Application of Davidenko's Method to Rigorous Analysis of Leaky Modes in Circular Dielectric Rod Waveguides

Ki Young Kim*, Heung-Sik Tae* and Jeong-Hae Lee**

Abstract - Numerical solutions to complex characteristic equations are quite often required to solve electromagnetic wave problems. In general, two traditional complex root search algorithms, the Newton–Raphson method and the Müller method, are used to produce such solutions. However, when utilizing these two methods, the choice of the initial iteration value is very sensitive, otherwise, the iteration can fail to converge into a solution. Thus, as an alternative approach, where the selection of the initial iteration value is more relaxed and the computation speed is high, Davidenko's method is used to determine accurate complex propagation constants for leaky circular symmetric modes in circular dielectric rod waveguides. Based on a precise determination of the complex propagation constants, the leaky mode characteristics of several lower-order circular symmetric modes are then numerically analyzed. In addition, no modification of the characteristic equation is required for the application of Davidenko's method.

Keywords: circular dielectric rod waveguide, circular symmetric modes, Davidenko's method, leaky mode.

1. Introduction

Leaky waves have recently received much attention, as they play a significant role in numerous antenna applications [1] and microwave/millimeter-wave integrated circuits [2], while also explaining a variety of physical phenomena, such as Wood's anomalies [3], Smith–Purcell Radiation [4], Čerenkov radiation [5], prism coupling [6], and so on. In particular, leaky waves in microwave/millimeter-wave integrated circuits have been extensively studied for over a decade, including the analysis of leakage effects and the discovery of their novel physical phenomena (see, for example, [2] and references therein). These two aspects of leaky mode studies strongly depend on the determination of the complex propagation constant, as the phase and attenuation constants involved in the complex propagation constant are two of the most important parameters revealing the properties of leaky waves. Accordingly, an accurate determination of the phase and attenuation constants is crucial prior to the analysis of leaky modes for specific guiding structures.

Since a complex characteristic equation cannot be solved analytically, traditional complex root search algorithms, such as the Newton-Raphson method or the Müller method [7], have been utilized to obtain the complex roots for a complex characteristic equation. However, these iterative methods require a careful selection of the initial starting point for iteration. If the initial values are not chosen properly, the iteration fails. As an alternative approach, Davidenko's method has been successfully applied to the dispersion analysis of guided waves, such as lossy moving waveguides [8], lossy nonlinear waveguides [9], lossy waveguides including gyrotropic media [10], surface plasmon polaritons [11], a cylindrical substrate–superstrate layered medium [12], surface-wave modes in microstrip antennas [13], and so on. Since Davidenko's method has the advantage of being more relaxed in regards to selection of the initial values, it is anticipated that it can also be applied to the analysis of leaky modes when using a complex propagation constant. Furthermore, the current authors have already reported a brief analysis of the leaky TM mode of a circular dielectric rod using Davidenko's method [14].

Accordingly, the current paper uses Davidenko's method to determine the zeros of a complex characteristic equation applicable to the analysis of leaky modes for various guiding structures. For example, accurate normalized phase and attenuation constants for the circular symmetric modes (TM_{0n} and TE_{0n} modes) of a circular dielectric rod waveguide are efficiently obtained when using Davidenko's method. In addition, the leaky mode characteristics of this structure, which are relatively unknown compared to those of microwave/millimeter-wave integrated circuits, are analyzed, and the difference in the leaky mode between the TM_{0n} and TE_{0n} modes are discussed based on a precise determination of the normalized phase and attenuation con-

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stants.

2. Review of Davidenko's Method

In principle, Davidenko's method transforms a set of n-coupled nonlinear algebraic equations with n unknowns into a set of n-coupled first-order ordinary differential equations with a scalar dummy variable. As the dummy variable approaches infinity, each unknown approaches a true value. It should be noted that Davidenko's method can only be applied to analytic functions. The following is a brief review of Davidenko's method. First, starting with the Newton-Raphson scheme, let $F(x)=0$ be a nonlinear algebraic equation where x is a root of this equation. In the Newton-Raphson method [7], the (n+1)th approximation of the root x of the equation $F(x)=0$ is as follows:

$$x_{n+1} = x_n - \frac{F(x_n)}{\dfrac{dF(x_n)}{dx}} \tag{1}$$

Equation (1) can be written as:

$$\frac{dF(x_n)}{dx} = -\frac{F(x_n)}{x_{n+1}-x_n} = -\frac{F(x_n)}{\Delta x_n} \tag{2}$$

where $\Delta x_n = x_{n+1} - x_n$ is the nth correction term between the (n+1)th and nth approximations. If $dF(x_n)/dx$ is too small, the nth correction term may diverge, meaning that the Newton-Raphson method will fail [10, 13]. This weak point in the Newton-Rhapson method is particularly serious when the value of the initial guess (x_0) is far from the root x of the given equation. Without loss of generality, if a small positive quantity factor, ξ ($0<\xi<1$) is included in (2), failure of the Newton-Raphson method can be avoided, and the modified form of (2) is as follows:

$$\frac{dF(x_n)}{dx} = -\frac{F(x_n)}{\Delta x_n}\xi \tag{3}$$

As such, even when the right hand side of (3) becomes small due to the small value of $dF(x_n)/dx$, the resultant small value on the right hand side can be mainly weighted to the factor ξ so that the correction term Δx_n does not have a large value. Consequently, the iteration will not fail, which is the essential feature of Davidenko's method. When taking the limit of both sides in (3) as $\xi \to 0$, the nth correction term Δx_n and factor ξ change into dx and dt, respectively. Thus, (3) becomes

$$\frac{dF(x)}{dx} = -\frac{F(x)}{dx}dt \tag{4}$$

where t is a scalar dummy variable independent of x. Rearranging and manipulating (4) as

$$\frac{dx}{dt} = -\frac{F(x)}{\dfrac{dF(x)}{dx}} = -\frac{dx}{d[\ln F(x)]} \tag{5}$$

then, equating the denominators of both sides in (5) as

$$dt = -d[\ln F(x)] \tag{6}$$

and integrating both sides of (6),

$$\int dt = -\int d[\ln F(x)] = -\ln F(x) + C_1 \tag{7}$$

$$\ln F(x) = -t + C_2 \tag{8}$$

where C_1 and C_2 are arbitrary integration constants, finally produces:

$$F(x) = Ce^{-t} \tag{9}$$

where C is also an arbitrary integration constant. Therefore, $F(x)=0$ is the independent scalar dummy variable when t approaches infinity, as mentioned previously.

Although the above procedure can also be applied to the general n-dimensional case [10], the current study focuses on the two-dimensional case of Davidenko's scheme, since the complex characteristic equation of leaky modes has two unknowns, *i.e.*, normalized phase and attenuation constants, which will be explained later. If the function $F(x)$ and its root x are considered as a complex analytic function and complex variable, *i.e.*, $F(x)=\mathrm{Re}\{F(x)\} + j\,\mathrm{Im}\{F(x)\}$ and $x = a + jb$, respectively, the first equality of (5) can be written as follows:

$$\frac{dx}{dt} = -J^{-1}F(x) \tag{10}$$

where J is a Jacobian matrix of the form

$$J = \begin{bmatrix} \mathrm{Re}\left\{\dfrac{\partial F(x)}{\partial a}\right\} & \mathrm{Re}\left\{\dfrac{\partial F(x)}{\partial b}\right\} \\ \mathrm{Im}\left\{\dfrac{\partial F(x)}{\partial a}\right\} & \mathrm{Im}\left\{\dfrac{\partial F(x)}{\partial b}\right\} \end{bmatrix} \tag{11}$$

Since the function $F(x)$ is assumed to be analytic, the total derivative of $F(x)$ with respect to x can be expressed using a Cauchy-Riemann relation as follows:

$$\begin{aligned} F_x(x) &\equiv \frac{\partial F(x)}{\partial x} = \mathrm{Re}\left\{\frac{\partial F(x)}{\partial a}\right\} + j\,\mathrm{Im}\left\{\frac{\partial F(x)}{\partial a}\right\} \\ &= \mathrm{Re}\left\{\frac{\partial F(x)}{\partial a}\right\} - j\,\mathrm{Re}\left\{\frac{\partial F(x))}{\partial b}\right\} \end{aligned} \tag{12}$$

Thus, the following relations are obtained:

$$\begin{cases} \mathrm{Re}\{F_x(x)\} = \mathrm{Re}\left\{\dfrac{\partial F(x)}{\partial a}\right\} \\ \mathrm{Im}\{F_x(x)\} = -\mathrm{Re}\left\{\dfrac{\partial F(x)}{\partial b}\right\} \end{cases} \tag{13}$$

Using (11), (12), and (13), the inverse form of the Jacobian matrix in (10) is

$$J^{-1} = -\frac{1}{\det J}\begin{bmatrix} \mathrm{Re}\{F_x(x)\} & -\mathrm{Im}\{F_x(x)\} \\ \mathrm{Im}\{F_x(x)\} & \mathrm{Re}\{F_x(x)\} \end{bmatrix} \tag{14}$$

with

$$\det J = \left[\mathrm{Re}\left\{\frac{\partial F(x)}{\partial a}\right\}\right]^2 + \left[\mathrm{Re}\left\{\frac{\partial F(x)}{\partial b}\right\}\right]^2 = |F_x(x)|^2 \tag{15}$$

Since the real and imaginary terms of the complex root x and complex function $F(x)$ can be expressed as the following column vectors, respectively:

$$x = \begin{bmatrix} a \\ b \end{bmatrix} \tag{16}$$

$$F(x) = \begin{bmatrix} \mathrm{Re}\{F(x)\} \\ \mathrm{Im}\{F(x)\} \end{bmatrix} \tag{17}$$

equation (10) can be expressed in a matrix form as follows:

$$\frac{d}{dt}\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{|F_x|^2}\begin{bmatrix} \mathrm{Re}\{F_x(x)\} & \mathrm{Im}\{F_x(x)\} \\ -\mathrm{Im}\{F_x(x)\} & \mathrm{Re}\{F_x(x)\} \end{bmatrix}\begin{bmatrix} \mathrm{Re}\{F(x)\} \\ \mathrm{Im}\{F(x)\} \end{bmatrix} \tag{18}$$

Finally, Davidenko's expression of a two-coupled first-order ordinary differential equation with an independent scalar variable t is obtained as follows:

$$\begin{cases} \dfrac{da}{dt} = -\dfrac{\mathrm{Re}\{F(x)\}\mathrm{Re}\{F_x(x)\} + \mathrm{Im}\{F(x)\}\mathrm{Im}\{F_x(x)\}}{|F_x(x)|^2} \\ \dfrac{db}{dt} = \dfrac{\mathrm{Re}\{F(x)\}\mathrm{Im}\{F_x(x)\} - \mathrm{Im}\{F(x)\}\mathrm{Re}\{F_x(x)\}}{|F_x(x)|^2} \end{cases} \tag{19}$$

The existence of the total derivative of $F(x)$ with respect to x, *i.e.*, $F_x(x)$ is a necessary condition for deriving Davidenko's expression, which means that the complex function $F(x)$ should be analytic. In other words, Davidenko's method can only be applied to an "analytic" characteristic function, as mentioned previously.

3. Application of Davidenko's Method to the Complex Characteristic Equation of a Circular Dielectric Rod Waveguide

Fig. 1 shows a cross-sectional view of the circular dielectric rod waveguide employed in the current work. Region 1 ($0 < \rho < r$) is a dielectric region where the magnetic and dielectric constants are μ_{r1} and ε_{r1}, respectively, whereas region 2 ($\rho > r$) is a free space region where the magnetic and dielectric constants are μ_{r2} and ε_{r2}, respectively. In the current study, the magnetic constant and the dielectric constant in region 2 are assumed to be in unity. The electric and magnetic axial field components of the circular symmetric modes (TM_{0n} and TE_{0n} modes) in each region can be expressed as follows:

$$\begin{cases} E_{z1} = A_{0n}J_0(k_1\rho)\exp[j(\omega t - \gamma z)] \\ H_{z1} = B_{0n}J_0(k_1\rho)\exp[j(\omega t - \gamma z)] \end{cases} \quad 0 < \rho < r \tag{20}$$

$$\begin{cases} E_{z2} = C_{0n}H_0^{(2)}(k_2\rho)\exp[j(\omega t - \gamma z)] \\ H_{z2} = D_{0n}H_0^{(2)}(k_2\rho)\exp[j(\omega t - \gamma z)] \end{cases} \quad \rho > r \tag{21}$$

where J_0 and $H_0^{(2)}$ are zero order Bessel and Hankel functions of the first and second kind, respectively, □ is the angular frequency, A_{0n}, B_{0n}, C_{0n}, and D_{0n} are complex amplitude constants, the subscript $0n$ denotes a circular symmetric mode, *i.e.*, TM_{0n} or TE_{0n} mode, and k_1 and k_2 are the complex transverse propagation constants in regions 1 and 2, respectively, and related to the complex axial propagation constant $\gamma(= \beta - j\alpha)$ as follows:

$$k_i^2 = k_0^2\mu_{ri}\varepsilon_{ri} - \gamma^2 \qquad (i = 1,\ 2) \tag{22}$$

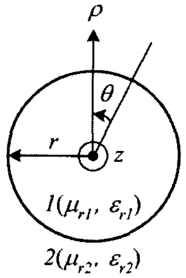

Fig. 1 Cross-sectional view of a circular dielectric rod waveguide.

Here, k_0 is the free space wave number. The normalized complex transverse and axial propagation constants can be respectively defined as follows:

$$\bar{k}_i = \frac{k_i}{k_0} = \frac{\text{Re}\{k_i\} + j\,\text{Im}\{k_i\}}{k_0} = \frac{\text{Re}\{k_i\}}{k_0} + j\frac{\text{Im}\{k_i\}}{k_0}$$
$$= \text{Re}\left\{\frac{k_i}{k_0}\right\} + j\,\text{Im}\left\{\frac{k_i}{k_0}\right\} = \text{Re}\{\bar{k}_i\} + j\,\text{Im}\{\bar{k}_i\} \quad (23)$$

$$\bar{\gamma} = \frac{\gamma}{k_0} = \frac{\beta - j\alpha}{k_0} = \frac{\beta}{k_0} - j\frac{\alpha}{k_0} = \bar{\beta} - j\bar{\alpha} \quad (24)$$

In (24), $\bar{\beta}$ and $\bar{\alpha}$ are the normalized phase and attenuation constants, respectively.

Substituting (23) and (24) into (22), produces the following relation [15]:

$$\begin{cases} \left(\text{Re}\{\bar{k}_i\}\right)^2 - \left(\text{Im}\{\bar{k}_i\}\right)^2 = \mu_{ri}\varepsilon_{ri} - \bar{\beta}^2 + \bar{\alpha}^2 \\ \text{Re}\{\bar{k}_i\}\text{Im}\{\bar{k}_i\} = \bar{\beta}\bar{\alpha} \end{cases} \quad (25)$$

For the TM_{0n} mode ($E_z \neq 0$ and $H_z = 0$), the constants B_{0n} and D_{0n} are chosen to be zero in (20) and (21). At the boundary $\rho = r$, the tangential field components in each region must be continuous, thereby resulting in a set of simultaneous equations with two unknowns as follows:

$$\begin{bmatrix} J_0(k_1 r) & -H_0^{(2)}(k_2 r) \\ \frac{\varepsilon_{r1}}{k_1} J_1(k_1 r) & -\frac{\varepsilon_{r2}}{k_2} H_1^{(2)}(k_2 r) \end{bmatrix} \begin{bmatrix} A_{0n} \\ C_{0n} \end{bmatrix} = 0 \quad (26)$$

The determinant of (26) must vanish to avoid a nontrivial solution, resulting in the following characteristic equation for the TM_{0n} mode:

$$Q(\bar{\gamma}) = \frac{\varepsilon_{r1}}{k_1}\frac{J_1(k_1 r)}{J_0(k_1 r)} - \frac{\varepsilon_{r2}}{k_2}\frac{H_1^{(2)}(k_2 r)}{H_0^{(2)}(k_2 r)} = 0 \quad (27)$$

Similarly, for the TE_{0n} mode ($E_z = 0$ and $H_z \neq 0$), the characteristic equation obtained is identical to that for the TM_{0n} mode, except for the material constants. Since the characteristic equation (27) cannot be solved analytically, Davidenko's method is used to obtain the normalized complex propagation constants satisfying (27).

By simply interchanging the complex root x in (16) and complex function $F(x)$ in (17) with the normalized complex propagation constant $\bar{\gamma}$ [1] in (24) and complex characteristic function $Q(\bar{\gamma})$ in (27), respectively, Davidenko's expression of the complex characteristic equation of this structure is obtained as follows:

$$\begin{cases} \frac{d\bar{\beta}}{dt} = -\frac{\text{Re}\{Q(\bar{\gamma})\}\text{Re}\{Q_{\bar{\gamma}}(\bar{\gamma})\} + \text{Im}\{Q(\bar{\gamma})\}\text{Im}\{Q_{\bar{\gamma}}(\bar{\gamma})\}}{|Q_{\bar{\gamma}}(\bar{\gamma})|^2} \\ \frac{d\bar{\alpha}}{dt} = \frac{\text{Re}\{Q(\bar{\gamma})\}\text{Im}\{Q_{\bar{\gamma}}(\bar{\gamma})\} - \text{Im}\{Q(\bar{\gamma})\}\text{Re}\{Q_{\bar{\gamma}}(\bar{\gamma})\}}{|Q_{\bar{\gamma}}(\bar{\gamma})|^2} \end{cases} \quad (28)$$

This is a two-coupled first-order ordinary differential equation with two unknowns, $\bar{\beta}$ and $\bar{\alpha}$, and an independent scalar dummy variable, t. In the current study, the set of equation (28) was implemented with the help of an internal function, "**NDSolve**" (based on the Runge–Kutta algorithm), from the software package MATHEMATICA 4.0 [16] and numerically solved, implying that the explicit analytic form of the complex characteristic function, $Q(\bar{\gamma})$ and its total derivative, $Q_{\bar{\gamma}}(\bar{\gamma})$ are not required, even when the analytic forms exist. The resulting normalized phase and attenuation constants are then substituted into the original characteristic function $Q(\bar{\gamma})$ in (27). The tolerances of the resulting values are checked by comparing them with the zeros for both the real and imaginary parts. For the procedure of solving (28), when t is set larger, the tolerance can have a smaller value, yet it takes longer to converge, therefore, the tolerance was arbitrarily set at 10^{-10} for both the real and imaginary parts. The normalized complex axial propagation constants are then utilized to obtain the normalized complex transverse propagation constants for each region using (22). The resulting complex axial and transverse propagation constants also satisfy the set of (25) and conditions for forward leaky waves, that is, $\bar{\beta} > 0$, $\bar{\alpha} > 0$, $\text{Re}\{\bar{k}_i\} > 0$, and $\text{Im}\{\bar{k}_i\} > 0$ [15].

[1]Here, for simplicity, the normalized complex axial propagation constant $\bar{\gamma}$ is assumed to be $\bar{\beta} + j\bar{\alpha}$ when deriving Davidenko's expression of (28). The resulting normalized attenuation constant for the leaky mode has a negative value, yet the normalized attenuation constant in (24) has a positive value. Therefore, a "–" sign needs to be included in the normalized attenuation constant obtained by Davidenko's expression of (28).

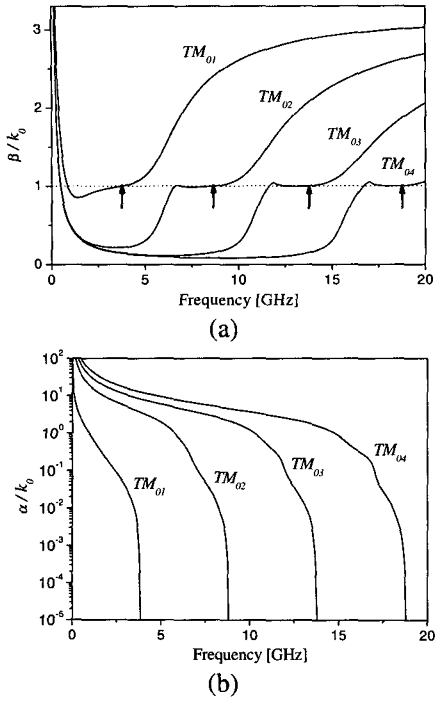

Fig. 2 (a) Normalized phase constant and (b) normalized attenuation constant of circular dielectric rod waveguide for four lower-order TM_{0n} modes. Arrows in (a) depict the cutoffs of the guided modes ($\varepsilon_{r1} = 10.0$ and $r = 10.0mm$).

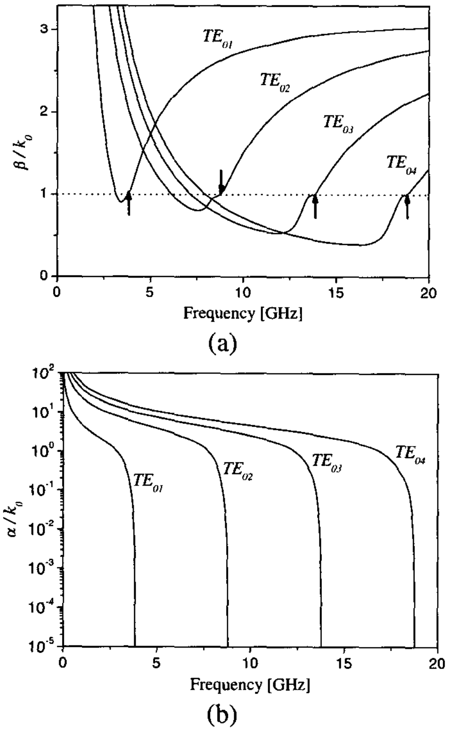

Fig. 3 (a) Normalized phase constant and (b) normalized attenuation constant of a circular dielectric rod waveguide for four lower-order TE_{0n} modes. Arrows in (a) depict the cutoffs of the guided modes. ($\varepsilon_{r1} = 10.0$ and $r = 10.0mm$)

4. Numerical Results and Leaky Mode Analysis of the Circular Dielectric Rod Waveguide

Figs. 2 and 3 show the normalized phase and attenuation constants of a circular dielectric rod waveguide for four lower-order TM_{0n} and TE_{0n} modes, respectively. The radius and dielectric constant of the rod were arbitrarily chosen to be 10.0mm and 10.0, respectively. As shown in Figs. 2 (a) and 3 (a), there were no crossing points among the normalized phase constants for the TM_{0n} mode, whereas there were several crossing points for the TE_{0n} mode. As regards the curves of the normalized attenuation constants in Figs. 2 (b) and 3 (b), there were no crossing points among the modes. The arrows shown in Figs. 2 (a) and 3 (a) depict the borders between the guided and leaky modes, *i.e.*, cutoff frequencies of the guided mode. In the guided mode region (above cutoff frequency), the normalized attenuation constants were zeros, since the dielectric material in the current study was assumed to have a real constant. As the frequencies decreased from the cutoff frequency for the guided mode, nonzero values were introduced for the normalized attenuation constants, implying the commencement points of the leaky mode. Here, the normalized attenuation constant was not derived from material loss but rather from leakage of the guided propagating power into the free space. The cutoff frequencies for the TM_{0n} and TE_{0n} modes were identical, i.e. 3.827, 8.785, 13.773, and 18.767 GHz for the first, second, third, and fourth modes, respectively. In the leaky mode region, several distinct modes were observed that exhibited their own unique properties. As shown in Figs. 2 (a) and 3 (a), as the frequency decreased from the cutoff frequency, the normalized phase constant decreased to a minimum point and then increased again until it exceeded unity and grew rapidly to infinity. Whereas, the normalized attenuation constant in Figs. 2 (b) and 3 (b) increased monotonically with a decrease in the frequency. In a lower frequency region, once a normalized phase constant exceeds unity, it becomes physically meaningless [17]. From Figs. 2 (a) and 3 (a), the upper limits of the nonphysical frequency regions of the TM_{0n} modes shifted toward lower frequencies as the modes became higher. The resulting nonphysical frequency regions were decreased, while those of the TE_{0n} modes were increased. More detailed numerical data are listed in Tables 1 and 2 for the TM_{0n} and TE_{0n} modes, respectively. In the frequency region higher than the nonphysical frequency region, a normalized phase constant below unity is physically meaningful. This frequency region is divided into two distinct regions; a reactive mode region ($\bar{\beta} < 1$ and $\bar{\beta} < \bar{\alpha}$), where the energy of the wave is stored as a form of reactive energy, and an antenna mode region ($\bar{\beta} < 1$

Table 1 Spectral ranges (and widths) of leaky TM_{0n} mode.

Mode	*Nonphysical mode*	*Reactive mode*	*1st Antenna mode*	*Spectral gap*	*2nd Antenna mode*	*Guided mode cutoff*
TM_{01}	0-0.850 GHz (0.850 GHz)	0.850-1.000 GHz (0.150 GHz)	Unavailable	Unavailable	1.000-3.827 GHz (2.827 GHz)	3.827 GHz
TM_{02}	0-0.486 GHz (0.486 GHz)	0.486-6.027 GHz (5.541 GHz)	6.027-6.590 GHz (0.563 GHz)	6.590-6.944 GHz (0.354 GHz)	6.944-8.785 GHz (1.841 GHz)	8.785 GHz
TM_{03}	0-0.481 GHz (0.481 GHz)	0.481-10.785 GHz (10.304 GHz)	10.785-11.636 GHz (0.851 GHz)	11.636-12.522 GHz (0.886 GHz)	12.522-13.773 GHz (1.251 GHz)	13.773 GHz
TM_{04}	0-0.480 GHz (0.480 GHz)	0.480-15.520 GHz (15.040 GHz)	15.520-16.698 GHz (1.178 GHz)	16.698-17.821 GHz (1.123 GHz)	17.821-18.767 GHz (0.946 GHz)	18.767 GHz

Table 2 Spectral ranges (and widths) of leaky TE_{0n} mode.

Mode	*Nonphysical mode*	*Reactive mode*	*Antenna mode*	*Guided mode cutoff*
TE_{01}	0-3.170GHz (3.170 GHz)	Unavailable	3.170-3.827 GHz (0.657 GHz)	3.827 GHz
TE_{02}	0-6.170 GHz (6.170 GHz)	6.170-7.896 GHz (1.726 GHz)	7.896-8.785 GHz (0.889 GHz)	8.785 GHz
TE_{03}	0-7.198 GHz (7.198 GHz)	7.198-12.789 GHz (5.591 GHz)	12.789-13.773 GHz (0.984 GHz)	13.773 GHz
TE_{04}	0-7.989 GHz (7.989 GHz)	7.989-17.601 GHz (9.612 GHz)	17.601-18.767 GHz (1.166 GHz)	18.767 GHz

and $\bar{\beta} > \bar{\alpha}$), where the energy of the guided wave is continuously leaked into the free space [18]. As listed in Tables 1 and 2, the spectral widths of both the reactive and antenna mode regions were increased as the modes became higher except for the TM_{01} antenna mode. It should be noted that there was no reactive mode for the TE_{01} mode, as the normalized phase constant always remained greater than the normalized attenuation constant for the entire frequency range considered. Descriptions of the TM_{01} antenna mode will be made available later.

Figs. 4 and 5 indicate enlarged scaled plots of the normalized phase constants for the TM_{0n} and TE_{0n} modes, respectively, near unity. The arrows depict the cutoff frequencies for the guided modes. As the frequency became higher, the normalized phase constants for the TM_{01} and TE_{0n} modes met the cutoff frequencies of the guided modes. Thus, it was found that the TE_{0n} mode had nonphysical, reactive, and antenna modes below the cutoff frequency of the guided mode, whose spectral widths increased as the mode became higher. In the case of the leaky TM_{0n} mode, except for the TM_{01} mode, the normalized phase constant exceeded unity again, as shown in Fig. 4, when the frequency was below the cutoff frequency of the guided TM_{0n} modes. The upper limit frequency of the TM_{01} antenna mode directly met the cutoff frequency of the guided TM_{01} mode. These narrow spectral ranges of the normalized phase constants that are greater than unity in the leaky TM_{0n} ($n > 1$) mode are identified as spectral gaps, where the normalized phase constant is physically meaningless [19], similar to the nonphysical frequency region.

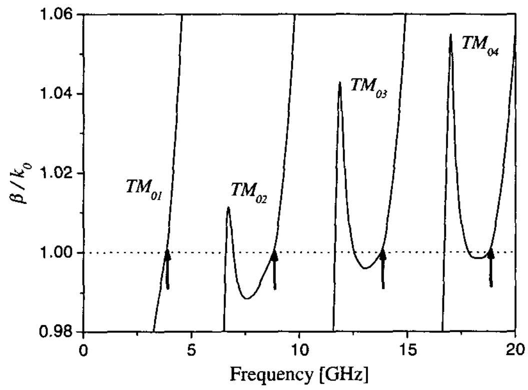

Fig. 4 Normalized phase constant of TM_{0n} mode with enlarged scale near unity. Arrows depict the cutoffs of the guided modes.

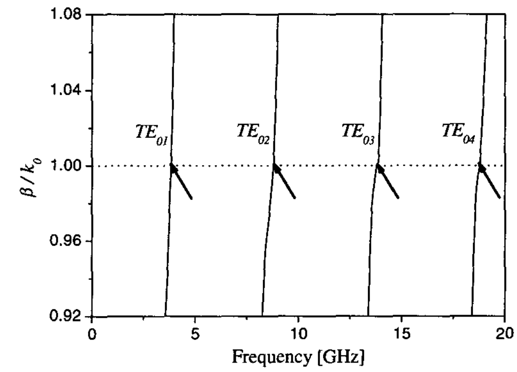

Fig. 5 Normalized phase constant of TE_{0n} mode with enlarged scale near unity. Arrows depict the cutoffs of the guided modes.

Plus, their spectral widths increased with the order of the mode, as seen in Table 1. However, the spectral gap region of this structure was inconsistent with the traditional transition region from a leaky ($\bar{\alpha} > 0$) to a guided ($\bar{\alpha} = 0$) mode, as it was found that the nonzero value of the normalized attenuation constants remained for the entire spectral gap region, as shown in Figs. 2 (b) and 4 [14]. In the frequency region above the spectral gap, there was a second antenna mode region, where the normalized phase constants became less than unity once again, as shown in Fig. 4. The spectral width of the second antenna mode decreased as the modes became higher. Since the previously observed spectral width of the TM_{01} antenna mode was broader than that of the TM_{0n} ($n > 1$) antenna mode and the second antenna mode exhibited a decreasing spectral width with the order of the mode, the antenna mode of the TM_{01} mode was categorized as a second antenna mode [14]. The upper limit of the second antenna mode region in the TM_{0n} ($n > 1$) mode met the cutoff frequency of the guided mode.

5. Conclusions

A numerical solution is often required for the complex characteristic equations involved in solving various electromagnetic wave problems. As an alternative approach that has the advantages of a more relaxed initial guess and high computation speed, Davidenko's method was used to derive the complex propagation constant of a circular dielectric rod. Several lower-order circular symmetric leaky modes in circular dielectric rod waveguides were then analyzed using the normalized phase and attenuation constants obtained with Davidenko's method. As a result, the leaky modes existing below the cutoff frequency of the guided mode were classified as nonphysical mode, reactive mode, antenna mode, and spectral gap. Accordingly, this successful application of Davidenko's method can contribute to the rigorous analysis of complex modes, including the guidance and leakage characteristics of circular guiding structures, such as dielectric tubes and Goubau lines, as well as planar-type guiding structures, such as an NRD guide.

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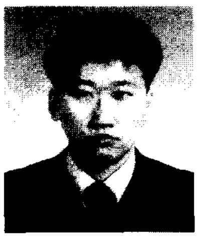

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