Strain Decomposition Method in Hull Stress Monitoring System for Container Ship

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Abstract
The hull monitoring systems of container ships with four long-base gages give enough information for identifying the hull girder loads such as bending and torsional moments. But such a load-identification for container ships has not been known.
In this paper, a load-identification method is suggested in terms of a linear matrix equation that the measured strain vector equals to the multiplication of the transformation matrix and the desired strain component vector. The equation is proved to be mathematically complete by the property of positive-definite determinant of the transformation matrix.
The method is applied to a hull stress monitoring system for 8100TEU container ship during sea trial, and the estimated external loads illustrate reasonable results in comparison with the pre-estimated results. This moment decomposition concept has also been tested in real operation conditions. The typical phenomena over the Suez Canal illustrated very suitable results comparing with the physical understandings.
Henceforth, one can effectively use the proposed concept to monitor the hull girder loads such as bending and torsional moments.

**Keywords: load estimation, strain decomposition, hull monitoring, container ship**

1 Introduction

Hull stress monitoring systems with long-base strain gages have been used to monitor the stress-states of hull structure both during navigation and loading/unloading operation. For most ships such as oil tankers, bulk carriers and LNG carriers, etc., the gages are attached on the main decks only, and thus just vertical bending moment can be calculated from the measured strains. Witmer and Lewis(1995) established the theoretical and experimental backgrounds of such a system.

In case of container ships, the strains due to warping deformation should be measured because the ships have large deck openings that cause large warping deformation. In order to monitor this warping, American Bureau of Shipping (1995) has suggested 4 long-base strain gages at the midship cross-section, 2 at main upper deck and 2 at bottom deck. The system suggested by ABS and others can capture enough strains to identify hull stresses due to bending and torsional moments including warping deformation, but such a load-identification has not been known because of the lack of understanding of the relations between load components.
This paper will propose a decomposition method of these strains. The relations between bending moments and corresponding strains can be easily found in terms of Young's modulus and cross-sectional moments, but the warping moments require the complex warping function consisting of many geometrical parameters. Lee (1981) has proposed a warping function of a container ship by the mathematical formulation by assuming that additional members and double hull structure are simplified. However, Fujitani (1990) has suggested the finite element method to give the distribution of warping function on the cross-section.

After decomposition, the vertical/horizontal moments can be estimated by using section properties and the decomposed strains. For torsional moment, Hill (1943) proposed a relation between strain and torsional moment for simple cross-sections. For general cases of container ships, Kim et al (2000) derived mathematical relations but the additional gages at the adjacent cross-section are required to obtain the third derivative of twisting angle. To avoid this drawback, one can use empirical or approximate formulas describing torsional moment and warping strain. Paik et al
(2001) described the distribution of St. Venant stress and warping stress as a sine function along the ship length. However, Kim et al (2000) and Lee (1981) showed that the ship length could be the distance between fore perpendicular and the front of accommodation space for container ships.

Chapter 2 describes the strains-stresses/moments relations, briefly. The mathematical verification and application will be illustrated in chapter 3 and chapter 4, respectively.

2 The relations between strains and moments

Horizontal bending, vertical bending and torsional moments are the main terms to be considered in the hull stress monitoring systems with four long-base strain gages at the mid ship of a container ship. Figure 1 illustrates a simplified arrangement of gages at the mid ship for a container ship. The measured strain components are $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ counter-clock-wise and a stress at the $i$-th sensor can be written by the following components.

$$\sigma_i = E\varepsilon_i = E(\varepsilon_{iy} + \varepsilon_{iz} + \varepsilon_{iw} + \varepsilon_{iT})$$

Here, $\sigma$ and $E$ stand for stress and Young’s modulus, respectively, and $\varepsilon_{iy}, \varepsilon_{iz}, \varepsilon_{iw}, \varepsilon_{iT}$ are the strains produced by horizontal, vertical and torsional moments. Table 1 illustrates the detailed relations between strains and corresponding loads. Though the derivation of warping relations can be found in the book of Kim et al (2000), the simplified theory is summarized in the Appendix to avoid misunderstanding of the related terminology.

Besides, Paik et al (2001) described that the contribution of torsional warping moment is larger than that of pure torsional moment at the mid ship for a container ship. Henceforth, the warping component could be essential in hull stress monitoring system for container ships.

3 Decomposition of strains

The measured strain vector, $\varepsilon_m$, at the long-base strain gages can be expressed as follow

$$\varepsilon_m = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \end{bmatrix}^T$$
Table 1: Relations between strains and corresponding loads

<table>
<thead>
<tr>
<th>Load types</th>
<th>Strain relations</th>
<th>Strains-moments</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal moment</td>
<td>$\varepsilon_{1y} = -\varepsilon_{y}$, $\varepsilon_{2y} = \varepsilon_{y}$</td>
<td>$M_y = -E Z_{yy} \varepsilon_{iy}$, $i = 1$ or $2$</td>
<td>See Figure 2.</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{3y} = y B_{yT} \varepsilon_{y}$</td>
<td>$M_y = E Z_{yy} \varepsilon_{iy}$, $i = 2$ or $3$</td>
<td>$\sigma_{iy} = E \varepsilon_{iy}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{4y} = -y B_{yT} \varepsilon_{y}$</td>
<td>$Z_{yy} :$ cross-sectional moment at the $i$-th sensor</td>
<td>$i = 1, 2, 3, 4$</td>
</tr>
<tr>
<td>Vertical moment</td>
<td>$\varepsilon_{1z} = \varepsilon_{z}$, $\varepsilon_{2z} = \varepsilon_{z}$</td>
<td>$M_z = E Z_{zz} \varepsilon_{iz}$, $i = 1$ or $2$</td>
<td>See Figure 3.</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{3z} = -z B_{zT} \varepsilon_{z}$</td>
<td>$M_z = -E Z_{zz} \varepsilon_{iz}$, $i = 3$ or $4$</td>
<td>$\sigma_{iz} = E \varepsilon_{iz}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{4z} = -z B_{zT} \varepsilon_{z}$</td>
<td>$Z_{zz} :$ cross-sectional moment at the $i$-th sensor</td>
<td>$i = 1, 2, 3, 4$</td>
</tr>
<tr>
<td>Torsional moment</td>
<td>$\varepsilon_{1w} = \varepsilon_{w}$, $\varepsilon_{2w} = -\varepsilon_{w}$</td>
<td>$T_{iw} = K_{iw} \left</td>
<td>\varepsilon_{iw} \right</td>
</tr>
<tr>
<td>(warping)</td>
<td>$\varepsilon_{3w} = U_1 U_3 \varepsilon_{w}$</td>
<td>$K_{iw} = U_i (L_H / \pi) \tan(\pi x / L_H)$</td>
<td>$\sigma_{iw} = E \varepsilon_{iw}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{4w} = -U_1 U_3 \varepsilon_{w}$</td>
<td></td>
<td>$i = 1, 2, 3, 4$</td>
</tr>
<tr>
<td>Torsional moment</td>
<td>$\varepsilon_{1T} = \varepsilon_{2T} = \varepsilon_{T}$</td>
<td>$T_T = G J \gamma I$</td>
<td>See Figure 5.</td>
</tr>
<tr>
<td>(St. Venant)</td>
<td>$\varepsilon_{3T} = \varepsilon_{4T} = \varepsilon_{T}$</td>
<td>$\gamma = \sqrt{\frac{3GJ}{I}} \sqrt{\varepsilon_T}$</td>
<td>$\sigma_T = E \varepsilon_T$</td>
</tr>
</tbody>
</table>

Here, $\varepsilon_i$ can be estimated by the superposition of components written in Table 1. Therefore, the measured strains can be written as follow

$$\varepsilon_m = \tilde{A} \tilde{\varepsilon}$$  \hspace{1cm} (3)

$$\tilde{A} = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ y B_{yT} / y_T & -z B_{zT} & U_1 / U_3 & 1 \\ -y B_{yT} / y_T & -z B_{zT} & -U_1 / U_3 & 1 \end{bmatrix}$$  \hspace{1cm} (4)

$$\tilde{\varepsilon} = \begin{bmatrix} \varepsilon_y \\ \varepsilon_z \\ \varepsilon_w \\ \varepsilon_T \end{bmatrix}^T$$  \hspace{1cm} (5)

Here, $\tilde{\varepsilon}$ stands for the strain vector consisting of strain components at the 1st sensor. Hence, the decomposed strain component vector can be obtained by the following

$$\varepsilon = \tilde{A}^{-1} \varepsilon_m$$  \hspace{1cm} (6)

The existence of solution is proved by the determinant of matrix $\tilde{A}$.

$$|\tilde{A}| = 4 \left( U_1 / U_3 + y_B / y_T + \left( z_B / z_T \right) \left( U_1 / U_3 + \left( y_B / y_T \right) (z_B / z_T) \right) \right)$$  \hspace{1cm} (7)

This determinant is positive-definite since all variables in the equation present distances. Henceforth, the vector $\varepsilon$, can always be obtained.
4 Applications

4.1 Model ship and related properties

Figure 6 shows simplified cross-section of a model container ship (8100TEU) for the property estimation. In this cross-section, long-base strain gages are located at the nodes of 47, 46, 17 and 35.

![Cross-section of a container ship](image)

**Figure 6**: Simplified cross-section of a container ship for property estimation

**Table 2**: Geometrical parameters and permissible moments of a model ship

<table>
<thead>
<tr>
<th>Geometrical parameters</th>
<th>Value</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_B/z_T$</td>
<td>6.99m/12.58m</td>
<td></td>
</tr>
<tr>
<td>$y_B/y_T$</td>
<td>19.08m/19.46m</td>
<td></td>
</tr>
<tr>
<td>$J_w$</td>
<td>$1.0 \times 10^{-5} m^6$</td>
<td>Second moment of cross-section</td>
</tr>
<tr>
<td>Neutral axis</td>
<td>12.066m</td>
<td>$z$ direction</td>
</tr>
<tr>
<td>Shear center</td>
<td>-11.3m</td>
<td>$z$ direction</td>
</tr>
<tr>
<td>$U$ (at nodes 46 and 47)</td>
<td>160$m^2$</td>
<td>Warping function by FEM method (Fujitani 1990)</td>
</tr>
<tr>
<td>$U$ (at nodes 17 and 35)</td>
<td>168$m^2$</td>
<td></td>
</tr>
</tbody>
</table>

**Permissible moments**

<table>
<thead>
<tr>
<th>Still water bending</th>
<th>866,700 ton·f·m</th>
<th>Harbor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>595,000 ton·f·m</td>
<td>Seagoing (hogging)</td>
</tr>
<tr>
<td></td>
<td>-5,000 ton·f·m</td>
<td>Seagoing (sagging)</td>
</tr>
<tr>
<td>Torsional moment</td>
<td>56,460 ton·f·m</td>
<td>Harbor</td>
</tr>
<tr>
<td></td>
<td>18,030 ton·f·m</td>
<td>Seagoing</td>
</tr>
</tbody>
</table>

4.2 Calibration of hull stress monitoring system

The ABS regulation for system calibration requires the calm sea and hogging condition. Under these circumstances, the related strain levels are adjusted according to the calculated strain levels.
Figure 7: Results after calibration (Ship speed, RPM of engine, still water bending and torsional moments normalized by permissible moments at harbor condition, respectively)

Based on structural analysis. The calculated bending and torsional moments by incorporating the weight distribution of hull and ballast tanks, etc. are as follow.

Bending moment in calibration condition = 523,723 ton·m
(60.4% w.r.t. permissible still water bending moment in harbor condition)

Torsional moment in calibration condition = 0 ton·m
(0.0% w.r.t. permissible torsional moment in harbor condition)

Figure 7 illustrates the still water bending and torsional moments after calibration by adjusting longitudinal strain induced by the above conditions under zero engine RPM. The estimated vertical bending moment illustrates about 60% and torsional moment shows some fluctuations for about ±10% due to some disturbances by waves and difference between calculation and real ship’s conditions.

4.3 Behaviors during the sea trial

After the system calibration, sea trial was performed to investigate the system performance. Figure 8 illustrates the results and one can see about 90% of vertical bending moment with respect to the permissible bending moment under the seagoing conditions. This is quite reasonable result for the container ship’s sea trial conditions. For torsional moment, one can see about 25% of torsional moment with respect to the permissible torsional moment under the seagoing conditions. The variation level is larger than that in calibration conditions because of the interaction between ship’s speed and waves.
Figure 8: Results during sea trial (Ship speed, RPM of engine, still water bending and torsional moments normalized by permissible moments at the seagoing condition, respectively)

4.4 Behaviors during the navigating the Suez Canal

The proposed method was installed in a hull stress monitoring system for 8100TEU container ship that has navigated through Suez Canal. Figure 9 illustrates the route from the Mediterranean sea (at 2003-11-10) to the Indian ocean (2003-11-16). During this voyage, the ship entered the canal at about 1900min. and left at about 3750min. Within the canal, the ship kept about 10knots speed due to the regulation and also stopped to wait for suitable tide and to avoid other ships. Figure 10 shows these phenomena in speed and RPM plots. Though the still water bending moment

Figure 9: Navigating route through the Suez Canal (Route from Mediterranean sea (at 2003-11-10) to Indian ocean (at 2003-11-16))
represents some fluctuations due to the draft change at the entering stage, the bending moments are adjusted in normal conditions. For the torsional moment, however, the signal patterns illustrate quite similar behaviors to normal conditions because the canal cannot produce twisting force.

![Graphs of speed of ship, RPM of engine, still water bending moment, and torsional moment](image)

**Figure 10**: Navigating results over the Suez Canal (Ship speed, RPM of engine, still water bending and torsional moments normalized by permissible moments at the seagoing condition, respectively)

### 5 Concluding remarks

A strain decomposition method is suggested for hull stress monitoring system with four long-base strain gages installed at the midship of a container ship. The method is written as a linear matrix equation that the decomposed strain vector equals to the multiplication of the inverse of the transformation matrix and the measured strain vector. The existence of solution is verified by the derivation of positive-definite property of the determinant of the transformation matrix.

The application to a real container ship shows the reasonable results for bending moment estimation with small perturbation. For torsional moment, the estimated result illustrates about 10% fluctuation because of some disturbances by waves and the inaccurate ship’s properties in calibration state. However, one can see that the fluctuation level increases under sea trial conditions due to the interaction between ship’ speed and waves.

The real navigation through the Suez Canal shows that the still water bending moments are varied at the entering stage due to the draft condition of the Canal. However, the torsional moments are not affected by the draft because the draft change cannot make twisting force to the hull structure.

Based on these verifications, the proposed method has been proved to be useful for the decomposing strains to obtain the bending and torsional moments for container ship.
Appendix A Warping strain due to the torsional moment

Warping stress is represented in terms of warping function, $U$ (Kim et al 2000).

$$
\sigma_{w} = E \varepsilon_{w} = E U \frac{dw}{dx} \quad (A1)
$$

$$
\frac{dw}{dx} = \frac{d}{dx} \left( \frac{d\gamma}{dx} \right) \quad (A2)
$$

Here, $\gamma$ stands for the twisting angle of the cross-section at $x$ and the warping function has the following relations (Figure 4).

$$
U_1 = -U_2, \quad U_3 = -U_4 \quad (A3)
$$

In order to apply these relations to a container ship with symmetrical mid ship section, many additional members can be simplified. Lee (1981) introduced a simplified method to calculate the warping constant by simplifying the additional members as well as double hull structure. Fujitani (1990) proposed the more exact solution based on the finite element method than the results by Lee (1981)’s method. Hence, this paper uses Fujitani (1990)’s method to estimate warping function.

Meanwhile, the warping relations for a container ship with symmetrical cross-section can be written as

$$
\varepsilon_{1w} = \varepsilon_w \quad (A4)
$$

$$
\varepsilon_{2w} = -\varepsilon_w \quad (A5)
$$

$$
\varepsilon_{3w} = U_1 / U_3 \varepsilon_w \quad (A6)
$$

$$
\varepsilon_{4w} = -U_1 / U_3 \varepsilon_w \quad (A7)
$$

The torsional moment that makes the warping stress/strain is defined by the differentiation of bimoment derived by the integration of the warping stress and warping function with respect to the
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\[
T = -\frac{dB_i}{dx}, \quad Bi = \int_0^l \sigma_w U'(\xi) t d\xi = \Gamma \frac{dw}{dx} \quad (A8)
\]

\[
\Gamma = EJ_w, \quad J_w = \int_\xi U^2 t d\xi \quad (A9)
\]

\[
\therefore T = -\Gamma \frac{d^2 w}{dx^2} \quad (A10)
\]

Here, \( \Gamma \) stands for warping constant and \( J_w \) presents the second moment of cross-section at shear center.

This is, the torsional moment by warping requires the differentiation \( dw/dx \) of with respect to the length variable \( x \). Henceforth, this requires two adjacent long-base strain gages at adjacent cross-sections. This drawback can be resolved by using an empirical formula for torsional moment in terms of estimated warping strain. Paik et al (2001) showed that the warping moment is distributed in terms of sine function between fore and aft perpendicular. But, Kim et al (2000) and Lee (1981) described that the torsional moment is distributed between fore perpendicular and the front of accommodation for a container ship. In this study, the latter approach is applied and the related equation can be written as

\[
T_w = K_{iw} |\varepsilon_{iw}|, \quad i = 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \quad (A11)
\]

Here, \( K_{1w} = K_{2w} \) and \( K_{3w} = K_{4w} \) from the assumption of the geometrical symmetry, and the works of Kim et al (2000) can be written as follow.

\[
K_{iw} = \frac{\Gamma}{U_i (L_H/\pi) \tan(\pi x/L_H)} \quad (A12)
\]

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