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ANALYSIS OF THE MOTION OF A TETHER-PERTURBED SATELLITE

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ABSTRACT

The motion of each satellite in a tethered satellite system is non-Keplerian in the Earth's gravitational field. In this paper, the tether perturbation force is formulated and compared with the perturbation force due to the Earth's oblateness. Also, the center of mass motion of the tethered satellite system is analyzed. The tether perturbing force on the one of satellites in a tethered satellite system is much bigger than the Earth's oblateness perturbation. The two-body motion approximation of the center of mass is acceptable to describe the motion of the system, when the libration is small.

Keywords: tethered satellite, orbit, perturbation

1. INTRODUCTION

Since the concept of tethered satellite systems was proposed, numerous types of tethered satellite systems and their dynamic behaviors have been investigated. Dynamics and control problems such as deployment and retrieval of tethers (Rupp 1978, Keshmiri & Misra 1996, Pradhan, Modi, & Misra 1995), elastic oscillation of tethers (Yu 1996, Luongo & Vestroni 1994, Pignataro, Luongo, & Pasca 1991), and electrodynamics of tethers (Williamson, Banks, & Raitt 1987, Martinez-Sanchez & Hastings 1987, Bonifazi et al. 1987) have been studied by using various tether models. Although the motion of a tethered satellite system is coupled orbital and attitude motion, the attitude dynamics and control of tethered satellite have generally been treated as decoupled from the dynamics of the center of mass motion around the Earth. The decoupling of attitude motion from orbital motion is usually acceptable because the effects of dynamic coupling of attitude motion with orbital motion (translational motion) are usually not very significant (Modi & Misra 1977, Bainum & Evans 1976). However, when the orbital motion of tethered satellite systems is considered, the dynamic coupling becomes the principal interest (Moran 1961, Yu 1964).

Although, the analytical investigations of the orbital motion of tethered satellite systems have been numerous (Beletskii & Levin1985, De Matteis 1992, Warnock & Cochran 1993), there are still some characteristics of tethered satellite systems that may cause significant problems. The main reason for academic and practical interest in tethered satellite systems is that their dynamical characteristics are different from those of conventional satellites. If the geometry of a satellite system is not symmetric, or its mass is not uniformly distributed, the nominal orbital motion is no longer Keplerian. Its relatively large geometric size and unique configuration cause the motion of a tethered satellite system (TSS) to differ from that of other space objects in the Earth's orbit (Cochran et al.

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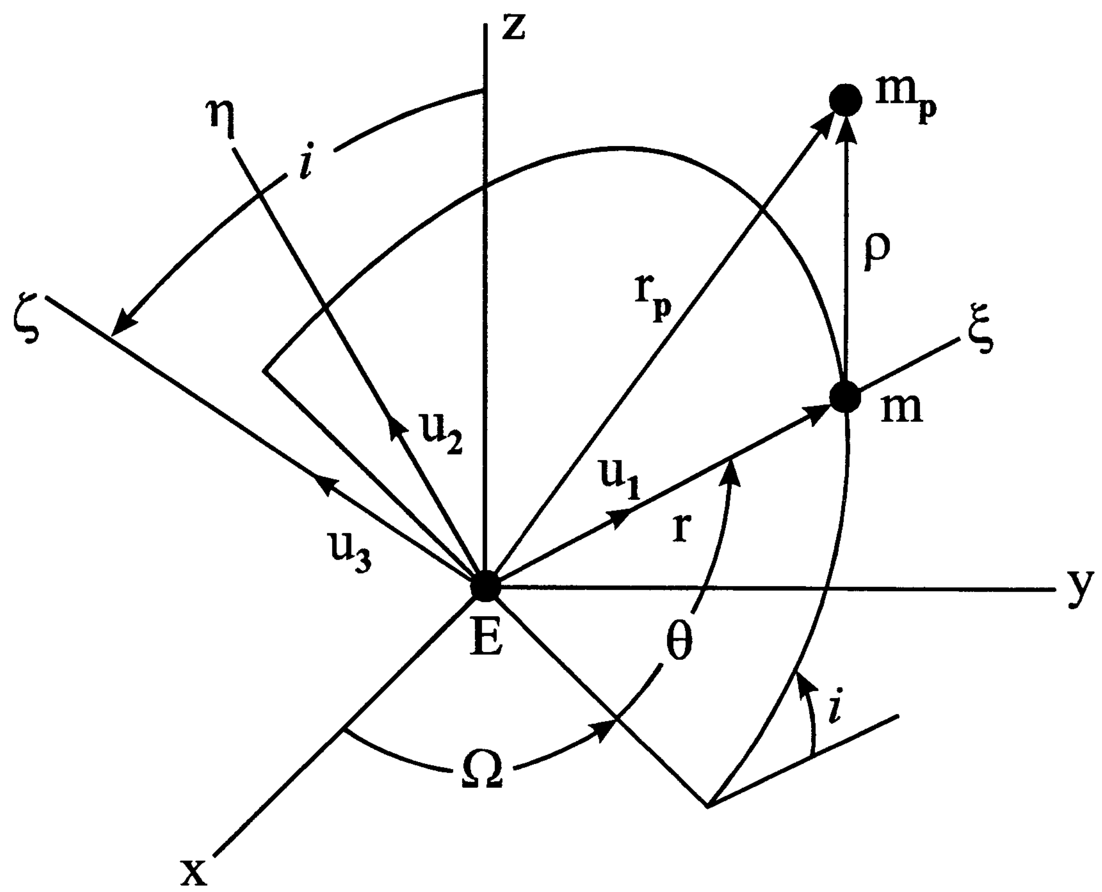

Figure 1. Two-body Tethered Satellite System.

1996, Cho, Cochran, & Cicci 1998). These exclusive characteristics may be treated as a new source of perturbation of a nominal Keplerian orbit. Tethered satellites and their motion may be considered as the source of perturbation on the orbital motion of TSS.

In this paper, a model of a two-satellite tethered system with point mass satellites connected by a massless rod tether is described and the tether-perturbing force on the satellites in the TSS is analyzed.

2. TETHERED SATELLITE SYSTEM MODEL

The coordinate systems and position vectors used to describe the motion of a tethered satellite system are depicted in Figure 1. For our purpose, we may take the origin of the reference frame E_{xyz} as the center of the Earth. We may also model the Earth as a point mass. In Figure 1, the plane $\xi\eta$ is the osculating plane in which m is moving at some point of time. The orientation of the osculating plane is defined by Ω and i, the longitude of the ascending node and inclination, respectively. The vector $\mathbf{r}$ defines the position of the perturbed satellite, m, with respect to $\mathbf{E}$ and the position of the other satellite, m_p, with respect to $\mathbf{E}$ is defined by $\mathbf{r}_\mathrm{p}$. The equations of motion for the satellites m and m_p in a Newtonian gravity field are

$$\ddot{\mathbf{r}} = -\mu\frac{\mathbf{r}}{r^3} + \frac{\mathbf{F}_\mathrm{T}}{m} \tag{1}$$

$$\ddot{\mathbf{r}}_\mathrm{p} = -\mu\frac{\mathbf{r}_\mathrm{p}}{r_p^3} - \frac{\mathbf{F}_\mathrm{T}}{m_p} \tag{2}$$

where μ is the gravitational constant in the Earth's gravitational field and the forces of the satellite on the other satellite are neglected. On the right-hand-sides of the Eqs. (1) and (2), $\mathbf{F}_\mathrm{T}$ is the force on the satellite m due to the tether. The force $\mathbf{F}_\mathrm{T}$ is a new disturbing force, which must be introduced in the equations of a tethered satellite system. In Figure 1, the vector ρ is the relative position vector of m_p with respect to m. By using Eqs. (1) and (2) we may then write

$$\ddot{\rho} = \Delta\mathbf{g} - \frac{M}{m\,m_p}\mathbf{F}_\mathrm{T}, \tag{3}$$

where $M = m + m_p$ and $\Delta\mathbf{g}$, the effect of gravity difference in the acceleration at m and m_p, is

$$\Delta\mathbf{g} = -\mu\left[\frac{\mathbf{r}_\mathrm{p}}{r_p^3} - \frac{\mathbf{r}}{r^3}\right]. \tag{4}$$

Figure 2 shows the angles θ_2 and θ_3. θ_2 and θ_3 are in-plane and out-of-plane motion of the body m_p with respect to m, respectively. In Figure 2, unit vector $\mathbf{e}_{123}$ represents relative coordinate system with respect to the rotating frame unit vector $\mathbf{u}_{123}$. From Figures 1 and 2, and Eqs. (1)-(3), the motion of m in $\mathbf{u}_{123}$ and the relative motion of the m_p in $\mathbf{e}_{123}$ coordinate system may be expressed, respectively, as

$$\ddot{\mathbf{r}} + \dot{\lambda} \times \mathbf{r} + 2\lambda \times \dot{\mathbf{r}} + \lambda \times (\lambda \times \mathbf{r}) = -\mu\frac{\mathbf{r}}{r^3} + \frac{\mathbf{F}_\mathrm{T}}{m} \tag{5}$$

$$\ddot{\rho} + \dot{\omega} \times \rho + 2\omega \times \dot{\rho} + \omega \times (\omega \times \rho) = \Delta\mathbf{g} - \frac{\mathbf{F}_\mathrm{T}(m + m_p)}{m\,m_p}. \tag{6}$$

In Eq. (5), $\boldsymbol{\lambda}$ is the angular velocity of the $\mathbf{u}_{123}$ coordinate system. In Eq. (6), ω is the angular velocity of the $\mathbf{e}_{123}$ coordinate system. From the vector differential Eq. (5), the following three scalar equations for the motion of the body m are obtained (Cochran et al. 1996, Cho, Cochran, & Cicci 1998):

$$\ddot{r} = (\lambda_2^2 + \lambda_3^2)r - \frac{\mu}{r^2} + \mathbf{u}_1 \cdot \frac{\mathbf{F}_\mathrm{T}}{m} \tag{7}$$

$$\dot{\lambda}_3 = \frac{1}{r}\left(-2\lambda_3\dot{r} - \lambda_1\lambda_2 r + \mathbf{u}_2 \cdot \frac{\mathbf{F}_\mathrm{T}}{m}\right) \tag{8}$$

$$\dot{\lambda}_2 = \frac{1}{r}\left(-2\lambda_2\dot{r} + \lambda_1\lambda_3 r + \mathbf{u}_3 \cdot \frac{\mathbf{F}_\mathrm{T}}{m}\right). \tag{9}$$

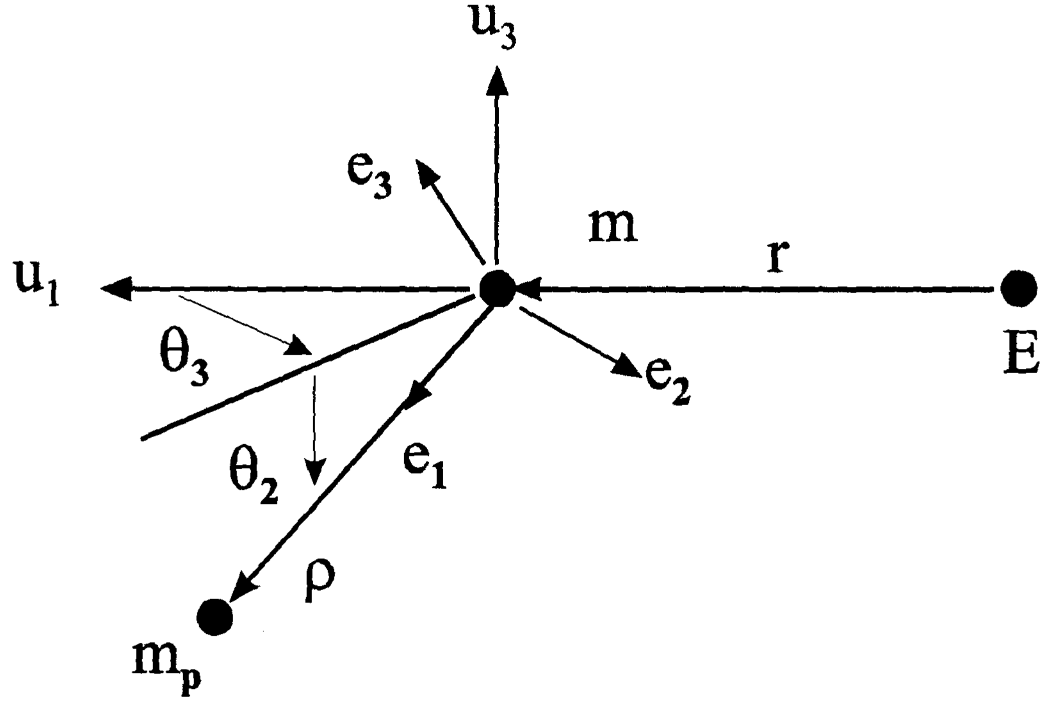

Figure 2. Relative Motion of m_p w.r.t. m.

The kinematics equations that relate the component of $\boldsymbol{\lambda}$ are

$$\dot{\Omega} = \lambda_1 \sin\theta / \sin i \tag{10}$$

$$\dot{i} = \lambda_1 \cos\theta \tag{11}$$

$$\dot{\theta} = \lambda_3 - \lambda_1 \sin\theta \cos i / \sin i. \tag{12}$$

Note that, since we have described the motion of the satellite m in the osculating plane $\xi\eta$, both $\mathbf{r}$ and $\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{e}}_1 + \boldsymbol{\lambda} \times \mathbf{r}$ must in that plane. Hence, $\lambda_2 \equiv \dot{\lambda}_2 \equiv 0$. Then, by using Eq. (9) we may write λ_1 as

$$\lambda_1 = \frac{1}{\lambda_3 r}\mathbf{u}_3 \cdot \frac{\mathbf{F_T}}{m}. \tag{13}$$

Similarly, the equations of motion for the perturbing satellite m_p can be written as

$$\ddot{\rho} = (\omega_2^2 + \omega_3^2)\rho + \mathbf{e}_1 \cdot \Delta\mathbf{g} + F_T \frac{M}{m\, m_p} \tag{14}$$

$$\dot{\omega}_3 = \frac{1}{\rho}(-2\omega_3\dot{\rho} - \omega_1\omega_2\rho + \mathbf{e}_2 \cdot \Delta\mathbf{g}) \tag{15}$$

$$\dot{\omega}_2 = \frac{1}{\rho}(-2\omega_2\dot{\rho} + \omega_1\omega_3\rho - \mathbf{e}_3 \cdot \Delta\mathbf{g}) \tag{16}$$

$$\dot{\theta}_2 = \omega_2 + \lambda_1 \sin\theta_3 \tag{17}$$
$$\dot{\theta}_3 = (\omega_3 - \lambda_1 \sin\theta_2 \cos\theta_3)/\cos\theta_2 - \lambda_3. \tag{18}$$

The gravity-gradient, $\Delta\mathbf{g}$ is in Eqs. (14)-(16) can be obtained approximately by using a series expansion. Then the expression for $\Delta\mathbf{g}$ is,

$$\Delta\mathbf{g} \cong \frac{\mu}{r^3}\rho \begin{bmatrix} 3\cos^2\theta_2 \cos^2\theta_3 - 1 \\ -3\cos\theta_2 \cos\theta_3 \sin\theta_3 \\ 3\cos\theta_2 \cos^2\theta_3 \sin\theta_3 \end{bmatrix} \tag{19}$$

Equations (7)-(19) govern the motion of m (r, Ω, i, and θ) and motion of m_p (θ_2 and θ_3) completely.

3. TETHER - PERTURBED ORBITAL MOTION

3.1 Tether Perturbing Force on Satellite m

In Eq. (1), the disturbing force, $\mathbf{F}_\mathrm{T}$ perturb the Keplerian motion of the satellite m. The tether perturbing force vector, $\mathbf{F}_\mathrm{T}$, in $\mathbf{e}_{123}$ coordinate system may be expressed as

$$\mathbf{F}_\mathrm{T} = \begin{bmatrix} F_T \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

If the tether is modeled as being inextensible, then the tether force on the satellite m may be obtained from the requirement that $\ddot{\rho}$= 0 in Eq. (14). Thus, we find that

$$F_T = \frac{m\,m_p}{M}\rho\left\{(\omega_2^2 + \omega_3^2) + \frac{\mu}{r^3}(3\cos^2\theta_2\cos^2\theta_3 - 1)\right\} \tag{21}$$

Since the tether is inextensible, we may use Eq. (21) when the right-hand side of Eq. (21) is greater than zero and $\mathbf{F}_\mathrm{T} = 0$ otherwise. Perturbations due to the tether and tethered mass may be analyzed by considering the direction of the perturbation. A vector form of the tether perturbation force in the $\mathbf{u}_{123}$ coordinate system obtained from Eq. (20) is

$$\mathbf{F}_\mathrm{T} = \begin{bmatrix} F_T \cos\theta_2 \cos\theta_3 \\ F_T \cos\theta_2 \sin\theta_3 \\ -F_T \sin\theta_2 \end{bmatrix} \tag{22}$$

Note that the unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ of the rotating coordinate system, $\mathbf{u}_{123}$, form the osculating orbital plane of satellite m, and the vector $\mathbf{u}_3$ represents the out-of-plane direction of the osculating orbital plane. From Eq. (22), it is clear that no out-of-plane perturbation due to the tether occurs when $\theta_2 = 0$. If both θ_2 and θ_3 are zero, only a radial component of the perturbation exists. In the latter case, the magnitude of the perturbing force in the radial direction is

$$F_T = \frac{m\,m_p}{M}\rho\left[\dot{\theta}^2 + \frac{2\mu}{r^3}\right] \tag{23}$$

3.2 Effect of the Earth's Oblateness on TSS

The components of the accelerations due to the Earth's oblateness of satellite m in $\mathbf{u}_{123}$ coordinate system shown in Figure 1 may be written as (Roy 1982)

$$a_{u1} = -\frac{3}{2}\mu J_2 \frac{R_e^2}{r^4}\left\{1 - \frac{3}{2}\sin i(1 - \cos 2\theta)\right\} \tag{24}$$

$$a_{u2} = -\frac{3}{2}\mu J_2 \frac{R_e^2}{r^4}(\sin^2 i \sin 2\theta) \tag{25}$$

$$a_{u3} = -\frac{3}{2}\mu J_2 \frac{R_e^2}{r^4}(\sin i \cos i \cos\theta). \tag{26}$$

where R_e is the Earth's radius. From Eq. (24), an estimate of the maximum magnitude of a_{J2} in radial direction may be approximated as

$$a_{J2max} \approx J_2 \frac{3\mu}{r^2}. \tag{27}$$

From Eq. (21), an estimate of the maximum magnitude of tether perturbing acceleration in radial direction may be obtained approximately as

$$a_{Tmax} \approx \frac{m_p}{M}\rho\frac{2\mu}{r^3}. \tag{28}$$

Also we can write the ratio of a_{J2max} to a_{Tmax}, e.i. a_{J2max}/a_{Tmax} , from Eqs. (27) and (28) as

$$R_F = \frac{3J_2}{2}\frac{M}{m_p}\frac{r}{\rho}. \tag{29}$$

Equation (29) shows that the magnitude of the tether perturbing acceleration is much larger than the Earth's oblateness effect, since r/ρ is generally very large.

3.3 Tether Perturbed Motion of the Center of Mass

Equations of motion for the center of mass may be derived by using basic principles and referring to Figure 1. If the tether is modeled as a massless rod then by using geometry and the definition of the center of mass, the position vectors of m and m_p may be expressed as

$$\mathbf{r} = -\frac{m_p}{M}\rho + \mathbf{r}_c \tag{30}$$

$$\mathbf{r}_p = \frac{m}{M}\rho + \mathbf{r}_c, \tag{31}$$

respectively. In Eqs. (30) and (31), $\mathbf{r}_c$ is the position vector of the center of mass. By differentiating Eqs. (30) and (31) with respect to time twice and adding, we can obtain

$$\ddot{\mathbf{r}}_c = \frac{\ddot{\mathbf{r}} + \ddot{\mathbf{r}}_p}{2} + \left(\frac{m_p}{M} - \frac{m}{M}\right)\frac{\ddot{\rho}}{2}. \tag{32}$$

By using $\ddot{\rho} = \ddot{\mathbf{r}}_p - \ddot{\mathbf{r}}$ and Eqs. (1) and (2), we can rewrite Eq. (32) as

$$\ddot{\mathbf{r}}_c = -\frac{\mu}{\mathrm{M}}\left(\mathrm{m}\frac{\mathbf{r}}{\mathrm{r}^3} + \mathrm{m}_\mathrm{p}\frac{\mathbf{r}_\mathrm{p}}{\mathrm{r}_\mathrm{p}^3}\right). \tag{33}$$

Equation (33) is the equation of motion of the center of mass. However, it still contains $\mathbf{r}$ and $\mathbf{r}_p$. The vectors $\mathbf{r}$ and $\mathbf{r}_p$ may be replaced by using Eqs. (30) and (31) and $1/r_p^3$ and $1/r^3$ rewritten as series expansion in ρ and r_c. We may then write the equation of motion for the center of mass of TSS as

$$\ddot{\mathbf{r}}_c = -\frac{\mu}{r^3}\mathbf{r}_c + \frac{3}{2}\frac{m\, m_p}{M}\left(\frac{\rho}{r_c}\right)^2 \frac{1}{r_c^3}\mathbf{r}_c + \cdots . \tag{34}$$

In Eq. (34), the first term in the right-hand side is the same gravitational force as in the two-body problem and the second term is a perturbation force due to the tether and end masses. It is noticed that this perturbation term in Eq. (34) is, generally, very small since ρ/r_c is small for any tethered satellite of current interest.

4. CONCLUSION

Generally, orbital motion and attitude motion of a satellite in the Earth's orbit are not strongly coupled and can be treated separately for purposes of analysis. However, relatively large geometrical scale of the configuration of a tethered satellite system makes it less reasonable to consider the motion of the center of mass (translational motion) and the motion about it (attitude motion) as separate problems. The motion of each satellite in a TSS is non-Keplerian in the Earth's gravitational field because of the tether perturbing forces. The tether perturbing force is formulated and it is noticed that the tether perturbing force on the one of the satellite in TSS is much bigger than the Earth's oblateness perturbation. This characteristic may cause problems to those who are detecting, identifying and tracking space objects with the conventional orbit determination method. The analysis of the motion of the center of mass of TSS shows that the two-body motion approximation by using the center of mass motion is feasible when the libration is small.

REFERENCES

Bainum, P. M., & Evans, K. S. 1976, AIAA Journal, 14, 26
Beletskii, V. V., & Levin, E. M. 1985, Acta Astronautica, 12, 285
Bonifazi, C., Musi, P., Cirri, G., & Cavallini, M. 1987, Proceedings of the International Conference on Space Tethers For Science in the Space Station Era, 14, 245
Cochran, J. E., Jr., Cho, S., Cheng, Y-M., & Cicci, D. A. 1996, in Advances in the Astronautical Sciences Series, ed. G. E. Powell et al. (San Diego: Univelt Inc.), p.757
Cho, S., Cochran, J. E., Jr., & Cicci, D. A. 1998, in Advances in the Astronautical Sciences Series, ed. J. W. Middour et al. (San Diego: Univelt Inc.), p.21
De Matteis, G. 1992, Journal of Guidance, Control, & Dynamics, 15, 621
Keshmiri, M., & Misra, A. K. 1996, Journal of Guidance, Control, & Dynamics, 19, 75
Luongo, A., & Vestroni, F. 1994, Journal of Sound & Vibration, 175, 259
Martinez-Sanchez, M., & Hastings, D. E. 1987, Journal of Astronautical Sciences, 35, 75
Modi, V. J., & Misra, A. K. 1977, Journal of the Astronautical Sciences, 25, 271
Moran, J. P. 1961, ARS Journal, 31, 1089
Pignataro, M., Luongo, A., & Pasca, M. 1991, Journal of Guidance, Control, & Dynamics, 14, 326
Pradhan, S., Modi, V. J., & Misra, A. K. 1995, Journal of the Astronautical Sciences, 43, 179
Roy, A. E. 1982, Orbital Motion (Bristol: Adam Hilger Ltd.)
Rupp, C. C. 1978, Journal of Astronautical Sciences, 26, 1

Williamson, P. R., Banks, P. M., & Raitt, W. J. 1987, Proceedings of the International Conference on Space Tethers For Science in the Space Station Era, 14, 186
Yu, E. Y. 1964, AIAA journal, 2, 553
Yu, S. 1996, Journal of Guidance, Control, & Dynamics, 19, 1195
Warnock, T. W., & Cochran, J. E. Jr. 1993, Journal of Astronautical Sciences, 41, 165