Substation Reliability Assessment Considering Non-Exponential Distributions And Restorative Actions

Gwang Won Kim* and Kwang Y. Lee**

Abstract - Reliability assessment of power systems has been an important topic for the past several decades. It is becoming even more important nowadays as the power market moves toward a new competitive environment. This paper deals with two topics on reliability assessment. The first is how to select probability distributions and determine their parameters to model the probabilistic events in a power system. The second is how to consider restorative actions in the assessment, which directly influence reliability indices. This paper proposes simple but convincing alternative solutions on the two topics. In the case study, this paper shows the influences of the probability distributions that are used in power system modeling.

Keywords: reliability assessment, substation, Monte Carlo simulation, fitness test, non-exponential distribution.

1. Introduction

The most crucial requirement of a power system is to supply quality electric energy economically to customers without interruption. However, non-interuption is virtually impossible due to the tremendous investment needed to maintain the highest level of reliability. This is the primary reason why a power system reliability assessment is necessary. Using the assessment, the most economical and effective investment levels can be determined. A vulnerable power system may cause enormous economical damage as was seen in the New York City blackout in 1977, with economical losses amounting to 350 million dollars [1].

The study of power system reliability assessment started in the mid 60s. At that time, the main study topic was building an accurate reliability model [2, 3]. Endrenyi introduced three different states - normal, fault, and switching state – to the modeling of equipment [4]. They evolved to a more precise model where a fault is subdivided into a passive and an active state [5]. Analytic methods have been mainly utilized all through assessment history owing to their computational simplicity. However, they have fatal limitations. Only exponential distribution was used in modeling, and it is also hard to consider complex situations such as restorative actions. Therefore, studies have been undertaken recently to overcome the difficulties using the Monte Carlo simulation [6, 7 & 8].

This paper proposes two things: one, a simple but convincing method for building accurate probability models for each event based on the operation data log; and two, a probable method for reflecting restorative actions in reliability assessments. Until now, the two topics have not been explored deliberately as they are considered less important compared to other topics, and results from statistics are considered as satisfactory. However, there is still room for improving assessment accuracy through the specific study of topics targeting power systems only.

In a case study, the influences of different probability models are shown using exponential and Weibull distributions in modeling repair times of equipment. The case study is performed using Monte Carlo simulation because it is the only method that allows various kinds of random variables as well as complex situations such as restorative actions and maintenances.

2. Probability Model

Since not only times between faults but also times for repair or maintenance are different from case to case, random variables (RVs) are needed to model such probabilistic processes. Until now, exponential distribution has been the most popular RV in modeling the probabilistic processes related to power system reliability assessment:

$$f(t) = \lambda e^{-\lambda t} \tag{1}$$

As for the exponential RV, its mean is $1/\lambda$ and λ is the failure rate (or repair rate) of the process. This constant failure rate (or repair rate) is an advantage of the exponen-

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tial RV because of the simplicity of calculation. This simplicity has made the use of the exponential RV accepted as a reliability assessment tool for the last several decades. Power engineers want to know reliability indices such as the loss of load expectation (LOLE) or the loss of load frequency (LOLF). Since such indices are only expectations or averages of the events concerned, the indices should be the same whatever RVs are utilized in the assessment as long as the means of the RVs are the same. Therefore, there has been no need to introduce complex RVs in the assessment. However, in some cases, especially in the new competitive environment, not only the reliability indices but also their distributions can be very useful information in a power system. For example, the probability that a blackout continues over one hour on a specific load point can be crucial reliability data. This kind of information depends on the shapes of RVs as well as their means. Therefore, there is a need for more elaborate RVs that can model the probabilistic processes accurately.

The shape of the exponential RV is very restricted because it has only one parameter that determines shape. RVs with two parameters can change their shapes with more flexibility. Gamma, Weibull, Normal, and Log-Normal RVs are representative of two parameter RVs, which are defined respectively by the following probability density functions:

$$f(t)=\frac{t^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}}\exp\left(-\frac{t}{\beta}\right),\ \text{if } t\geq 0 \quad (2)$$

$$f(t)=\alpha t^{\beta-1}\exp\left(-\frac{\alpha t^{\beta}}{\beta}\right),\ \text{if } t\geq 0 \quad (3)$$

$$f(t)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) \quad (4)$$

$$f(t)=\frac{1}{\sqrt{2\pi\sigma^2}t}\exp\left(-\frac{(\ln t-\mu)^2}{2\sigma^2}\right),\ \text{if } t\geq 0 \quad (5)$$

Many useful RVs such as Chi-square, Erlangian, Rayleigh, and even exponential RVs are derived from these RVs. These are good candidates for event modeling in reliability assessment.

The parameters of each RV need to be properly determined based on an operation data log. The method of momentum estimator (MME) is one method that can be used. Since the concerned RVs have two parameters, they can be uniquely determined using the mean and variance of data in a log in the sense of MME. The following four equations are used in this paper to calculate each parameter of the RVs, Gamma, Weibull, Normal, and Log-Normal RVs, respectively, where m and v are mean and variance of data in a log:

$$\alpha=\frac{m}{\beta},\quad \beta=\frac{v}{m} \quad (6)$$

$$\alpha=\beta\left(\frac{\Gamma(1/\beta+1)}{m}\right)^{\beta},\quad \frac{\Gamma(2/\beta+1)}{\Gamma(1/\beta+1)}=\frac{v}{m(m+1)} \quad (7)$$

$$\mu=m,\quad \sigma^2=v \quad (8)$$

$$\mu=\ln(m)-\frac{v}{2},\quad \sigma^2=\ln\left(\frac{v}{m(m+1)}\right) \quad (9)$$

In the case of Weibull RV, parameter β needs to be determined using a numerical calculation such as the Newton-Raphson method. And, only the positive region has physical meaning in the Normal RV.

After calculating each parameter, the best RV that describes a data log most accurately needs to be selected through a fitness test. The χ^2 in the well-known Chi-Square test can be a criterion of the fitness test,

$$\chi^2=\sum_{k=1}^{K}\frac{(N_k-m_k)^2}{m_k} \quad (10)$$

where K is the number of disjoint intervals, N_k is the observed number of outcomes, and m_k is the expected number.

To confirm usefulness of χ^2 in reliability assessment, 10,000 data points are prepared for test or sample data according to Weibull distribution with α=β=2 within the range of 0 to 5. Then, parameters of each RVs are calculated by (6) to (9) and tabulated in Table 1. Fig. 1 shows their distributions together with the sample data, where the size of the domain subinterval is 0.1. According to Fig. 1, Weibull distribution is the closest to the sample data. However, Chi-Square test claims, in Table 1, that Gamma distribution is the best. In addition, the χ^2 for the Normal distribution is too large while its shape differs little from the sample data. These unusual results are caused by m_k in the denominator of (10). If N_k is not zero and m_k is nearly zero at any subinterval, χ^2 becomes very large. It is not realistic that one data point influences a fitness result so much. Therefore, it is more meaningful to use the area between the two curves as a fitness test criterion. Table 1 also contains this area error for each RV, which agrees with Fig. 1.

Table 1 also verifies that the parameter calculation method of this paper given in (6) – (9) is accurate; Parameters of Weibull RV are 2.051 and 2.054, both are very close to 2, the original value for the sample data.

Table 1 Parameters and fitness test results

Item	Gamma	Weibull	Normal	Log-Nor.
$\alpha(\mu)$	3.725	2.051	0.884	-0.228
$\beta(\sigma^2)$	0.237	2.054	0.210	0.238
χ^2	741.8	5121.8	2.46×10^7	1.18×10^5
area-err	113.3	23.4	152.1	233.4

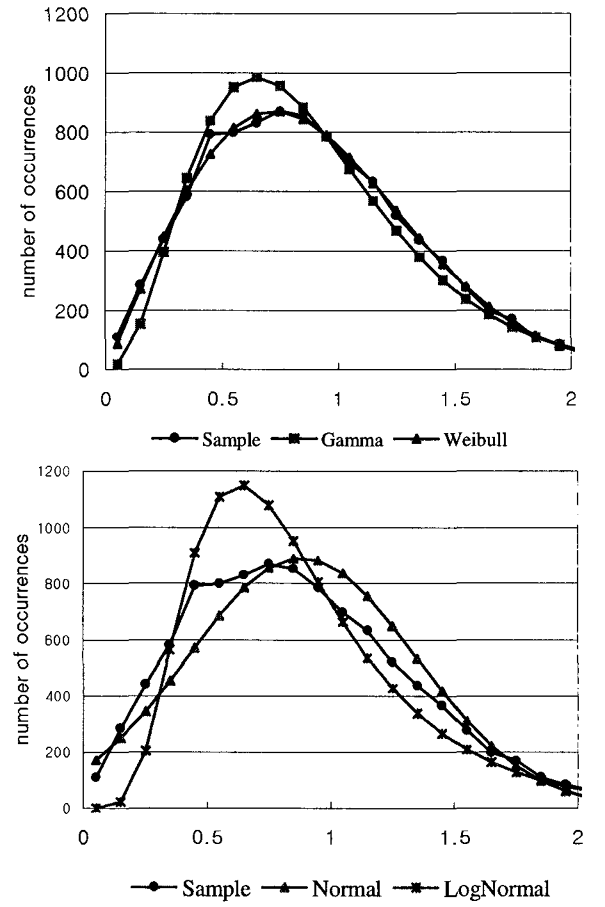

Fig. 1 Distributions of the sample data and its estimated models.

3. Reliability Assessment of a Substation

The reliability assessment of a substation is to examine the connectivity between load points (feeders) and source points (transmission lines). A substation is composed of much equipment. If any of this equipment fails, it can influence the connectivity. Any fault that brings about a blackout should be followed-up by appropriate restorative actions to recover the connectivity. A reliable substation not only contains reliable equipment but must also have reliable restorative strategies and appropriate redundancies to support the restoration. Therefore, a strong reliability assessment technique needs to also consider the restorative actions. In addition, a proper consideration of periodical maintenance also improves the accuracy of the assessment.

Until now, Monte Carlo simulation has been the only method to consider the restorative actions as well as various non-exponential distributions in modeling. The following is a list of items that may be considered in Monte Carlo simulation:

- passive and active failures
- maintenance
- restorative action
- various non-exponential random variables

There are three kinds of sampling methods developed for Monte Carlo simulation; state sampling, state duration sampling, and system state transition sampling. Among them, state duration sampling is the most appropriate to keep the advantages of Monte Carlo simulation: there are no restrictions in using various RVs, and reliability indices about frequency can be calculated easily.

Table 2 An example of a fault-blackout table

Faulted element	Blackout load point (LD_1, LD_2)	
	Before restoration	After restoration
T_1	0 1	1 1
T_2	1 1	1 1
T_3	1 0	1 1
B_1	0 1	1 1
B_2	1 0	1 1
...	...	...
B_1 and B_2	0 0	0 0
...	...	...

Since, in reliability assessment, the interest is to recognize whether blackout areas exist or not, the precise restorative schemes that are needed in automatic restoration are not necessary. Therefore, this paper proposes a fault-blackout table (FBT) to consider restoration actions easily in the state duration sampling. The proposed FBT contains only the relationship between faulted element(s) and blackout points before and after restoration. Table 2 is a part of FBT for a substation in Fig. 2.

Since the sample substation contains only two load points, the above FBT has two digits in the second and third columns, where 0 means blackout and 1 means non-blackout. According to the FBT, if a fault occurs at transformer 1 (T_1), load point 1 (LD_1) is in blackout after the fault and is restored after restorative actions. In case of a double fault at T_1 and T_2, the resultant blackout information is simply a result of bit-wise logical AND between the rows of T_1 and T_2. However, in the case of a double fault in circuit breakers 1 and 2 (B_1, B_2), the resultant blackout information is different from the result of the AND operation

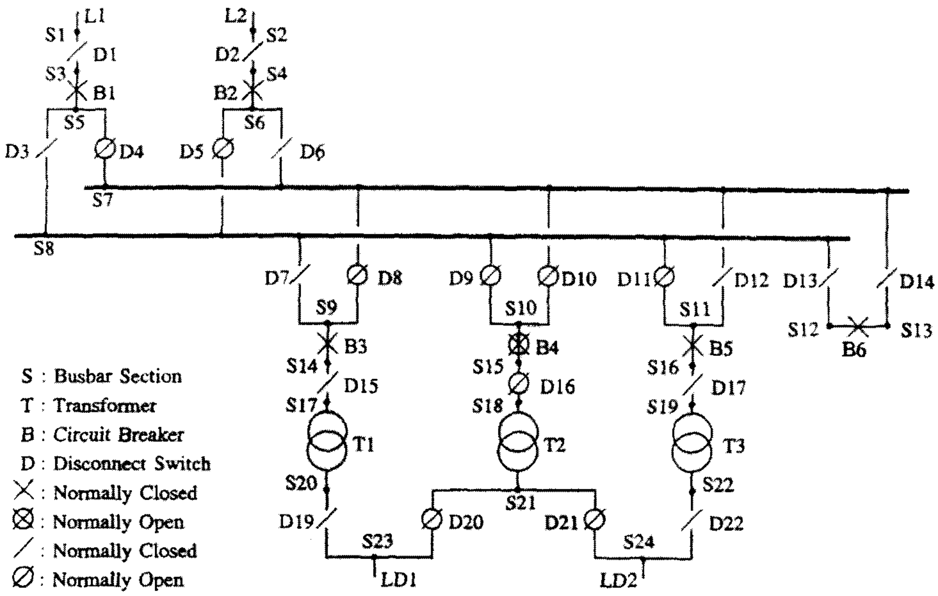

Fig. 2 One line diagram of Model A.

between the rows of B_1 and B_2. Therefore, FBT needs to contain separate information for such double faults as shown in Table 2. Consequently, the FBT contains the data corresponding to all single faults and some double faults whose effects cannot be obtained from single fault cases. In this paper, more than double simultaneous faults are not considered because they are very rare. Since the proposed FBT contains only essential information, it is very simple to implement. In addition, blackout points before restoration as well as after restoration can be determined directly from the proposed FBT.

4. Case Study

This case study has two objectives: one, to show the necessity of including restorative actions; and two, to show the effectiveness of using non-exponential RVs in the assessment.

4.1 System Description

Figs 2, 3, and 4 show the structure of substation models A, B, and C, respectively. Each model has two source points and two load points. The meaning of each symbol is shown in detail in Fig. 2.

Table 3 contains *a-priori* reliability data for each device, quoted from the reference [6] except variances of repair times, which were not used because only exponential RVs were used in that study. This case study considers the Weibull RV as well as the exponential RV in modeling repair times.

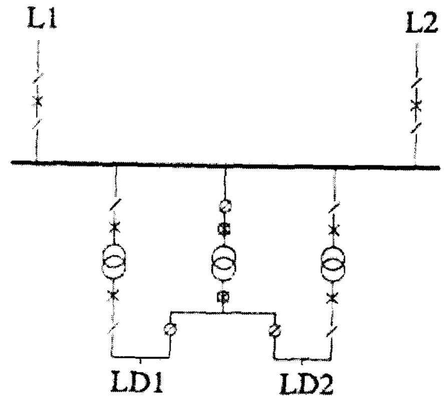

Fig. 3 One line diagram of Model B.

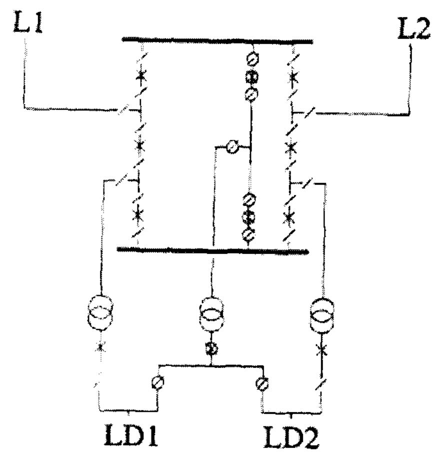

Fig. 4 One line diagram of Model C.

Table 3 Prior reliability information

Element	*a*	*b*	*c*	*d*	*e*
CB	0.010	0.01	1	12	6
Bus	0.025	-	2	25	12
Trans.	0.100	-	1	150	75
TR line	1.000	-	1	10	5

a: active failure rate [occ./yr]
b: passive failure rate [occ./yr]
c: switching time for restoration [hr]
d: mean time to repair [hr]
e: variance of repair time [hr]

In this case study, three different FBTs for each model are used to consider the effects of restorations. Maintenance is performed based on the following assumptions:

- On transformers, maintenance is performed for ten hours about once per every five years.
- On circuit breakers, maintenance is performed for five hours about once per every decade.

There will be no maintenance if any element in a substation is in fault or maintenance state.

In this case study, the loss of load expectation (LOLE) in hours per year, the loss of load frequency (LOLF) in occurrences per year, and the average outage duration per outage in hours are utilized as reliability indices.

4.2 Reliability Assessment Results

Since models A, B, and C are symmetrical for each load point, the reliability indices for LD_1 and LD_2 should be identical. Table 4 shows the reliability indices for LD_1 performed by Monte Carlo simulation for 200,000 years when all reliability indices converge satisfactorily.

Table 4 Reliability indices of Load 1.

Index	Exponential			Weibull		
	A	B	C	A	B	C
LOLF	0.164	0.195	0.139	0.165	0.196	0.138
LOLE	0.190	0.842	0.139	0.191	0.843	0.138
a	1.15	4.32	1.00	1.15	4.31	1.00

LOLF: loss of load frequency [occ./yr]
LOLE: loss of load expectation [hr./yr]
a: average outage duration per outage [hr/occ.]

In Table 4, the results are independent of RVs used because both RVs have the same means. The results, however, depend on model types. Model C is most reliable because it has the most redundancies, whose average outage duration per outage is exactly one hour. That means every blackout is recovered in one hour by restorative actions. The index without considering restorative actions would be much different from the current value, therefore it is crucial to consider restorative actions in the assessment.

Table 5 Probability density of the average outage duration per outage of Load 1.

Time (hour)	Exponential			Weibull		
	A	B	C	A	B	C
0 – 1	58.09	58.54	68.29	57.02	59.15	68.81
1 – 2	39.09	27.71	31.71	40.09	27.11	31.19
2 – 3	2.82	0.64	0.00	2.89	0.00	0.00
3 – 4	0.00	0.60	0.00	0.00	0.00	0.00
4 – 5	0.00	0.41	0.00	0.00	0.00	0.00
5 -	0.00	12.10	0.00	0.00	13.74	0.00

Monte Carlo simulation can offer not only the reliability indices but also their distributions as shown in Table 5. From Table 5, it can be recognized that the probability of a blackout over one hour is about 42% in model A. This can be crucial information for power companies, especially in a competitive environment.

If a blackout occurs, it is recovered by restorative actions by repair of faulted equipment. According to Table 3, switching times are much smaller than repair times. Therefore, it is not easy to examine the effect of Weibull RVs if an outage is recovered by restorative actions; Weibull RVs are used only in repair time modeling in this case study. The only exception is a bus fault in Model B. Since Model B has only one bus, it has no redundancy to replace the faulted bus. This is why only Model B has non-zeros after five hours in Table 5. Since the average bus repair time is 25 hours, there needs to be a graph whose time span is at least 30 hours, like Fig. 5, to examine the results of the different RV models. Fig. 5 shows the influences of different RVs; in case of a blackout caused by a bus fault in Model B, recovery probabilities are relatively high around 27 hours if repair time probabilities comply with the Weibull distribution.

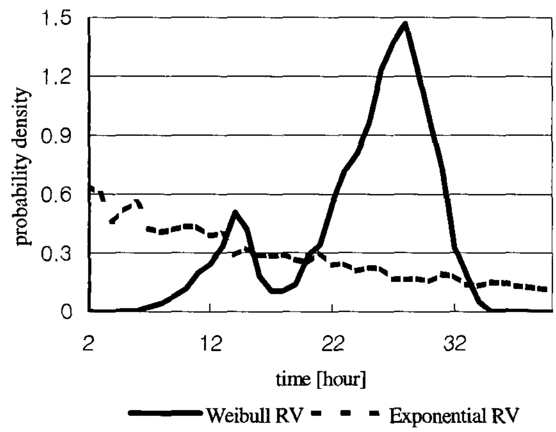

Fig. 5 Probability density comparison of average outage duration per outage in hours.

5. Conclusion

Since the popular exponential distribution has only one parameter, it is restricted in representing the probabilistic nature in a power system. On the other hand, probability distributions with two parameters such as Gamma, Weibull, Normal, and Log-Normal are much more flexible in representing their shapes. This paper proposes a systematic method for selecting appropriate probability distributions and their parameters to model the probabilistic nature in a power system for reliability assessment; first, determine parameters of each distribution based on mean and variance of a data log; and second, select the most appropriate distribution among the candidates through a fitness test with the area-error criterion.

This paper also proposes an efficient way of considering restorative actions in reliability assessment; the data tables that are utilized for this purpose are very simple to make and easy to use in a reliability assessment.

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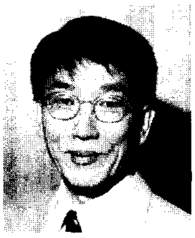

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