP(q)}{dq} = p\{1-G(q)\} \geq 0$$

If we differentiate $v(q)$ with respect to q,

1) Note that the purchaser could be either an assembler or a retailer. When the purchaser is an assembler, there could be the case that more than one unit of the item is needed to assemble one unit of the product. In this case, the proof can be easily proved. If the purchaser is a retailer, the proof can be applied without modification.

$$\frac{dv(q)}{dq} = p^2\{(q-E[d'])^2 g(q) + 2G(q)(q-E[d'])(G(q)-1) + 2(1-G(q))\int_0^q G(x)dx + 2(q-E[d'])G(q)(1-G(q)) - (q-E[d'])^2 g(q)\} = 2p^2(1-G(q))(q-E[d'])$$

Therefore,

$$\frac{dv(q)}{dq} = 2p^2(1-G(q))\int_0^q G(x)dx \geq 0$$

As q increases, $P(q)$ and $v(q)$ increase.

We can show that $\frac{d^2P(q)}{dq^2} = -pg(q) \leq 0.$ Therefore, $P(q)$ is an increasing concave function of q. If both parties are risk averse, the optimal order quantity is determined by solving $P'(q) = c + \frac{\alpha\beta}{\alpha+\beta} v'(q)$. The second term, which is $\frac{\alpha\beta}{\alpha+\beta} v'(q)$, is strictly positive. If the one is risk neutral and the other is risk averse, the optimal order quantity is determined by solving $P'(q) = c$. If both the supplier and the purchaser are risk averse, the purchaser orders less quantity than when the one is risk neutral and the other is risk averse, since $c + \frac{\alpha\beta}{\alpha+\beta} v'(q) > c$ and $P(q)$ is an increasing concave function of q. When both parties are risk neutral, the optimal order quantity is determined by solving $P'(q) = c$, therefore we will have the same result. [*QED*]

4.2 Bargaining Power

We have discussed that the supplier made a first move by suggesting a contract (w^*, t^*, F_p^*) to the purchaser. Then, the purchaser decides whether to take the contract or not. If he wants to take the contract, he decides on q^*. It means that the supplier has more bargaining power and that he takes all the remaining utility after the minimum reservation utility is given to the purchaser. In the opposite case when the purchaser has more bargaining power, the reverse would happen. If the purchaser wants to maximize his utility given the reservation utility of the supplier, he also has to maximize the utility of the supply chain. It will lead to the same result as before except the constraint for the reservation utility of the supplier. Let z' be the reservation utility of the supplier. The new constraint will be as follows.

$$(w-c)q + (1-t)P(q)\alpha - F_s - (1-t)^2 v(q) \geq z' \quad (13)$$

Therefore, the purchaser should choose F_s^* that satisfies the following equation.

$$F_s^* = (w^*-c)q + (1-t^*)P(q^*) - \alpha(1-t^*)^2 v(q^*) - z'$$

Note that the purchaser has more bargaining power and determines the fixed cost for the supplier.

The bargaining power can be explained in terms of the reservation utility. If the reservation utility of the purchaser (i.e. z) increases, he has more bargaining power. If z becomes very large, it could require a negative F_p^* which means that the supplier have to pay a lump sum money to the purchaser to complete the contract. Even though it would be the case, it is Pareto optimal.

4.3 Cost of Information System

We discussed that the risk sharing contract in the form of profit sharing is Pareto optimal for the risk averse purchaser and supplier. However, the contract between the purchaser and the supplier is not always a profit sharing contract. The reason would be that they have to have the perfect information on each other's cost structure. The supplier also needs to know exactly how much profit the purchaser makes. Actually, it is the problem of gathering and monitoring the information on each other. Furthermore, there exist incentives for the purchaser to deceive his ex-ante profit. If this problem is solved, the profit sharing contract is possible. Hewlett-Packard and Apple require that their retailer's POS (point of sales) system send the sales data to their information systems when the sales occur. The requirement of sell-through data is used to reduce the bullwhip effect. In a retail industry, many firms adopt new inventory management programs such as VMI (vendor managed inventory), continuous replenishment, and consignment. All these programs enable the supplier to monitor the exact sales data continuously through the information system or sometimes the manual system. If both parties in the supply chain agree to adopt the information system that can monitor the sales, the profit sharing contract is possible. However, such information systems require investments. They are included in the model as fixed costs such as F_s and F_p.

Assume that there are no other fixed costs except costs associated with the information system in the model discussed earlier. F_s is a fixed cost required for the supplier to make his information system handle the communication with the purchaser. Also, F_p a fixed cost required for the purchaser to make his information system handle the communication with the supplier. Since they are fixed costs, they are not included in the equations to obtain w^*, t^*, and q^*. However, they play a role in determining whether the constraint for the reservation utility can be satisfied or not. If the fixed costs are large, the constraints (equation (9) and (13)) for the reservation utility are not satisfied unless the reservation utility is small enough. If the fixed costs are too large, it would be impossible to make a contract even though the reservation utilities are zero.

Assume that the required fixed costs (F_s and F_p) are for other capital investments only and they are not for investments in information systems. Assume that the extra fixed cost F_i is need for the purchaser to modify its information system and the supplier does not have to invest in its information system. If the new fixed costs ($F_p + F_i$ instead of F_p) associated with the contract can satisfy the reservation utility constraints (constraint (9)), the Pareto optimal contract can be established and the expected utilities of the supplier and the purchaser are as follows.

$$
\begin{aligned}
EU_s(q^*, w^*, t^*) &= (w^* - c)q^2 + (1 - t^*)P(q^*) \\
&\quad - F_s^* - \alpha(1 - t^2)^2 v(q^*) \\
EU_p(q^*, w^*, t^*) &= t^*P(q^*) - q^*w^* - F_p^* \\
&\quad - F_i - \beta t^{*^2} v(q^*)
\end{aligned}
$$

The expected utility of the supply chain is the sum of the expected utilities of both parties. It is as follows.

$$EU(q^*, t^*) = P(q^*) - cq^* - F_s^* - F_p^* - F_i - \{\alpha(1-t^*)^2 + \beta t^{*2}\} v(q^*) = P(q^*) - cq^* - F_s^* - F_p^* - F_i - \frac{\alpha\beta}{\alpha+\beta} v(q^*)$$

Assume that the profit sharing ratio is 1, *t=1*. It means that the purchaser keeps all the profit and there is no profit sharing. When there is no profit sharing, there is no need for the investment in the information system of the purchaser and the expected utility of the supply chain is as follows.

$$EU(q, t)_{t=1} = P(q) - cq - F_s - F_p - \beta v(q)$$

Let $q^*_{t=1}$ is an optimal order quantity when there is no profit sharing. $q^*_{t=1}$ can be obtained by solving the following equation.

$$P'(q) - c - \beta v'(q) = 0$$

Note that q^* is obtained by solving the equation (4′), which is

$$P'(q) - c + \frac{\alpha\beta}{\alpha+\beta} v'(q) = 0.$$

Two equations are different and it means that $q^*_{t=1}$ is different from q^* except the case when $\beta=0$. When the purchaser is risk neutral ($\beta=0$), he takes all the risk associated with the demand uncertainty and it is true that $q^*_{t=1} = q^*$.

If $EU(q^*_{t=1}, 1)_{t=1} \geq EU(q^*, t^*)$ is satisfied, it is Pareto optimal not to share the profit. This is the case, which satisfies the following inequality.

$$P(q^*_{t=1}) - cq^*_{t=1} - F_s^* - F_p^* - \beta v(q^*_{t=1}) \geq P(q^*) - cq^* - F_s^* - F_p^* - F_i - \frac{\alpha\beta}{\alpha+\beta} v(q^*)$$

The inequality can be rewritten as follows.

$$F_i \geq P(q^*) - P(q^*_{t=1}) + c(q^*_{t=1} - q^*) + \beta\{v(q^*_{t=1}) - \frac{\alpha}{\alpha+\beta} v(q^*)\}$$

If the fixed cost for the information system is big enough to satisfy the above inequality, it is Pareto optimal not to share the profit and the purchaser takes all the risk even though he is risk averse ($\beta > 0$). Therefore, the following theorem is proved.

Theorem 3 : If the fixed cost for the purchaser's information system is big enough to satisfy the inequality, $F_i \geq P(q^*) - P(q^*_{t=1}) + c(q^*_{t=1} - q^*) + \beta\{v(q^*_{t=1}) - \frac{\alpha}{\alpha+\beta} v(q^*)\}$, it is Pareto optimal not to share the profit and the purchaser takes all the risk even though he is risk averse ($\beta > 0$).

5. Numerical Example

Let the distribution of the demand is uniform between 0 an 5. Also assume that the contribution margin is 10 ($p=10$) and the cost of the item supplied to the purchaser is 3 ($c=3$). The ex-ante profit of the purchaser is as follows.

$$P(q) = p(q - \int_0^q G(x)dx) = 10(q - \int_0^q \frac{x}{5} dx) = 10(q - \frac{q^2}{10})$$

The variance of the ex-ante profit is obtained as follows.

$$v(q) = p^2\{\int_0^q (x - E[d'])^2 dG(x) + (q - E[d'])^2(1 - G(q))\}$$

$$= 100\{\int_0^q \frac{1}{5}(x-(q-\frac{q^2}{10}))^2 dx + (1-\frac{q}{5})\frac{q^2}{10}\} = 10q^2+\frac{14}{3}q^3-2q^4+\frac{q^5}{5}$$

For different combinations of α and β, we obtained the Pareto optimal contract (w^*, t^*, F_p^*). We have to solve equation (4′) and find optimal q^*'s. Mathematica (v2.0) is used to solve the equation. The constraint for the reservation utility can be rewritten as follows.

$$F_p^* + z \ \leq \ tP(q^*) - q^* w - \beta t^2 v(q^*)$$

We can obtain the maximum possible value of $F_p^* + z$, which is the right hand side of the above inequality. The sum of the reservation utility and the value of F_p^* can be as big as the numbers in the sixth column in the following tables. The results are shown in <Table 1> and <Table 2>.

<Table 2> shows the Pareto optimal contracts obtained after the values of α and β in <Table 1> are interchanged. Therefore, the values of t^* and w^* are different from the values in <Table 1>, but the values of q^* are the same as the values in <Table 1>, since we are solving equation (4′) to obtain the values of q^* and the values of $\frac{\alpha\beta}{\alpha+\beta}$ in each row of the tables are the same. It is obvious the values of $F_p^* + z$ in <Table 1> are different from <Table 2> since we use equation (12) to obtain the values of $F_p^* + z$.

We can observe that if the purchaser or the supplier becomes more risk averse, he orders less quantity as suggested in Theorem 2. Since $w^* = t^* c$, it is obvious that, as t^* decreases, w^* decreases in <Table 1>. As t^* increases in <Table 2>, w^* increases.

〈Table 1〉 Pareto Optimal Contracts when $\alpha = 0.1$

α	β	t^*	q^*	w^*	$F_p^* + z$
0.1	0.06	0.625	2.532	1.875	5.231
0.1	0.07	0.588	2.442	1.765	4.759
0.1	0.08	0.556	2.365	1.667	4.364
0.1	0.09	0.526	2.299	1.579	4.029
0.1	0.1	0.500	2.242	1.500	3.742
0.1	0.11	0.476	2.192	1.429	3.493
0.1	0.12	0.455	2.148	1.364	3.276
0.1	0.13	0.435	2.110	1.304	3.083
0.1	0.14	0.417	2.075	1.250	2.912

〈Table 2〉 Pareto Optimal Contracts when $\beta = 0.1$

α	β	t^*	q^*	w^*	$F_p^* + z$
0.06	0.1	0.375	2.532	1.125	3.139
0.07	0.1	0.412	2.442	1.235	3.331
0.08	0.1	0.444	2.365	1.333	3.491
0.09	0.1	0.474	2.299	1.421	3.627
0.1	0.1	0.500	2.242	1.500	3.742
0.11	0.1	0.524	2.192	1.571	3.843
0.12	0.1	0.545	2.148	1.636	3.931
0.13	0.1	0.565	2.110	1.696	4.008
0.14	0.1	0.583	2.075	1.750	4.077

It is also interesting to note that $F_p^* + z$ increases, as t^* increases in <Table 1> and <Table 2>. It means that the ex-post profit of the purchaser should be increased if he has to make a larger capital investment or if his reservation utility increases. Since we already discussed the case when there is no demand uncertainty and the case when one of the purchaser and the supplier is risk neutral, numerical examples for these cases are not treated.

6. Summary and Further Research

We proposed a profit sharing contract which both the supplier and the purchaser would be better off. The contract specifies the supplier's selling price for the item, the amount to be purchased by the purchaser, the amount of required capital investment, and that the supplier's proportion of the purchaser's profit. We have proved that the proposed contract can be Pareto optimal for both the supplier and the purchaser under the demand uncertainty. It was shown that the Pareto optimal contract can be obtained even though the decisions are made in a decentralized manner. The profit sharing enables both the supplier and the purchaser to share the risk associated with the demand uncertainty. The effect of risk attitude of the members of the supply chain is discussed. We examined various aspects of the risk sharing contract such as risk attitude, bargaining power, and cost of information system. The different risk attitude changes the optimal parameters and decision variables. Especially, we proved that, when both the supplier and the purchaser are risk averse, the purchaser orders less quantity than when the one is risk neutral and the other is risk averse. If the fixed cost for the information system is big enough to satisfy a certain condition, it is Pareto optimal not to share the profit and the purchaser takes all the risk even though he is risk averse.

As we have discussed earlier, there exist two sources of risk in a supply chain : price uncertainty and demand uncertainty. However, our model assumes that the uncertainty in a supply chain comes only from the demand. The price uncertainty is not treated in our risk sharing contract. In general, it would be better off to share the risk with companies in the supply chain, when there exists risk. It will be an interesting but challenging subject if price uncertainty and demand uncertainty can be considered at the same time in a risk sharing contract. One limitation would be that our model is only valid under perfect information. Even though firms in a supply chain have fully agreed to share their sales data, there still is a possibility to deceive the other. If there exists information asymmetry between the purchaser and the supplier, we need more elegant mechanism of the contract to make it Pareto optimal.

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