

TRACKING CONTROL DESIGN USING SLIDING MODE TECHNIQUES FOR SATELLITE FORMATION FLYING

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ABSTRACT

Satellite formation flying is currently an active area of research in the aerospace engineering. So it has been researched by various authors. In this study, a tracking controller using sliding mode techniques was designed to control a satellite for the satellite formation flying. In general, Hill's equations are used to describe the relative motion of the follower satellite with respect to the leader satellite. However the modified Hill's equations considering the J_2 perturbation were used for the design of sliding mode controller. The extended Kalman filter was applied to estimate the state vector based on the measurements of relative distance and velocity between two satellites. The simulation results show that the follower satellite tracks the desired trajectory well by thruster operations based on the sliding mode control law.

Keywords: satellite formation flying, sliding mode control, Hill's equations, Kalman filter

1. INTRODUCTION

Satellite formation flying (SFF) is the placing of micro-satellites into nearby orbits to form a cluster for the same mission. In the recent it has become a topic of significant interest in the aerospace engineering. Specifically, NASA has identified satellite formation flying as an enabling technology for future missions and launched the Earth Observing-1 (EO-1) in November 2000, which was the first satellite demonstrating SFF technology and was scheduled to acquire a stereo image with Landsat 7. Formation flying system has several benefits compared to the single spacecraft system that has equivalent functions: low cost for launch and mass production, larger aperture size, greater launch flexibility, higher system reliability and easier expandibility.

It is important to maintain a formation under perturbations to achieve missions successfully because perturbations derive satellites within a cluster to separate. These perturbations include spherical harmonics of the Earth, atmosphere drag, solar radiation pressure and luni-solar attraction. Among these perturbations, the dominant factor is a second order zonal harmonic of the Earth due to oblateness which is called as the J_2 perturbation. It causes three important effects on SFF: nodal regression, and drifts in the perigee and the mean anomaly. In general, Clohessy-Wiltshire equations are used to describe the relative motion and control strategies between satellites within a cluster, which are known as Hill's equations. However Hill's equations cannot capture the J_2 perturbation effect because they are derived under the assumption that the reference orbit is circular,

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the Earth is spherically symmetric, and the target satellite is very close to the reference orbit. The modified Hill's equations were developed by Schweighart & Sedwick (2002), which describe the relative motion of satellites in the presence of the J_2 perturbation and are surprisingly similar in form to Hill's equations. These equations were shown to have only a maximum modeling error of 0.4% over all inclinations, radii, and cluster configurations.

The practical implementation of SFF missions depends on the accurate control of the relative positions and orientations among satellites within a cluster for formation configuration, formation keeping and collision avoidance. Most SFF control law has been designed on the base of the simplified relative dynamic equations such as Hill's equations. In addition, it can be designed using the Gaussian variational equations and the inertial equations of cartesian coordinates. Schaub *et al.* (2000) demonstrated the design of control laws based on the feedback of the errors in the Gaussian variational and the inertial equations. Their results show that the Gaussian variational equations are suitable to the feedback control design of satellites with pulse-type thrusters, whereas the inertial equations are suitable to one of satellites with continuous thrusters. Hill's equations have formed the basis for the application of various linear control techniques to the SFF control problem. Kumar & Seywald (1995) treated the problem of fuel-optimal station-keeping of two satellites in very low Earth orbits based on Hill's equations. Ulybyshev (1998) proposed the formation keeping strategy for satellite constellations in circular orbits, which includes the use of a discrete-time LQR (Linear Quadratic Regulator) for feedback control based on Hill's equations. Kapila *et al.* (2000) developed a mathematically rigorous control design framework for linear control using Hill's equations and proposed a pulse-based control architecture for lowering the fuel consumption in SFF problem.

Control designs based on Hill's equations require high fuel consumption and can imperil the formation flying mission with long duration and large separation between satellites since Hill's equations disregard the perturbation and nonlinear terms on the relative motion dynamics. So many nonlinear control theories have been researched in SFF. Queiroz *et al.* (2000) developed a nonlinear adaptive control law for the relative position tracking of multiple satellites. Gurfil *et al.* (2003) proposed a nonlinear adaptive neural control methodology for deep-space SFF. Currently, most nonlinear control schemes for SFF use full state feedback controllers, which require both position and velocity sensors. Wong *et al.* (2002) designed a adaptive output feedback tracking control in the absence of velocity measurements.

In this study, a tracking controller using sliding mode techniques is designed to control a satellite for SFF. The modified Hill's equations considering the J_2 perturbation were used for the design of sliding mode controller. Sliding mode control law causes chattering phenomenon because it is a discontinuous control. Dead-zone was used to avoid the chattering. The extended Kalman filter (EKF) was applied to estimate the state vector based on the measurements of relative distance and velocity between two satellites.

2. SLIDING MODE CONTROL LAW

Sliding mode control is intrinsically robust against parametric uncertainties and unmodeled dynamics such as external disturbances. The control always drives the system towards the sliding plane against disturbances.

Consider the second order linear dynamic system

$$\begin{aligned}\dot{x} &= \bar{A}(t)x(t) + \bar{B}(t)u(t) \\ \text{or} \\ \ddot{q} &= A(t)q(t) + B(t)q(t)\end{aligned}\tag{1}$$

where $x = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T = [q, \dot{q}]^T$ is the state vector and $u = [u_x, u_y, u_z]^T$ is the control input vector. The desired trajectory is represented by $q_d(t)$, and the tracking error is defined as $e(t) = q_d(t) - q(t)$. Furthermore, let us define a sliding plane

$$s(e, t) = \dot{e}(t) + K_1 e(t) + K_2 e_s(t) \quad (2)$$

where $e_s(t) = \int e(t) dt$

Differentiating Eq. (2) with the aid of Eq. (1) yields

$$\begin{aligned} \dot{s}(e, t) &= \{\ddot{q}_d(t) + K_1 \dot{q}_d(t) + K_2 q_d(t) - A(t)q(t) - K_1 \dot{q}(t) - K_2 q(t)\} - B(t)u(t) \\ &= \tilde{u}(t) - B(t)u(t) \end{aligned} \quad (3)$$

where $\tilde{u} = \ddot{q}_d(t) + K_1 \dot{q}_d(t) + K_2 q_d(t) - A(t)q(t) - K_1 \dot{q}(t) - K_2 q(t)$.

Since the order of Eq. (2) is lower than the dynamic system, the right hand side of Eq. (2) does not involve the control input, but Eq. (3) does it. This makes the sliding mode control solution easy to obtain when a quadratic function of $s(e, t)$ is used as a Lyapunov function (Yeh et al. 2002). A positive definite Lyapunov function can be defined as

$$V = \frac{1}{2} s^T(e, t) s(e, t) \quad (4)$$

Differentiating the Lyapunov function gives

$$\begin{aligned} \dot{V} &= s^T(e, t) \{\tilde{u} - B(t)u(t)\} \\ &= s^T(e, t) B(t) \{B^{-1}(t)\tilde{u}(t) - u(t)\} \end{aligned} \quad (5)$$

The derivative of the Lyapunov function should always be negative (for $s(e, t) \neq 0$) or zero (for $s(e, t) = 0$) for asymptotical stability of the closed-loop dynamics. If we define the asymptotically stabilizing control law $u(t)$ as Eq. (6), the above requirement can be satisfied as:

$$u(t) = B(t)^{-1} \tilde{u}(t) + \hat{u}(t) \quad (6)$$

with

$$\hat{u}(t) = \begin{pmatrix} \mu \operatorname{sgn}(B^T(t)s(e, t)) & \text{for } s(e, t) \neq 0 \\ 0 & \text{for } s(e, t) = 0 \end{pmatrix}$$

where μ have the same dimension of the control input vector and its component is positive. Sgn means a signum function. According to Eq. (6), control law $u(t)$ drives the system to the sliding plane through a reaching mode, whose magnitude is discontinuous and variable. This control law is not suitable to the general satellite control system since the thruster magnitude is not adjustable. The new control law can be introduced based on Eq. (6) to overcome this problem. To implement a discontinuous and constant control input, the new control law is defined as

$$u(t) = \mu \operatorname{sgn}(B^T(t)s(e, t)) \quad (7)$$

where $\mu_i > B_i^{-1}(t)\tilde{u}(t)$. This control law satisfies the condition of Lyapunov's stability and then can give an asymptotical stability for the closed-loop system.

The sliding mode control law causes chattering phenomenon since it is discontinuous across the sliding plane. Chattering describes rapid control signal switching between positive and negative values because the control input enforces the system reach to the sliding plane. The way to avoid

chattering is to establish dead-zone which keep thruster from operating frequently. Applying dead-zone to Eq. (7) of the control law gives

$$u_i(t) = \begin{pmatrix} \mu_i & \text{if } \delta_i < \mathbf{B}_i^T(t)s(e, t) \\ 0 & \text{if } -\delta_i \leq \mathbf{B}_i^T(t)s(e, t) \leq \delta_i \\ -\mu_i & \text{if } \mathbf{B}_i^T(t)s(e, t) < -\delta_i \end{pmatrix} \quad (8)$$

where μ is the magnitude of thrusters and should be selected to satisfy Eq. $\mu_i > |\mathbf{B}_i^{-1}(t)\bar{u}(t)|$. δ_i is small positive values, which can be tuned by trial and error.

3. PROBLEM FORMULATION

In this section, we discuss the modified Hill's equations as a dynamic model, the EKF for the estimation of state variables and the method of determining the sliding plane using a pole placement.

A. Dynamic Model

A rotating local-vertical-local-horizontal (LVLH) frame is used to visualize the relative motion with respect to the reference satellite. The x-axis points in the radial direction, the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points in the along-track direction. The modified Hill's equations developed by Schweighart & Sedwick (2002) are derived in the LVLH frame, which describe the relative motion of satellites under the effect of the J_2 perturbation. If two satellites are coplaner in the modified Hill's equations, it is not necessary to correct the cross-track motion. The necessary parameters to these equations are the radius and the inclination of the reference orbit, relative position and the velocity of z component to describe the relative motion. These equations are given by:

$$\begin{aligned} \ddot{x} - 2(\omega c)\dot{y} - (5c^2 - 2)\omega^2 x &= F_x \\ \dot{y} + 2(\omega c)\dot{x} &= F_y \\ \ddot{z} + k^2 z &= F_z \end{aligned} \quad (9)$$

where,

$$\begin{aligned} s &= \frac{3J_2 R_e^2}{8r_{ref}^2} [1 + 3\cos(2i_{ref})] \\ c &= \sqrt{1 + s} \\ k &= \omega\sqrt{1 + s} + \frac{3\omega J_2 R_e^2}{2r_{ref}^2} [\cos(i_{ref})]^2 \end{aligned}$$

where ω , i_{ref} and r_{ref} are the mean motion, the inclination and the radius of the reference orbit, respectively. R_e is the mean equatorial radius of the Earth and J_2 is the zonal potential coefficient of second order. $[F_x, F_y, F_z]^T$ can be the external disturbance or control force. These equations show that the in-plane (xy-plane) motion is decoupled from the cross-track motion. Solving the above equations yield the closed-form solutions

$$\begin{aligned} x &= x_0 \cos(nt\sqrt{(1-s)}) + \frac{\sqrt{(1-s)}}{2\sqrt{(1+s)}} y_0 \sin(nt\sqrt{(1-s)}) \\ y &= -\frac{2\sqrt{(1+s)}}{\sqrt{(1-s)}} x_0 \sin(nt\sqrt{(1-s)}) + y_0 \cos(nt\sqrt{(1-s)}) \end{aligned}$$

$$z = z_0 \cos(kt) + \frac{\dot{z}_0}{k} \sin(kt) \quad (10)$$

Herein, the subscript 0 is used for the initial values. In this study, control problem is to design a control law which derives a satellite to track a desired trajectory. So these closed-form solutions will be used for the generation of a desired trajectory. For convenience, we introduce a new time variable ($\tau = \omega t$) to establish the sliding plane using a pole placement. Since $dx/dt = \omega(dx/d\tau)$ and $d^2x/dt^2 = \omega^2(d^2x/d\tau^2)$, the modified Hill's equations of Eq. (9) based on the new time variable can be rewritten by

$$\begin{aligned} \ddot{x} - 2(c)y - (5c^2 - 2)x &= F_x/\omega^2 = u_x \\ \ddot{y} + 2(c)\dot{x} &= F_y/\omega^2 = u_y \\ \ddot{z} + k^2/\omega^2 z &= F_z/\omega^2 = u_z \end{aligned} \quad (11)$$

where differential equations are for the new time variable. Rewriting these equations based on Eq. (1), $\bar{A}(t)$ and $\bar{B}(t)$ are given by

$$\bar{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ (5c^2 - 2) & 0 & 0 & 0 & 2c & 0 \\ 0 & 0 & 0 & -2c & 0 & 0 \\ 0 & 0 & -k^2/\omega^2 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{B}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The process noise is considered for implementing the EKF. So the system equation is defined as

$$\dot{x} = \bar{A}x(t) + \bar{B}u(t) + w(t) \quad (12)$$

where random variable w represents the process noise which is assumed to be uncorrelated in time (white noise). Thus, the process noise can be characterized by the normal probability distribution

$$w(t) \sim N(0, Q) \quad (13)$$

where Q is the process noise covariance matrix.

B. Measurement Model

It is required to measure or estimate the state variables of relative motion for a tracking control. The relative position can be obtained from a spaceborne GPS (Global Positioning System) receiver since it gives a position for each satellites. An additional sensor is necessary for taking velocity information besides a GPS receiver. For general satellite system two sensors, position and velocity, are required for relative position and velocity information, which can increase the cost and mass of SFF. In this study, a relative distance and a velocity are considered as measurements. The EKF is used for the estimation of the state variables because a measurement model is nonlinear, even though a dynamic model is linear.

Measurement equations can be defined using a new time variable as

$$\begin{aligned} h(x) &= \begin{bmatrix} \rho \\ \dot{\rho} \end{bmatrix} + \nu = \begin{bmatrix} \rho \\ \left[\frac{d\rho}{d\tau} \right] \end{bmatrix} + \nu \\ &= \begin{bmatrix} \sqrt{x^2 + y^2 + z^2} \\ \frac{1}{\rho} \left(x \frac{dx}{d\tau} + y \frac{dy}{d\tau} + z \frac{dz}{d\tau} \right) \end{bmatrix} + \nu \end{aligned} \quad (14)$$

where ρ and $\dot{\rho}$ are measurements of relative distance and velocity between two satellites in LVLH frame. Random variable ν represents the measurement noise which is assumed to be white noise. Thus, the measurement noise can be characterized by the normal probability distribution

$$\nu(t) \sim N(0, R) \quad (15)$$

where R is the measurement noise covariance matrix.

The measurement Jacobian is defined by partial derivatives of measurement equation with respect to the state variables, which is written using a new time variable by

$$\begin{aligned} \frac{\partial h(x)}{\partial x} &= \frac{\partial}{\partial x} \begin{bmatrix} \rho \\ \dot{\rho} \end{bmatrix} \\ &= \begin{bmatrix} \frac{x}{\rho} & \frac{y}{\rho} & \frac{z}{\rho} & 0 & 0 & 0 \\ \left[-\frac{x\dot{\rho}}{\rho^2} + \frac{\dot{x}}{\rho}\right]\omega & \left[-\frac{y\dot{\rho}}{\rho^2} + \frac{\dot{y}}{\rho}\right]\omega & \left[-\frac{z\dot{\rho}}{\rho^2} + \frac{\dot{z}}{\rho}\right]\omega & \frac{x}{\rho} & \frac{y}{\rho} & \frac{z}{\rho} \end{bmatrix} \end{aligned} \quad (16)$$

C. Sliding Plane Design

In the previous section, the sliding plane was defined without determination of parameters (K_1 , K_2). This study is based on the second order design method (Yeh et al. 2002) for determining parameters of the sliding plane. If a new time scale is considered, the sliding plane can be rewritten by

$$s(e, \tau) = \dot{e}(\tau) + K_1 e(\tau) + K_2 e_s(\tau) \quad (17)$$

The sliding plane can be established using a pole placement since it is similar to a proportional-integral-derivative control. Two poles of a characteristic equation with a 0.8 damping ratio and a 0.1 natural frequency are given by

$$p_1 = 0.08(-1 + j6), \quad p_2 = 0.08(-1 - j6) \quad (18)$$

When the above damping ratio and the natural frequency are applied to a satellite with 800 km altitude, the setting time which tracking error decays is about 8 orbits. The characteristic equation is established with the above poles

$$(\sigma - p_1)(\sigma - p_2) = \sigma^2 + 0.16\sigma + 0.01 \quad (19)$$

From comparison of Eq. (17) with Eq. (19), the parameters of the sliding plane are $K_1 = 0.16$ and $K_2 = 0.01$. Thus, the sliding mode control law with a dead-zone is rewritten by

$$u_i(t) = \begin{bmatrix} \mu_i & \text{if } \delta_i < \dot{e}(t) + 0.16e(t) + 0.01e_s(t) \\ 0 & \text{if } -\delta_i \leq \dot{e}(t) + 0.16e(t) + 0.01e_s(t) \leq \delta_i \\ -\mu_i & \text{if } \dot{e}(t) + 0.16e(t) + 0.01e_s(t) < -\delta_i \end{bmatrix} \quad (20)$$

For the modified Hill's equations and control system with a fixed magnitude of thruster such as pulse-type actuators, the magnitude of the thrusters should satisfy the following condition:

$$\mu_i > |\tilde{u}_i(t)| = |\ddot{x}_d + K_1 \dot{e}(t) + K_0 e(t) - A(t)|_i \quad (21)$$

Table 1. Numerical data for the simulation.

Altitude (Reference)	800 km
Inclination (Reference)	35.4 degree
Simulation time (τ)	100 radian
Initial position and velocity [m, m/s]	$q_d(0)$ (0, 2000, 0), (1000 ω , 0, 0)
	$q_a(0)$ (500, -1000, 500), (50 ω , -50 ω , 0)
	$q_e(0)$ (400, 900, 400), (55 ω , -55 ω , 0)
Magnitude of thruster [m/s ²]	(3E-3, 3E-3, 6E-4)
Dead-zone	[5E-3; 5E-3; 5E-3] ω
Q (Process noise covariance)	$10^{-8}I_{6 \times 6}$
R (Measurement noise covariance)	$10^{-4}I_{2 \times 2}$
P (0) (Initial estimate error covariance)	$[q_a(0) - q_e(0)][q_a(0) - q_e(0)]^T$

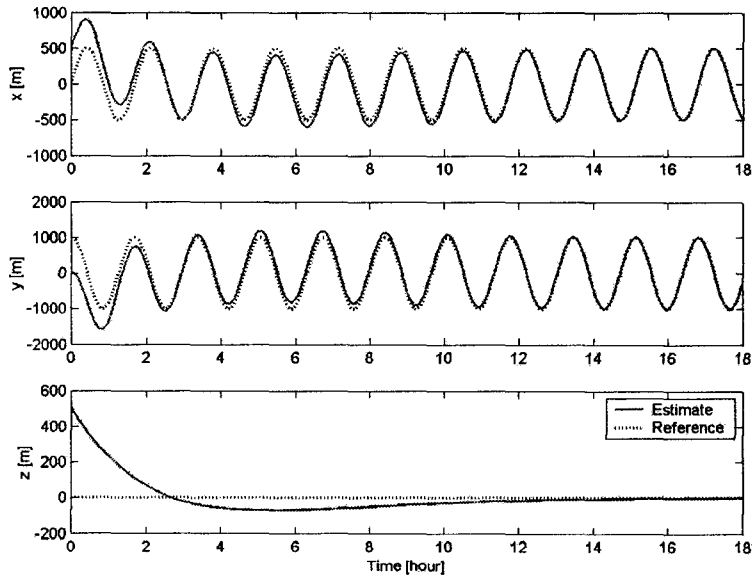


Figure 1. Reference and estimate trajectories.

4. NUMERICAL SIMULATION AND RESULTS

The sliding mode control law described in Eq. (20) is simulated using the modified Hill's equations and the EKF. The sliding mode controller works to minimize the difference between the estimated relative position and the desired position which is based on solutions of the modified Hill's equations. As mentioned in previous sections, the control input is generated by pulse-type thrusters whose magnitude is constant not variable. The initial estimate error covariance should be given to implement the EKF, which is updated based on the Kalman gain and the measurement Jacobian. In practice, the process noise covariance Q and measurement noise covariance R matrices might change with time or measurement, however here we assume they are constant.

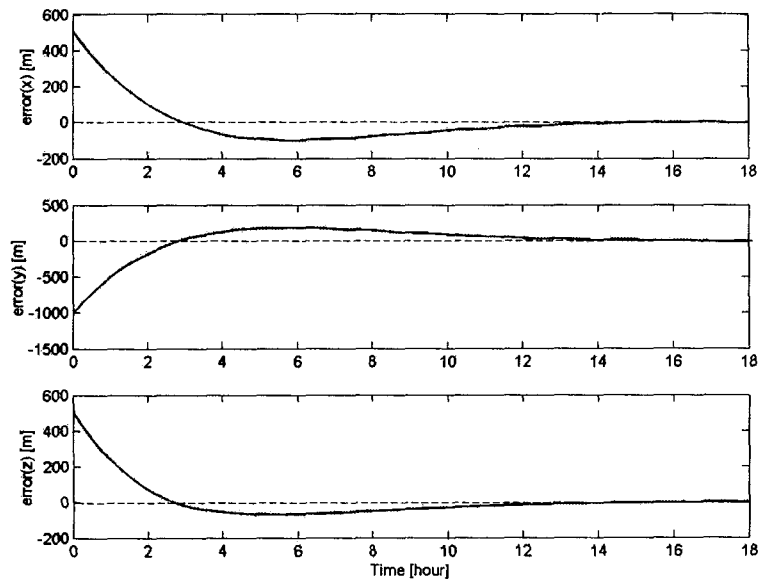


Figure 2. Position tracking errors between reference and estimate.

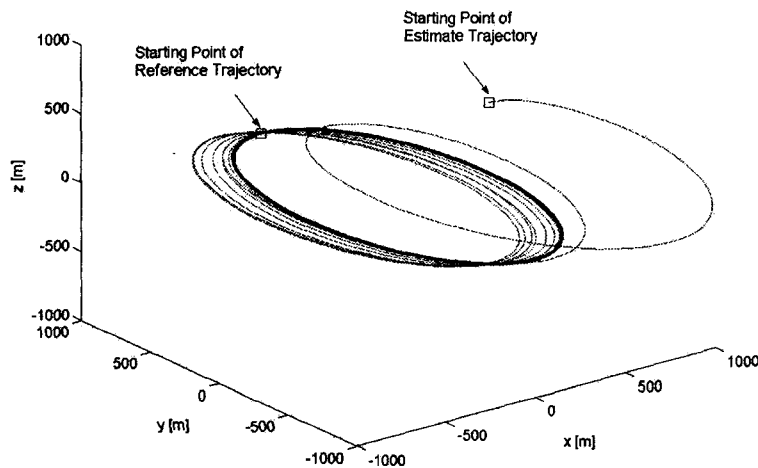


Figure 3. Estimate trajectory in 3-D space with reference trajectory.

All numerical data is given in the Table 1 for the simulation of the sliding mode control law. $q_d(0)$ is the initial values of desired trajectory and $q_a(0)$ is for actual trajectory which is used to generate the measurements of relative distance and velocity. $q_e(0)$ is the estimated initial values to implement the EKF.

Figure 1 show that the follower satellite tracks the desired trajectory well by thruster operations based on the sliding mode control law. Figure 2 is the difference between the estimated relative position and the desired position. The period of the reference satellite with 800 km altitude is about 1.7 hour and thus the setting time is approximately 13.4 hour. The designed control law have to make

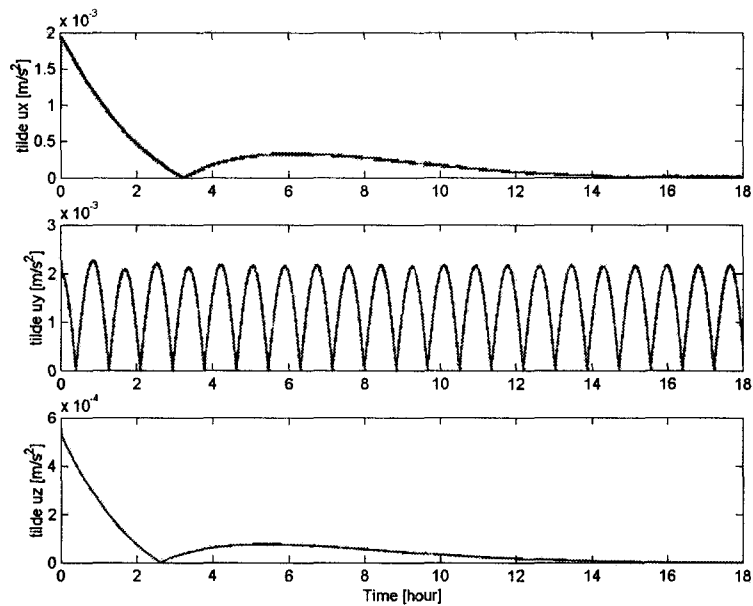


Figure 4. $|\tilde{u}(t)|$ (\tilde{u}_x , \tilde{u}_y and \tilde{u}_z mean \bar{u}_x , \bar{u}_y and \bar{u}_z in Eq. (21)).

position errors decrease to zero after the setting time. It is shown from Fig. 1 and 2 that position error is approximately zero after 14 hour which is similar to the setting time. To make position error converge to zero more quickly, thrusters with an higher propellant power are required. However the system stability can not be guaranteed without changing the sliding plane because the magnitude of thrusters is fixed. As mentioned in the previous sections, dead-zone plays a role to prevent chattering and frequent operations of thrusters. If the size of dead-zone is larger, position errors may not decrease and have high ripples even though the fuel is saved. Therefore, it is necessary to combine many factors adequately such as the sliding plane, the magnitude of thrusters and dead-zone. Figure 3 show the trajectory of the follower satellite in the 3-D space.

From the condition of Lyapunov stability Eq. (21), the magnitude of thrusters should be selected larger than $|\tilde{u}_i(t)|$ for $i = x, y, z$. Figure 4 shows that $|\tilde{u}_i(t)|$ are lower than the thruster magnitudes which are assigned in Table 1. This means that the desired system is stable since the condition of Lyapunov stability is satisfied.

5. CONCLUSIONS

In this paper, the tracking control law was designed using the sliding mode techniques to derive a satellite to the desired trajectory. The modified Hill's equations were used for a dynamic model and generation of the desired trajectory. Dead-zone was applied to prevent chattering and frequent operations of thrusters which were assumed to be a pulse-type. The state vector was estimated by the EKF based on the measurements of relative distance and velocity between two satellites.

Since most controllers are based on the Hill's equations which cannot capture the J_2 perturbation effect, they require much fuel consumption for the formation keeping or configuration. However, this tracking control law is more effective for the control of SFF, because it was designed using the

modified Hill's equations considering the J_2 perturbation. This control system based on the sliding mode control can be inherently robust against external disturbances. However, the main drawback is a chattering which can be decreased by applying dead-zone.

Simulation results were provided to demonstrate the performance of the controller and filter. It was proven from results that the designed controller worked well to drive a satellite to the desired trajectory.

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