

# GA-BASED PID AND FUZZY LOGIC CONTROL FOR ACTIVE VEHICLE SUSPENSION SYSTEM

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**ABSTRACT**—Since the nonlinearity and uncertainties which inherently exist in vehicle system need to be considered in active suspension control law design, this paper proposes a new control strategy for active vehicle suspension systems by using a combined control scheme, i.e., respectively using a genetic algorithm (GA) based self-tuning PID controller and a fuzzy logic controller in two loops. In the control scheme, the PID controller is used to minimize vehicle body vertical acceleration, the fuzzy logic controller is to minimize pitch acceleration and meanwhile to attenuate vehicle body vertical acceleration further by tuning weighting factors. In order to improve the adaptability to the changes of plant parameters, based on the defined objectives, a genetic algorithm is introduced to tune the parameters of PID controller, the scaling factors, the gain values and the membership functions of fuzzy logic controller on-line. Taking a four degree-of-freedom nonlinear vehicle model as example, the proposed control scheme is applied and the simulations are carried out in different road disturbance input conditions. Simulation results show that the present control scheme is very effective in reducing peak values of vehicle body accelerations, especially within the most sensitive frequency range of human response, and in attenuating the excessive dynamic tire load to enhance road holding performance. The stability and adaptability are also showed even when the system is subject to severe road conditions, such as a pothole, an obstacle or a step input. Compared with conventional passive suspensions and the active vehicle suspension systems by using, e.g., linear fuzzy logic control, the combined PID and fuzzy control without parameters self-tuning, the new proposed control system with GA-based self-learning ability can improve vehicle ride comfort performance significantly and offer better system robustness.

**KEY WORDS :** Genetic algorithm, Active vehicle suspension, PID, Fuzzy logic control

## 1. INTRODUCTION

Compared with conventional passive suspensions, active suspensions are very effective in improving vehicle ride comfort and handling stability. However, a key task for active suspension design is to determine a control law, which is capable of giving good system performance and better robustness. During the past two decades, various approaches to derive the control scheme have been proposed by many researchers (Abdelhaleem and Crolla, 2000; Tanaka and Sano, 1994; Elmadany and Abduljabbar, 1999; Li *et al.*, 1998; Chou, 1998; Yoshimura *et al.*, 1999; Ghazi *et al.*, 1997; D'Amato and Viassolo, 2000; Yu and Crolla, 1998; Li *et al.*, 2002; Arslan and Kaya, 2001; Li and Shieh, 2000; Kandel and Langholz, 1993; Yu *et al.*, 2001).

The optimal control and robust control have been introduced for active suspension application and some good performances been achieved on the assumption of a

linear vehicle model (Abdelhaleem and Crolla, 2000; Tanaka and Sano, 1994; Elmadany and Abduljabbar, 1999). In fact, vehicle is a complicated, nonlinear system with uncertainties of itself. And also operating conditions are changeable, e.g., the changes of road irregular excitation inputs with the variation of road surface roughness and of vehicle speed. So the control approaches for active suspensions based on the linear assumption of vehicle model have difficulties in practical application for good performance and robustness. However, for some practical systems including nonlinear elements, which cannot be expressed accurately in mathematics, the fuzzy logic control, has been proved to be one of the most efficient and systematic approaches to deal with such kinds of problems in that its control capability arises from emulating human logic instead of accurate mathematical model. Some researchers have successfully applied fuzzy logic control to active suspension control law design and satisfactory performances have been achieved (Li *et al.*, 1998; Chou, 1998; Yoshimura *et al.*, 1999; Ghazi *et al.*, 1997; Yu *et al.*,

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2001). Li, Q. employed the fuzzy reasoning in a large-sized bus active suspension design and significantly reducing the vertical and pitch acceleration of vehicle body (Li *et al.*, 1998). Chou examined the effectiveness of grey-fuzzy control for active suspension by using a quarter car model (Chou, 1998). Yoshimura used a combined controller, i.e., a linear and fuzzy logic controller in active suspension system and good performances have been obtained (Yoshimura *et al.*, 1999). But in Yoshimuras controller, the linear feedback, in essence, is a single proportional control and the overshoot of response might be resulted by system inputs in some severe disturbance cases. Since the feedback of the integral and differential of the errors between system actual output and reference output can be useful in reducing the steady-state error and overshoot of system, these information should be utilized in order to ensure the control precision of system. Therefore, a properly designed PID controller can be employed instead of only using linear feedback control. This paper combines the advantages of both PID and fuzzy control to derive a control law for an active vehicle suspension.

In fuzzy controller design two important problems have to be solved firstly, one is that the fuzzy control rules are experience-oriented and the other is that some suitable parameters, such as membership functions, scaling factors and gain values, need to be determined in different cases by time-consuming trial-and-error procedure, especially in determining the membership functions. This implies that the control rules and membership functions are human dependent and may not be selected properly. In the previous papers (Li *et al.*, 1998; Chou, 1998; Yoshimura *et al.*, 1999; Ghazi *et al.*, 1997; Yu *et al.*, 2001), the determination of the membership functions is not presented in detail. In addition, the fuzzy logic controller may need to be adaptable when the system parameters change significantly. Obviously, it is desired that the system parameters can be selected more reasonably instead of only depending on designers subjective experience. Similar problems could occur in deriving the three key parameters for a PID controller. In order to overcome the problems, a self-learning algorithm can be introduced to the PID and fuzzy controller design for a general case. The genetic algorithm, which has received increasing interests owing to its advantages over conventional optimization, can be a powerful technique to self-tune the parameters for the hybrid active suspension controller.

Therefore, in this paper, a new control strategy to minimize vehicle body vertical and pitch accelerations for passenger comfort is proposed by using a combined control scheme, i.e., respectively using GA-based a PID controller and a fuzzy logic controller in two loops. One loop, using PID control, is to minimize vehicle body

vertical acceleration; and the other one, fuzzy logic controller, is to minimize pitch acceleration and meanwhile to attenuate vehicle body vertical acceleration further by tuning weighting factors. The combined controller parameters are tuned by a GA-based optimization technique to minimize the defined performance index function on-line. Although previous knowledge of membership functions and parameters of PID controller are not available, the optimal PID and fuzzy controller based on defined objectives still can be obtained. By using a four degree-of-freedom nonlinear vehicle model, the proposed control scheme is applied and simulations are carried out in different road disturbance input conditions. Simulation results show that the proposed control strategy is very effective in reducing peak values of vehicle body accelerations, especially within the most sensitive frequency range of human response and also with good stability even if the system is subject to a discrete event input, i.e., a sudden change of road conditions, such as a pothole, an obstacle or a step input. Compared and evaluated with conventional passive suspensions, the active vehicle suspensions respectively by using a linear and fuzzy logic controls, a normal PID and fuzzy control without parameters self-tuning, the new designed control scheme based on GA can improve vehicle ride comfort performance significantly and offer higher adaptability to severe changes of road conditions. Furthermore, the great reduction of dynamic tire load enhances the road handling performance.

Following issues are respectively presented in this paper, a) a half vehicle model is established considering the nonlinearity of suspension and tire. b) Two control schemes are proposed. And the design procedure for the normal PID and fuzzy controller and the GA-based PID and fuzzy controller are described. c) The proposed

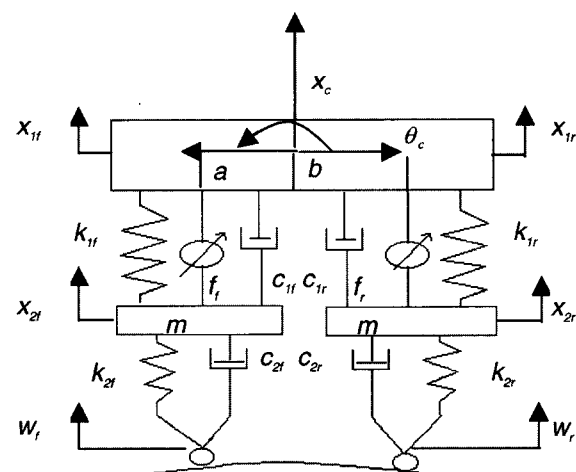


Figure 1. Half vehicle model.

control strategies are used effectively and simulation results are carried out to investigate performances improvements and robustness by comparing the results.

## 2. VEHICLE MODEL

A half vehicle model shown in Figure 1, is taken as example in the study. In the model, the nonlinearity of spring and damper is considered based on experimental data for a typical, practical car suspension. The motion equations for vehicle body, front and rear wheels are respectively given by,

$$m_c \ddot{x}_c + c_{2f}(\dot{x}_{2f} - \dot{w}_f) + k_{2f}(x_{2f} - w_f) + c_{1r}(\dot{x}_{1r} - \dot{x}_{2r}) + k_{1r}(x_{1r} - x_{2r}) - f_f - f_r = 0 \quad (1)$$

$$I_c \ddot{\theta}_c + c_{1r}(\dot{x}_{1r} - \dot{x}_{2r}) + k_{1r}(x_{1r} - x_{2r}) - c_{1f}(\dot{x}_{1f} - \dot{x}_{2f}) - k_{1f}(x_{1f} - x_{2f}) - f_r + f_f = 0 \quad (2)$$

$$m_f \ddot{x}_{2f} + c_{2f}(\dot{x}_{2f} - \dot{w}_f) + k_{2f}(x_{2f} - w_f) - c_{1f}(\dot{x}_{1f} - \dot{x}_{2f}) - k_{1f}(x_{1f} - x_{2f}) + f_f = 0 \quad (3)$$

$$m_r \ddot{x}_{2r} + c_{2r}(\dot{x}_{2r} - \dot{w}_r) + k_{2r}(x_{2r} - w_r) - c_{1r}(\dot{x}_{1r} - \dot{x}_{2r}) - k_{1r}(x_{1r} - x_{2r}) + f_r = 0 \quad (4)$$

in which,

$$x_c = \frac{x_{1f}b + x_{1r}a}{l}, \quad \theta_c = \frac{x_{1r} - x_{1f}}{l}, \quad l = a + b$$

where  $m_c$ ,  $m_f$ ,  $m_r$  are masses respectively for vehicle body, front wheel, and rear wheel,

$x_c$ ,  $x_{2f}$ ,  $x_{2r}$  are vertical displacements respectively for vehicle body C.G., front wheel, and rear wheel,  $x_1$  is vertical displacement of vehicle body,  $l$  is wheelbase,

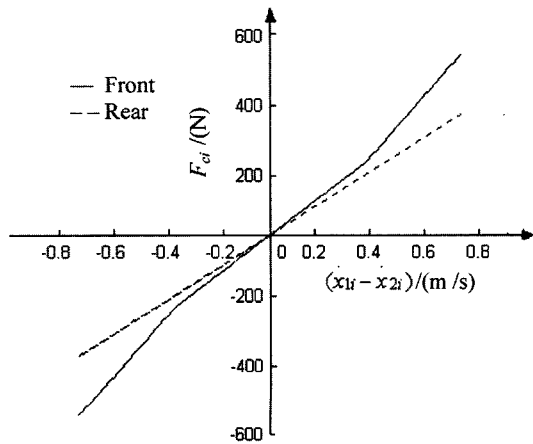


Figure 2. Damping forces of the front and rear suspensions.

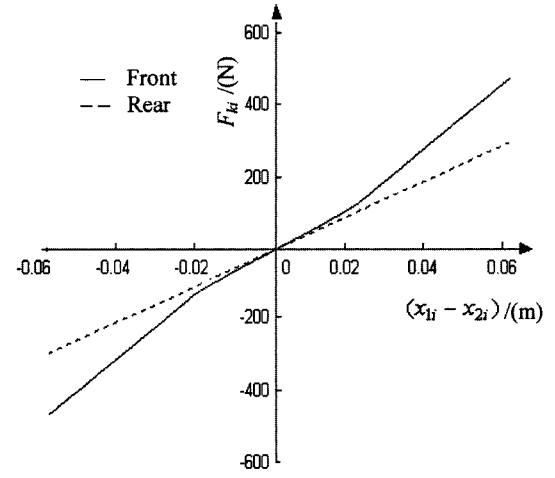


Figure 3. Spring forces of the front and rear suspensions.

$a$ ,  $b$  are distances respectively from front axle and rear axle to vehicle body C.G.,

$c_1$  is damping coefficient,  $k_1$  is spring stiffness,  $\theta_c$  is vehicle body pitch angle,  $f$  is suspension control force,  $w$  is road input displacement,  $f$ ,  $r$  are subscripts, respectively represents front and rear.

In simulations, the nonlinear damping coefficients  $c_{1f}$ ,  $c_{1r}$  and nonlinear spring stiffness  $k_{1f}$ ,  $k_{1r}$  are chosen based on practical experiment data. The relationships between damping forces  $F_{ci}$  and relative velocity between body and wheels, i.e.,  $\dot{x}_{1i} - \dot{x}_{2i}$  ( $i = f, r$ ), are shown in Figure 2. And the relationships between suspension spring restoring forces  $F_{ki}$  and relative displacement between body and wheels, i.e.,  $x_{1i} - x_{2i}$  ( $i = f, r$ ), are shown in Figure 3.

The road excitation input  $w_i$ , in form of a filtered white noise process, is used in the paper, with spectral density as,

$$S_w(\omega) = (\sigma^2/\pi) \alpha v / (\omega^2 + \alpha^2 v^2) \quad (5)$$

where  $S_w$  – power spectral density,  $\omega$  – wave number,  $\sigma^2$  – variance of the road input,  $v$  – vehicle speed,  $\alpha$  – positive constant.

Hence, the road input,  $w_i$ , can be given by,

$$\dot{w}_i + \alpha v w_i = \xi_i \quad (i = f, r) \quad (6)$$

in which  $\xi_i$  is a white noise process with zero mean value, whose covariance function can be described by,

$$\begin{aligned} E[\xi_i(t) \xi_j(t - \tau)] &= 2\pi A \delta(\tau) \quad (\text{when } i = j) \\ &= 2\pi A \delta(\tau - t_i) \quad (\text{when } i \neq j) \end{aligned} \quad (7)$$

in which  $\tau$  is time delay of rear wheel road input, equal to wheelbase/vehicle speed,  $A$  denotes the intensity of road input and  $\delta(\cdot)$  is the Dirac's delta function.

Combing the road input model, i.e., Equation (6) and vehicle model, i.e., Equation (1)–(4), the simulation model for active suspension system can be rewritten in a vector matrix form as below,

$$M\ddot{\mathbf{x}} + \mathbf{C}(t)\dot{\mathbf{x}} + \mathbf{K}(t)\mathbf{x} = \mathbf{D}\mathbf{f} + \mathbf{E}\mathbf{w} \quad (8)$$

where the state vector, active control and road excitation are respectively given by,

$$\mathbf{x} = [x_{1f} \ x_{2f} \ x_{1r} \ x_{2r}]^T, \quad \mathbf{f} = [f_f \ f_r]^T, \quad \mathbf{w} = [w_f \ \dot{w}_f \ w_r \ \dot{w}_r]^T$$

and

$$\mathbf{M} = \begin{bmatrix} m_c b/l & 0 & m_c a/l & 0 \\ I_c/l & 0 & -I_c/l & 0 \\ 0 & m_{2f} & 0 & 0 \\ 0 & 0 & 0 & m_{2r} \end{bmatrix} \quad \mathbf{C}(t) = \begin{bmatrix} c_{1f} & -c_{cf} & c_{1r} & -c_{1r} \\ ac_{1f} & -ac_{1f} & bc_{1r} & -bc_{1r} \\ -c_{1f}c_{1f} + c_{2f} & 0 & 0 & 0 \\ 0 & 0 & -c_{1r}c_{1r} + c_{2r} & 0 \end{bmatrix}$$

$$\mathbf{K}(t) = \begin{bmatrix} k_{1f} & -k_{1f} & k_{1r} & -k_{1r} \\ ak_{1f} & -ak_{1f} & bk_{1r} & -bk_{1r} \\ -k_{1f}k_{1f} + k_{2f} & 0 & 0 & 0 \\ 0 & 0 & -k_{1r}k_{1r} + k_{2r} & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ a & b \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_{2f} & c_{2f} & 0 & 0 \\ 0 & 0 & k_{2r} & c_{2r} \end{bmatrix}$$

### 3. GA-BASED PID AND FUZZY CONTROLLER DESIGN FOR ACTIVE VEHICLE SUSPENSION

#### 3.1. Normal PID and Fuzzy Controller Design

In suspension design, body acceleration and body attitude are obviously important for ride comfort and stability performances. In present paper, assuming a straight-running condition, the control strategy is to minimize vertical acceleration and pitch movement of vehicle body. Because of the nonlinearity and uncertainty in vehicle system and also of the difficulties in modeling, the paper proposed a control scheme, expressed as a sum of PID controller and fuzzy controller as below (Yu *et al.*, 2001),

$$f_i = u_{PIDi} + u_{Fi} \quad (i = f, r) \quad (9)$$

where  $u_{PIDi}$  is PID controller output for the reduction of vertical body acceleration and  $u_{Fi}$  is fuzzy controller output mainly accounting for minimizing pitch acceleration, but also along with the reduction of vertical body acceleration to some extent. The PID control is calculated

as,

$$u_{PIDi} = K_{Pi}(\ddot{X}_c - \ddot{x}_c) + K_{Ii} \int (\ddot{X}_c - \ddot{x}_c) dt + K_{Di} \frac{d}{dt}(\ddot{X}_c - \ddot{x}_c) \quad (i = f, r) \quad (10)$$

in which  $\ddot{X}_c$  is the output value of reference acceleration, which is taken as zero

$K_{Pi}$ ,  $K_{Ii}$ , and  $K_{Di}$  respectively denote proportional, integral and differential coefficients

The main function of the first term in Equation (10), i.e., the proportional part of the PID controller, is to speed up system response, implying to follow the reference body acceleration value rapidly. While the tasks of the second term, i.e., integral part, and third term, i.e., differential part, is to reduce system steady-state error and overshoot respectively. Fuzzy control output  $u_{Fi}$  is mainly for the reduction of vehicle body pitch and it can be obtained by fuzzy control algorithm described below,

$$u_{Fi} = G_{Fi} \cdot \gamma_i \quad (i = f, r) \quad (11)$$

in which  $\gamma_i$  are fuzzy controller output variable of front or rear suspension and  $G_{Fi}$  is gain value

For the derivation of fuzzy logic control output  $\gamma_i$ , the input variables  $\beta_{1i}$  and  $\beta_{2i}$  are respectively assumed as,

$$\beta_{1i} = (x_{1i} + \kappa_{1i}\theta_{ci})/\eta_{1i} \quad (i = f, r) \quad \beta_{2i} = (\dot{x}_{1i} + \kappa_{2i}\dot{\theta}_{ci})/\eta_{2i} \quad (i = f, r)$$

in which  $\kappa_{1i}, \kappa_{2i}$  are weighting factors and  $\eta_{1i}, \eta_{2i}$  are scaling factors

From the above equations, it can be seen that the fuzzy controller input variable also includes the terms of body displacement and body velocity. Correspondingly, by tuning the weighting factors, the vertical and pitch movements of vehicle body can be further controlled. By tuning the scaling factors, the input values of fuzzy controller are limited in the range of  $[-1, 1]$  in the implementation of the simulation algorithm (Yoshimura *et al.*, 1999).

Since the front and rear active suspension controllers are designed in same procedure, only one (front or rear suspension) fuzzy controller design is described. Therefore, the fuzzy controller input variables can be simply described as

$$\beta_1 = (x_1 + \kappa_1\theta_c)/\eta_1, \quad \beta_2 = (\dot{x}_1 + \kappa_2\dot{\theta}_c)/\eta_2$$

For a typical fuzzy controller with two inputs and one output, the control rules  $R^j$  can be expressed as,

$R^j$ : if  $\beta_1$  is  $A_1$  and  $\beta_2$  is  $B_1$ , then  $\gamma$  is  $C_j$  ( $j = 1, 2, \dots, n$ ) where  $\beta_1$  and  $\beta_2$  are input variables,  $\gamma$  is output variable,  $A_j, B_j, C_j$  are corresponding input and output fuzzy sets, whose membership functions are determined by  $a_1, a_2$  and  $a_{e1}, a_{e2}$ , respectively, and now presumed to be

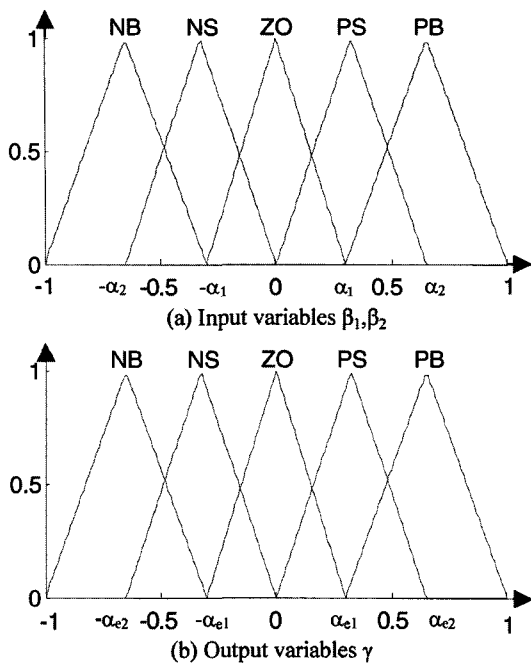


Figure 4. Incipient membership functions for the input and output variables.

identical and shown in Figure 4. The triangle membership function has been widely used in fuzzy controller design (Li *et al.*, 1998; Chou *et al.*, 1998; Yoshimura *et al.*, 1997; Yu *et al.*, 2001).

The design procedure of the fuzzy logic controller includes three main parts, i.e., fuzzification, fuzzy reasoning, defuzzification. Each part is respectively presented in detail as following.

(1) Fuzzification

The fuzzification process of input variable can be expressed by following equations,

$$\lambda_{1j} = \mu_{A_j}(\beta_1^0), \lambda_{2j} = \mu_{B_j}(\beta_2^0)$$

where  $\mu_{A_j}(\cdot), \mu_{B_j}(\cdot)$  respectively represent the membership functions of  $A_j$  and  $B_j$

$\lambda_{1j}, \lambda_{2j}$  are degrees of fitness of input value  $\beta_1^0, \beta_2^0$ , i.e., measurements of input variable  $\beta_1$  and  $\beta_2$

(2) Fuzzy reasoning

In present study, *Mandain* method is used in fuzzy reasoning process, given as,

$$\lambda_j = \lambda_{1j} \wedge \lambda_{2j}$$

where  $\lambda_j$  denotes the degree of fitness of fuzzy reasoning results, and symbol  $\wedge$  conducts minimum operation. In each control rule  $R^i$ , the membership function for output variable  $\gamma$  can be expressed as  $\lambda_j \hat{\mu}_{C_j}(\gamma)$ , in which  $\mu_{C_j}(\cdot)$  denotes the membership function of  $C_j$ .

(3) Defuzzification

For the improvement of reasoning and computation speed, the product-sum gravity method (Kandel and Langholz, 1993) is used to obtain the defuzzified value of output variable  $\gamma$ . The product-sum-gravity method gives the defuzzified value  $\gamma^0$  is,

$$\gamma^0 = \frac{\sum \gamma_j' \lambda_j S_{C_j}}{\sum \lambda_j S_{C_j}}$$

where

$S_{C_j}$  is area of  $\mu_{C_j}(\gamma)$ ,  $\gamma_j'$  is distance from center of gravity of  $S_{C_j}$  to the base point of zero

The empirical knowledge is used to construct the fuzzy control rules for suspension control system. And the fuzzy rules are described by language values as below (Chou *et al.*, 1998; Yoshimura *et al.*, 1997),

- (a) If  $\beta_1$  and  $\beta_2$  are both positive (negative) big, then  $\gamma$  negative (positive) big;
- (b) If the absolute value of  $\beta_2$  and  $\beta_1$  is both small, then the absolute value of  $\gamma$  is relatively small;
- (c) If  $\beta_1$  is positive big (negative big) and  $\beta_2$  is negative big (positive big), then the absolute value of  $\gamma$  is small;
- (d) If  $\beta_1$  is positive big (negative big), then  $\gamma$  is negative (positive) medium;
- (e) If the absolute of value  $\beta_1$  is small, then  $\gamma$  is small.

To summarize, the above fuzzy rules expressed by language values are shown in Table 1.

3.2. GA-based PID and Fuzzy Controller Design

Theoretically, the normal PID and fuzzy controller for active vehicle suspension design can obtain optimal results, if the controller parameters are selected properly, as described above. However, in the cases of variable conditions, e.g., in the changes of road condition or system parameters, how to adapt the parameters  $K_{P_i}, K_{I_i}$  and  $K_{D_i}$  of PID controller, and the scaling factors  $\eta_{1i}, \eta_{2i}$ , gain values  $G_{F_i}$  and membership function (determined by  $a_1, a_2, a_{e1}, a_{e2}$ ) of fuzzy controller is the most important to achieve optimal vehicle performances.

A GA can be an appropriate solution with respect to its powerful and global search technique based on the operation of natural genetics and the Darwinian ‘survival

Table 1. Fuzzy rules.

g	${}_1\beta$					
	NB	NS	ZO	PS	PB	
${}_2\beta$	NB	PB	PB	PS	PS	ZO
	NS	PB	PS	PS	ZO	NS
	ZO	PS	ZO	ZO	NS	NS
	PS	PS	ZO	NS	NS	NB
	PB	ZO	NS	NS	NB	NB

of the fittest' theory with a randomly structured information exchange (Arslan and Kaya, 2001; Li and Shieh, 2000). Given an optimization problem, GA encodes the parameters concerned into finite bite binary strings, called a chromosome. And this is the first and important part of a GA process. A chromosome population subsequently forms, each represent a possible solution to the optimization problem. Evaluating the fitness of each chromosome according to the performance index is an important link between the GA and the practical system. Three basic operations, i.e., 'reproduction', 'crossover', and 'mutation', similar to genetic evolution, are then performed. 'Reproduction' is a process by which the strings with higher probabilities will breed large number of their copies in the new generation. The 'crossover' involves exchanging corresponding portions of binary strings at a random selected portioning position of two chromosomes, which are chosen from the parent strings. This process can combine better qualities among the preferred good strings and extend the genetic search space. 'Mutation' is a process by which the chance for the GA to reach the optimal point is reinforced through just an occasional alteration of a value at a randomly selected bit position, such as flipping the state of bit form 1 to 0, or vice versa. In the following, each of the strings is decoded to be its decimal values of corresponding actual parameters and sent to objective function. At last, the string with the largest value of performance index is found and decoded to obtain the parameters to be optimized for controller.

Because of the features mentioned above, the GA is

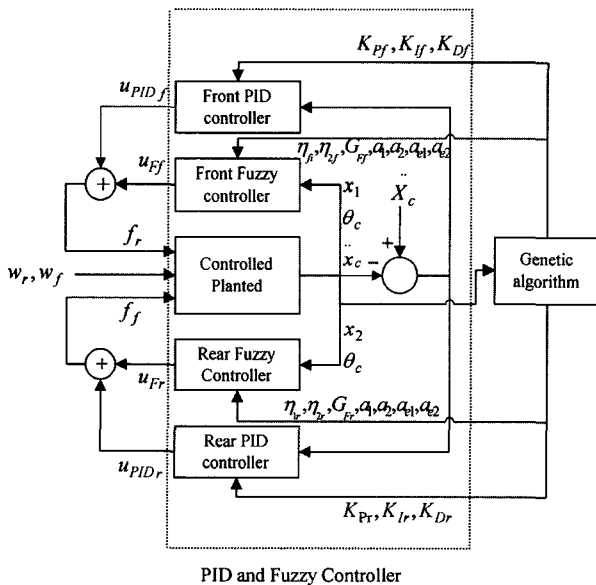


Figure 5. Block diagram of GA-based PID and fuzzy controller.

introduced to search the optimal values of  $K_{Pi}, K_{Ii}, K_{Di}, \eta_{1i}, \eta_{2i}, G_{Fi}$  and  $a_1, a_2, a_{e1}, a_{e2}$ . The proposed controller is designed based on the block diagram, shown in Figure 5. According to the optimizing process of GA, the design procedure for the PID and fuzzy controller for active vehicle suspension with the self-tuning parameters on-line is described. To obtain a suitable compromise between resolution accuracy and computational speed, the parameters  $K_{Pi}, K_{Ii}, K_{Di}, \eta_{1i}, \eta_{2i}, G_{Fi}$  and  $a_1, a_2, a_{e1}, a_{e2}$  which determine the shape of its corresponding membership functions of fuzzy controller, are encoded into a binary string of fixed length as following.

$$\underbrace{\overbrace{S_1}^{K_{Pf}}, \overbrace{S_2}^{K_{If}}, \overbrace{S_3}^{K_{Df}}, \overbrace{S_4}^{\eta_{1f}}, \overbrace{S_5}^{\eta_{2f}}, \overbrace{S_6}^{G_{Ff}}, \overbrace{S_7}^{K_{Pr}}, \overbrace{S_8}^{K_{Ir}}, \dots, \overbrace{S_9 \dots S_{11}}^{G_{Fr}}, \overbrace{S_{12}}^{a_1}, \overbrace{S_{13}}^{a_2}, \overbrace{S_{14}}^{a_{e1}}, \overbrace{S_{15}}^{a_{e2}}}_{string}}$$

Without losing generality and assuming that there are  $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9$  and  $N_{10}$  bits for each value of  $K_{Pi}, K_{Ii}, K_{Di}, \eta_{1i}, \eta_{2i}, G_{Fi}, a_1, a_2, a_{e1}$  and  $a_{e2}$  and respectively, so the whole length of the *string*, i.e., the chromosome in GA optimized solution process, have  $2N_1+2N_2+2N_3+2N_4+2N_5+2N_6+N_7+N_8+N_9+N_{10}$  bits For example, if

$$\begin{aligned} S_1, S_7 &= \underbrace{0 \dots 0100100}_{N_1 \text{ bits}} \\ S_2, S_8 &= \underbrace{0 \dots 0100000}_{N_1 \text{ bits}} \\ \dots & \dots \\ S_{15} &= \underbrace{0 \dots 0100101}_{N_9 \text{ bits}} \\ S_{16} &= \underbrace{1 \dots 0100100}_{N_{10} \text{ bits}} \end{aligned}$$

and then the corresponding decimal values for  $K_{Pi}, K_{Ii}, K_{Di}, \eta_{1i}, \eta_{2i}, G_{Fi}, a_1, a_2, a_{e1}$  and  $a_{e2}$  are given as

$$\begin{aligned} K_{Pi} &= X_{K_{Pi} \min} + \frac{d_{K_{Pi}}}{2^{N_1} - 1} (X_{K_{Pi} \max} - X_{K_{Pi} \min}) \\ K_{Ii} &= X_{K_{Ii} \min} + \frac{d_{K_{Ii}}}{2^{N_2} - 1} (X_{K_{Ii} \max} - X_{K_{Ii} \min}) \\ \dots & \dots \\ a_{e1} &= X_{a_{e1} \min} + \frac{d_{a_{e1}}}{2^{N_9} - 1} (X_{a_{e1} \max} - X_{a_{e1} \min}) \\ a_{e2} &= X_{a_{e2} \min} + \frac{d_{a_{e2}}}{2^{N_{10}} - 1} (X_{a_{e2} \max} - X_{a_{e2} \min}) \end{aligned}$$

where  $[X_{K_{Pi} \min}, X_{K_{Pi} \max}], [X_{K_{Ii} \min}, X_{K_{Ii} \max}], \dots, [X_{a_{e1} \min}, X_{a_{e1} \max}]$  and  $[X_{a_{e2} \min}, X_{a_{e2} \max}]$  respectively are the limit range for  $K_{Pi}, K_{Ii}, \dots, a_{e1}$  and  $a_{e2}$ ; and the  $d_{K_{Pi}}, d_{K_{Ii}}, \dots, d_{a_{e1}}$  and  $d_{a_{e2}}$  respectively represent the binary value for  $K_{Pi}, K_{Ii}, \dots, a_{e1}$  and  $a_{e2}$ .

In order to minimize both the vertical and pitch acceleration of vehicle body to achieve ride comfort, the present paper deploys performance index of active suspension by the following equation:

$$\max PI = \frac{1}{1 + W_1 \cdot \text{rms}(\ddot{x}_c) + W_2 \cdot \text{rms}(\ddot{\theta}_c)}$$

where  $\text{rms}(\ddot{x}_c)$  is root of mean square value (i.e., RMS value) of body C.G. vertical acceleration,  $\text{rms}(\ddot{\theta}_c)$  is root of mean square value of body C.G. pitch acceleration,  $W_1$  and  $W_2$  are the weighting factors respectively for  $\ddot{x}_c$  and  $\ddot{\theta}_c$ .

Besides encoding the parameters and derivation of performance index, several important genetic parameters in the GA-based searching procedure, e.g. the generation number, population size, the crossover rate and the mutation rate, must be chosen. In the present study, the selection for these parameters based on previous experience are described in next section.

#### 4. SIMULATION RESULTS AND ANALYSIS

Except for suspension spring stiffness and damping coefficient, which have been given in Figures 2 and 3, the

Table 2. Parameters used in simulations.

Notation		Value	Unit
$m_c$	Body mass	480	Kg
$I_c$	Pitch moment	620	Kg-m <sup>2</sup>
$K_{2f}$	Front tire stiffness	160	KN/m
$K_{2r}$	Rear tire stiffness	160	KN/m
$c_{2f}$	Front tire damping coefficient	2	KN/(m/s)
$c_{2r}$	Rear tire damping coefficient	2	KN/(m/s)
$m_{2f}$	Front tire mass	25	Kg
$m_{2r}$	Rear tire mass	25	Kg
$a$	Distance from front axle to body C.G.	0.871	m
$b$	Distance from rear Axle to body C.G.	1.469	m
$v$	Vehicle velocity	30	m/s
$\alpha$	Positive constant	0.04	1/m
$A$	Intensity of road	$10^{-6}$	m
$W_1$	Weighting factor	20	
$W_2$	Weighting factor	1	
$K_{1i}$	Weighting factors	14.2	i=f
		-16.2	i=r
$K_{2i}$	Weighting factors	10.8	i=f
		-11.0	i=r

Table 3. Parameters optimized by GA.

Parameters	Code as (bits)	Initial value	Searched value
$K_{Pf}$	14	390	263.75
$K_{If}$	10	10	8.76
$K_{Df}$	10	10	5.89
$\eta_{1f}$	8	0.02	530.00
$\eta_{2f}$	8	0.03	0.0200
$G_{Ff}$	14	190	148.77
$K_{Pr}$	14	350	277.02
$K_{Ir}$	10	15	10.60
$K_{Dr}$	10	15	8.91
$\eta_{1r}$	10	0.02	0.0061
$\eta_{2r}$	10	0.03	0.0091
$G_{Fr}$	14	170	163.28
$a_1$	8	0.30	0.1087
$a_2$	8	0.65	0.4508
$a_{e1}$	8	0.30	0.0495
$a_{e2}$	8	0.65	0.3098

plant parameters chosen in the simulations are presented in Table 2 and the selected controller parameters are presented in Table 3.

The parameters for the GA-based searching procedure are selected as follows:

- (1) Initial population size: 10
- (2) Max number of generation: 250
- (3) Crossover rate: 0.2
- (4) Mutation rate: 0.8

In the following numerical simulations, the GA-based PID and fuzzy logic controller is derived from 250 simulation runs having 10 populations by applying the genetic algorithm as shown in Figure 5. The optimizing procedure, i.e., the value of PI increasing (best and means

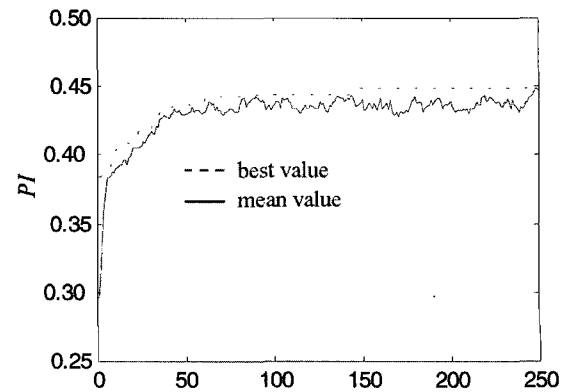


Figure 6. Fitness results for a GA optimizing process.

Table 4. RMS values of simulation results.

Performances		Case A	Case B	Case C	Case D	Units
$\ddot{x}_c$	Vertical acceleration	0.0423	0.0663	0.1054	0.241	m/s <sup>2</sup>
$\ddot{\theta}_c$	Pitch acceleration	0.0028	0.0046	0.0074	0.0187	rad/s <sup>2</sup>
$K_{2f}(x_{2f} - w_f)$	Front dynamic tire load	142	928	944	240	N
$K_{2r}(x_{2r} - w_r)$	Rear dynamic tire load	139	848	864	240	N
$x_{1f} - x_{2f}$	Front suspension working space	0.0063	0.0074	0.0062	0.0041	m
$x_{1r} - x_{2r}$	Rear suspension working space	0.0063	0.0078	0.0053	0.0040	m

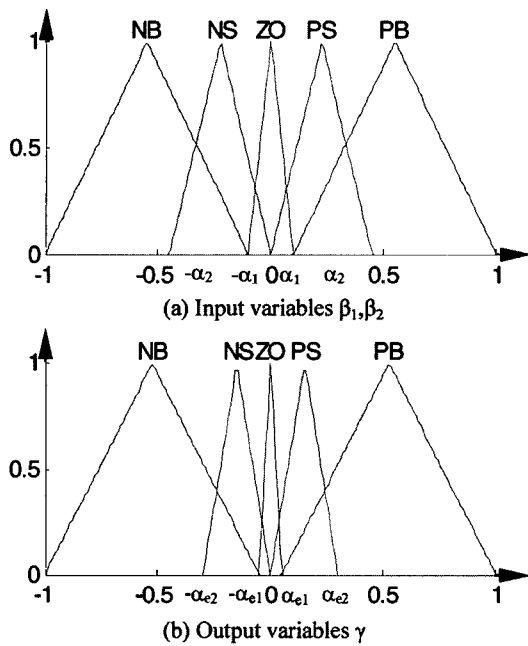


Figure 7. GA-based optimized membership functions for the input and output variables.

of the fitness for each generation) vs the number of generations, is illustrated in Figure 6 and the controller parameters optimized by GA are given in Table 3. Figure 7 presents the optimized membership functions of fuzzy controller.

In order to examine the effectiveness of the proposed control scheme, simulations are carried out in many different cases, e.g. in different road input conditions and with different control schemes. In the present paper, the results of the three control schemes, i.e., the linear and fuzzy control scheme, normal PID and fuzzy control scheme, are compared with the results of a conventional passive suspension system. The four different cases are respectively denoted as,

Case A: Active suspension by using the new proposed control scheme;

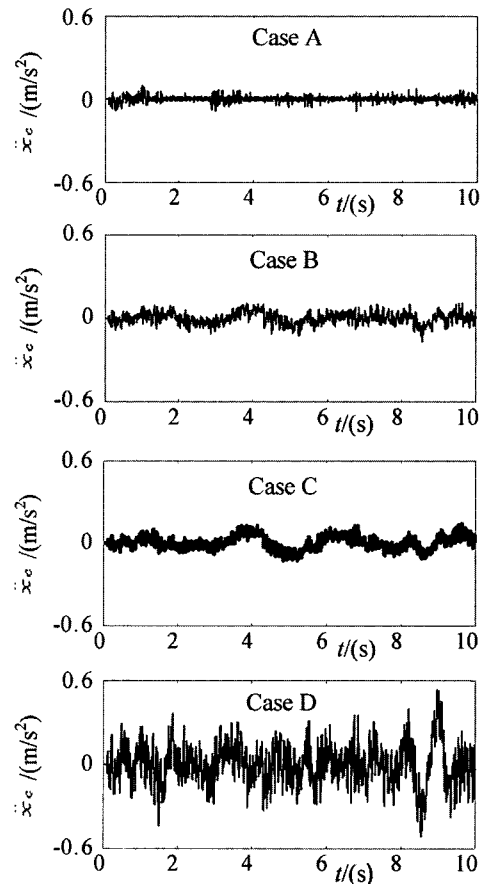


Figure 8. Vertical acceleration of vehicle body C.G. in different cases.

Case B: Active suspension by using normal PID and fuzzy control scheme;

Case C: Active suspension by using the linear and fuzzy logic control scheme;

Case D: Conventional passive suspension.

Simulation results in time domain, including vehicle body vertical acceleration and pitch acceleration, are compared for the four cases, respectively presented in Figure 8 and Figure 9. Compared with the results of the



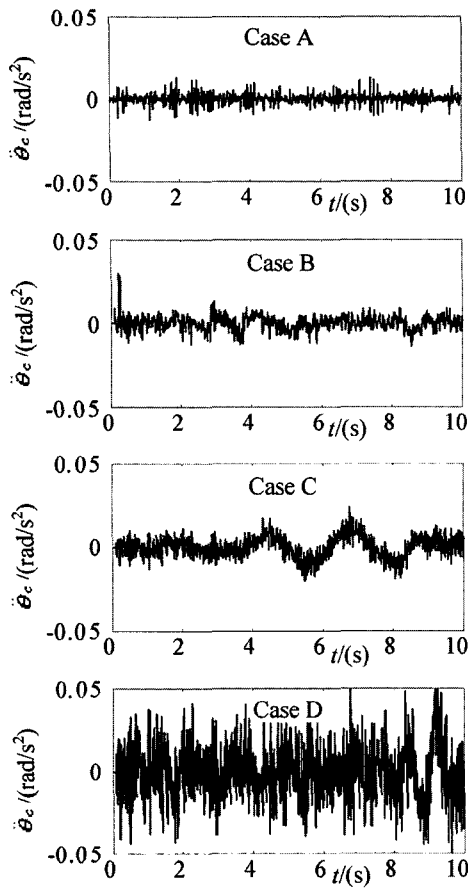


Figure 9. Pitch acceleration of vehicle body in different cases.

passive suspension system (in Figures 8(d) and 9(d)), the significant improvements can be obtained by proposed active suspension system (in Figures 8(a) and 9(a)). The frequency domain results in Figure 10 and Figure 11 show the satisfactory performances more clearly. In the most sensitive frequency range of human response, the power spectral densities both for vertical and pitch accelerations of Case A, B and C is reduced compared with results of Case D. Particularly the Case A shows more satisfactory performance. Whether in time or in frequency domain, it can be found that the new proposed scheme offers the best performance among the three control schemes.

The vehicle overall performances for suspension design are also compared in Table 4, including the RMS values of acceleration ( $\ddot{x}_c$ ), pitch acceleration ( $\ddot{\theta}_c$ ) of vertical body C.G., front and rear dynamic tire loads ( $K_{2i}(x_{2i}-w_i)$ ), front and rear suspension working space ( $x_{1i}-x_{2i}$ ). It can be seen that, compared with the passive system, the active ones reduce vertical and pitch body accelerations distinctly, and in particular, Case A shows

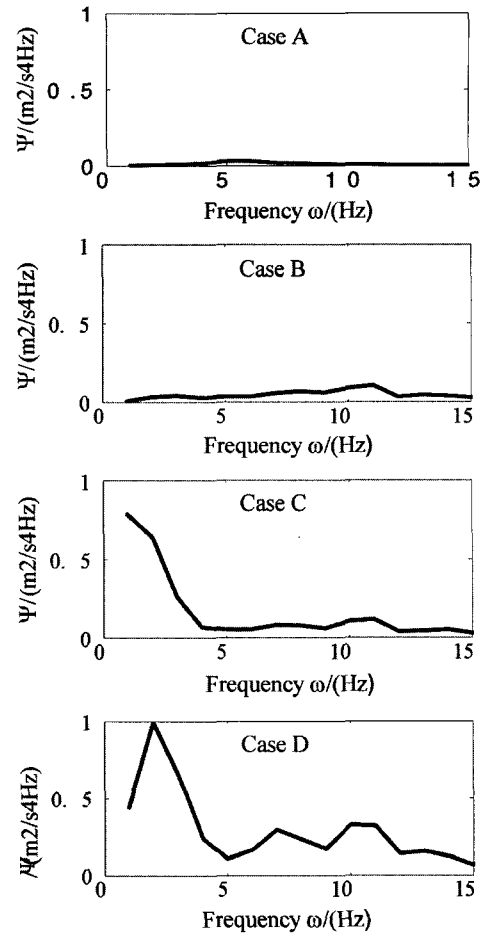


Figure 10. Spectral density of vehicle body C.G. vertical acceleration.

the best performance. Another promising results are that only in Case A, the dynamic tire load can be reduced greatly, so that this ensures tire road holding and then improves the road holding performance. Although the RMS values of suspension working space in Case A are relatively large, but no larger than 60% compared with the results in Case D and are still in an acceptable range. This result is consistent with the previous study conclusion, i.e., the benefits of active suspension will increase with the increase of available suspension working space. Compared with the system by using a linear and fuzzy logic controller, i.e., the scheme proposed by Yoshimura, the new designed active suspension could offer better performances, especially in reducing vertical and pitch acceleration, and improving road holding.

Furthermore, simulations are also carried out in some discrete event input conditions in order to examine the system robustness. Compared with passive suspension system, much better performance can be found even if the system is subject to a sudden change of road conditions,

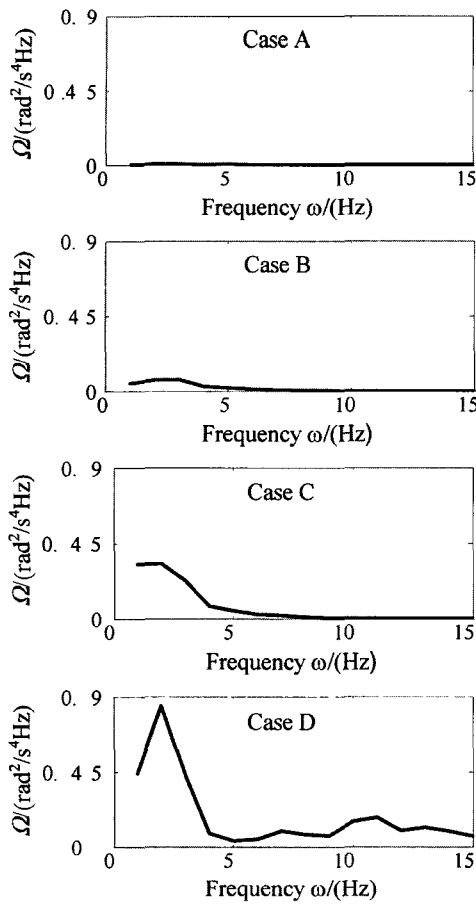


Figure 11. Spectral density of pitch acceleration of vehicle body.

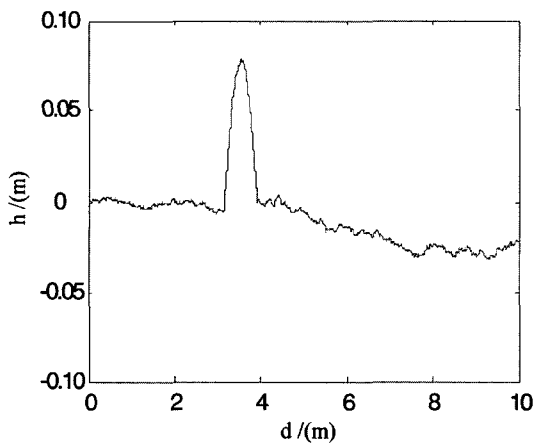


Figure 12. Typical road disturbance with a severe bump.

such as a pothole, an obstacle or a step input. While the vehicle running at the speed of 20 m/s on the road with a discrete obstacle, illustrated in Figure 12, with a bump

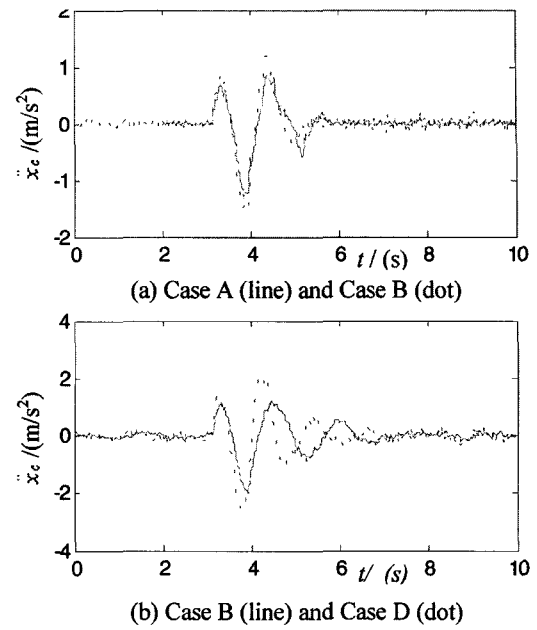


Figure 13. Vertical acceleration of vehicle body under severe road disturbance input.

with amplitude of 0.085 meters, the system response in four cases are shown in Figure 13. Because of the sudden bump disturbance, the amplitude of vertical acceleration  $\ddot{x}_c$ , changes very sharply. Compared with the results of passive suspension, the proposed active suspension can reduce the peak values of accelerations considerably and show more promising performance than that of Case B and Case C. The response of pitch acceleration also shows the similar results and is not given here.

### 5. CONCLUSIONS

A new control strategy is proposed for active vehicle suspension systems by using a GA-based combined control scheme, i.e., respectively using a PID controller and a fuzzy logic controller in two loops. The parameters of both PID controller and fuzzy controller are optimized on-line by GA to achieve optimal performance and adapt to variable conditions. By using a four degree-of-freedom nonlinear vehicle model, for example, the algorithm is implemented and simulations are carried out in different road disturbance input conditions. Simulation results in both time and frequency domains show that present control scheme offers the best performance. Compared with the other control scheme, i.e., by using linear and fuzzy control, by using normal combined PID and fuzzy control, the proposed new scheme is more effective in reducing peak values of vehicle body vibration caused by irregular road excitation, especially within low frequency

range, i.e., the most sensitive frequency range of human response. Furthermore, the great reduction of dynamic tire load RMS value improves the road holding and enhances handling performance. The system robustness is also examined in some severe road input conditions, such as a sudden change of road condition. Simulations in the road with a bump input show that the peak values of vertical acceleration and pitch acceleration can be respectively reduced greatly compared with the results of a passive system. Hence, the effectiveness and robustness of new proposed control strategy is proved.

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