

# An Improved Secondary Path Modeling Method by Modified Kuo Model

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## Abstract

Kuo *et al* proposed an on-line method for an adaptive prediction error filter for improving secondary path modeling performance in the modeling method of the secondary path. This method have some disadvantages, namely having to use additive noise with the result that noise control performance is not good since it is focused on the estimated performance of the secondary path. In this paper, we proposes a modified Kuo model using gain control parameter and delay. It uses a reference signal for additive noise to improve the problems in the existing Kuo model.

*Keywords:* Adaptive signal processing, Active noise control (ANC), Secondary path, Kuo model

## 1. Introduction

As for noise control methods, there are passive noise controls and active noise controls. Passive noise control model means that noise is absorbed by sound-absorbing materials, and the path is excluded by a soundproof panel or a soundproof wall[1-5]. This is effective for controlling noise over a frequency of 500 Hz but ineffective if under a 500 Hz frequency. The problems in this method are ineffective utilization of space and high costs for equipment, as the system is huge in size and installation.

Active noise control (ANC) method is the method by which noise is canceled by superposing a signal of equal amplitude and opposite phase on the primary noise. This system efficiently attenuates low frequency noise where passive noise control methods are either ineffective or tend to be very expensive or bulky. Therefore, this active noise

control system is being used in this paper for the above-stated reasons[1-12].

The most popular adaptive algorithm used for ANC is the Filtered-X LMS algorithm. This algorithm is a modification of the well-known LMS algorithm. It is relatively fast and robust, and is low-operating quantity, compared to other algorithm. This algorithm model transfers performance of the secondary path in advance and is influenced by the secondary path through modeling since the algorithm uses pre-simulation data[3-5]. However, the algorithm contains problems due to modeling errors between the transfer performance of the secondary path and the modeled transfer performance; and further, the active noise control system becomes unstable. To solve these problems, a method for updating the transfer function value of the secondary path was proposed by using an on-line modeling method. Therefore, the Kuo model is the modeling method for the secondary path; it extracts the value of noise eliminated only from the primary path errors in error signals, by using a prediction

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error filter[3-7].

With this method, although the performance of the secondary path modeling was improved, noise control performance was not improved. There is also the problem of using additive noise.

In this paper, we will propose a secondary path modeling method with a modified Kuo model for improving noise control performance, while maintaining modeling performance at the maximum, without additive noise as in the Kuo model.

## II. Active Noise Control System Using Existing On-line Modeling

### 2.1. The Eriksson Model

Among the secondary path modeling methods using the on-line method, Eriksson proposed a secondary path modeling method using random additive-noise for the first time. The Eriksson model is shown in Fig. 1[3-5].

The random noise generator of Fig. 1 is uncorrelated with the primary noise  $d(n)$ , and generates zero-mean white noise. The additive noise  $v(n)$  is mixed with  $y(n)$  that was created by convolution of the reference signal and the adaptive filter  $w(z)$  in order to be used for input in the secondary path. Therefore, the residual error in the ANC system  $e(n)$  is shown as Eq. (1).

$$e(n) = d(n) + y(n)*s(n) - v(n)*s(n) \quad (1)$$

where  $*$  refers to convolution, the  $s(n)$  is the coefficient transfer function  $S(z)$  in time  $n$  of the secondary path. To rewrite, as  $y'(n) = y(n)*s(n)$ ,  $v'(n) = v(n)*s(n)$

$$e(n) = d(n) + y'(n) - v'(n) \quad (2)$$

If  $d(n) + y'(n)$  without additive noise is  $f(n)$ , also

$$e(n) = f(n) - v'(n) \quad (3)$$

The  $n$ th coefficient  $s'(n)$  is updated by the LMS algorithm as shown Eq. (4).

$$s'(n+1) = s'(n) - \mu_2 v(n) e_s(n) \quad (4)$$

where  $\mu_2$  is a step size in order to adapt the modeled secondary path, and the error signal  $e_s(n)$  that is to input to  $S(z)$  is Eq. (5).

$$e_s(n) = e(n) - u(n) = f(n) - v'(n) - u(n) \quad (5)$$

If  $f(n)$  in the residual error in the Eriksson model, explained above, becomes larger, the modeled secondary path  $S'(z)$  will not be modeled exactly[3,5]. Also, the larger random-additive noise signal  $v(n)$  is demanded for accurate convergence performance. Performance in the entire system has decreased because the residual error in the system is increased by  $v(n)$  with a big value[5-7].

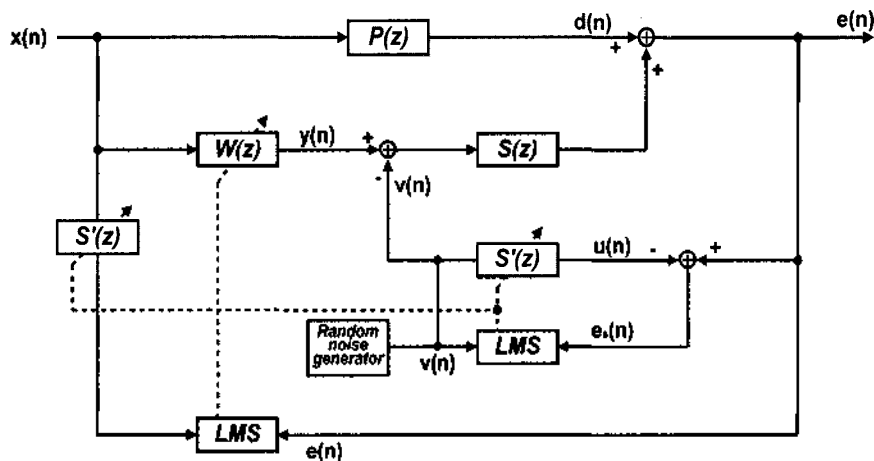


Figure 1. Eriksson model.

## 2.2. The Kuo Model

Kuo proposed a method that would improve the modeling performance of the secondary path by using a prediction error filter in the Eriksson model. This method decreases  $f(n)$  that affects the modeling performance of the secondary path[5,7]. The Kuo model is shown in Fig. 2. The adaptive prediction error filter  $B(z)$  in Fig. 2 is used for estimating errors  $d(n) + y(n) * s(n)$  included in residual error signals, and for reducing any errors. Assuming that the transfer performance  $S(z)$  of the secondary path can be modeled as a finite impulse response (FIR) filter for order  $M$ , from Fig. 2, the residual error signal measured by the control point can be expressed as

$$d(n) = f(n) + [y(n) + v(n)] * s(n) \quad (6)$$

To rewrite (6),  $d(n) + y(n) * s(n)$  can be replaced to  $f(n)$ , then

$$\begin{aligned} d(n) &= f(n) + v(n) * s(n) \\ &= f(n) + \sum_{m=0}^{M-1} s_m(n) v(n-m) \end{aligned} \quad (7)$$

where  $s(n)$  is the impulse response and  $s_m(n)$  is the  $m$ th impulse response of the secondary path  $S(z)$  at time  $n$ . The adaptive prediction error filter  $B(z)$  in Fig. 2 updates the coefficient value of the adaptive algorithm by using the reference signal  $e(n-\Delta)$  and error signal  $g(n)$ .

The error signal  $g(n)$  of the adaptive prediction error

filter is expressed as Eq. (8).

$$\begin{aligned} g(n) &= e(n) - e(n-\Delta) * b(n) \\ &= d(n) + y'(n) + v'(n) - [d(n-\Delta) + y'(n-\Delta) \\ &\quad + v'(n-\Delta)] * b(n) \end{aligned} \quad (8)$$

where  $b(n)$  denotes the  $n$ th coefficient value of  $t$ , the prediction error filter  $B(z)$ ,  $y'(n)$  indicates  $y(n) * s(n)$ , and  $v'(n)$  represents  $v(n) * s(n)$ . If  $d(n)$  is predictable, the undesired component  $f(n) = d(n) + y(n) * s(n)$  can be eliminated for  $\Delta > M$  after  $B(z)$  converges and thereby improves the convergent performance of the secondary path modeling.

Therefore, the coefficient vector  $b(n)$  can be updated as in Eq. (9),

$$b(n+1) = b(n) - \mu_2 e(n-\Delta) g(n) \quad (9)$$

where  $\mu_2$  is the step size for adapting the prediction error filter, and  $\Delta$  is the delay.

Assuming that the mean value of the additive error signal  $v(n)$  is zero, uniformly distributed white noise, which is not correlated with the interference  $f(n)$ , we can show that

$$\begin{aligned} E[e(n)e(n-\Delta)] &= E[f(n)f(n-\Delta)] \\ &+ \sum_{j=0}^{M-1} s_j(n) \sum_{j=0}^{M-1} s_j(n) \cdot E[v(n-j)v(n-i-\Delta)] \end{aligned} \quad (10)$$

where the first term is due to the interference  $f(n)$ , and

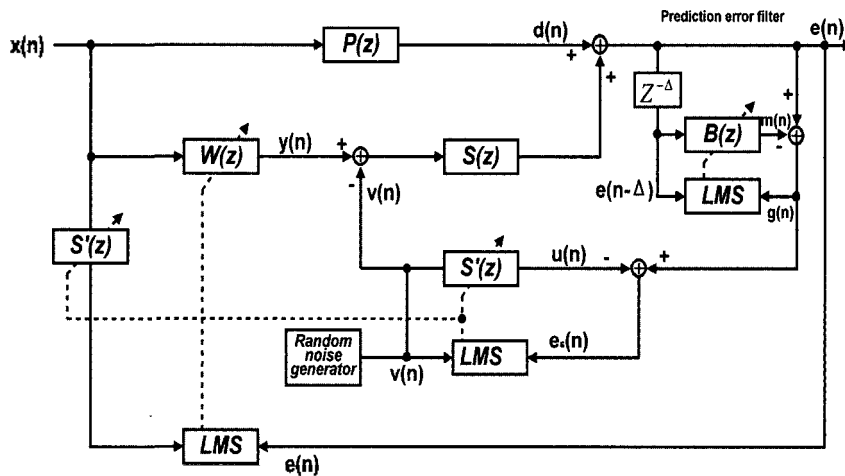


Figure 2. The ANC system with secondary path modeling using the prediction error filter (Kuo model).

the second term comes from the training signal  $u(n)$ .

If

$$\Delta > M \quad (11)$$

the expected value of the second term in Eq.(10) is

$$E[v(n-j)v(n-i-\Delta)] = 0, \text{ for } 0 \leq i, j < M \quad (12)$$

$$E[e(n)e(n-\Delta)] = E[f(n)f(n-\Delta)] \quad (13)$$

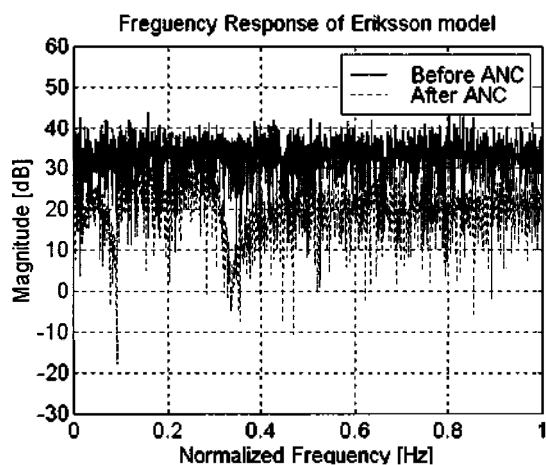
Therefore, the training signal component in  $e(n)$  and  $e(n-\Delta)$  will be uncorrelated for  $\Delta > M$ . As a consequence, the prediction filter  $B(z)$  will not be able to predict the training signal components in  $e(n)$ . The components of the interference  $f(n)$  that remain correlated after the delay  $\Delta$  will be predicted and canceled by the predictor  $B(z)$ . Therefore, the prediction error filter output can be approximated as follows[5-7].

$$g(n) \approx v(n) * s(n) \quad (14)$$

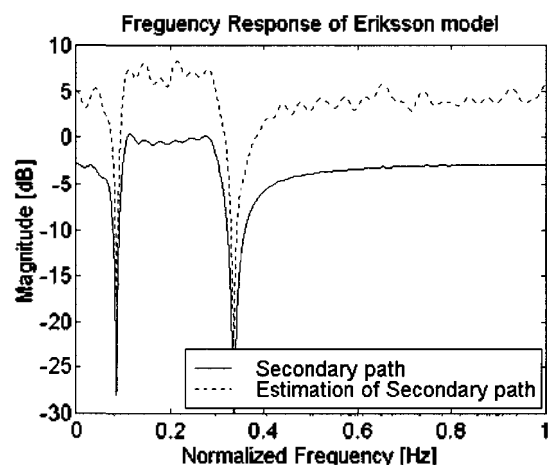
If the delay is  $\Delta < M$ , the second term in Eq. (10) is non-zero, it results in the cancellation of the training signal components by  $B(z)$ , thus affecting convergence of the modeling filter  $\hat{S}(z)$ .

### 2.3. Comparison with the Eriksson Model and the Kuo Model

We simulated the Eriksson model and the Kuo model



(a) Residual error in the Eriksson model



(b) Secondary path of the Eriksson model

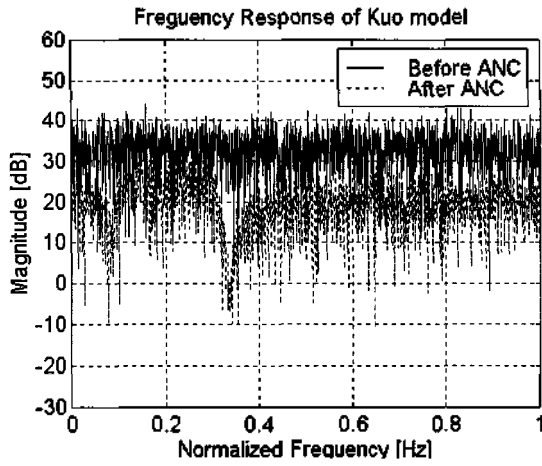
Figure 3. Frequency response of the Eriksson model and the Kuo model.

for comparison of performance. The simulation conditions were as follows. A reference signal was zero-mean white noise and average power of 1, and the adaptive filter was modeled as a FIR filter of order 128. The primary path, the secondary path and the modeled secondary path used the FIR filter of order 64. The prediction error filter was a FIR filter of order 25 and delay for the prediction error filter  $\Delta$  was set at 66 to satisfy Eq. (11).

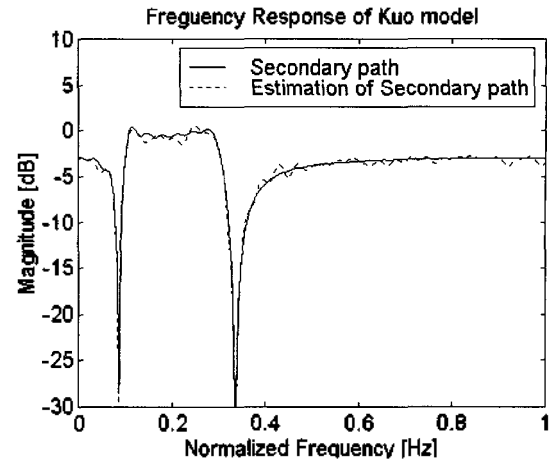
Fig. 3 shows the simulation results of noise control performance and the secondary path modeling performance of the system. Although noise control performance in the two models are almost the same, Kuo model is better than the Eriksson model in the secondary path modeling performance. It is suspected that the secondary path modeling performance has been improved by eliminating  $f(n)$ , included in the residual error by using the prediction error filter. Although the Kuo model improved the modeling performance of the secondary path well, it did not improve noise control performance very much. It has the problem whereby the system becomes larger because a random noise-additive generator should be used independently in order to eliminate noise[11,12].

### III. Modified Kuo Model

Although the Kuo model is improved in the convergence performance of the secondary path, the noise control performance of the system is not good. There is the



(c) Residual error in the Kuo model



(d) Secondary path of the Kuo model

Figure 3. Frequency response of the Eriksson model and the Kuo model. (Continue)

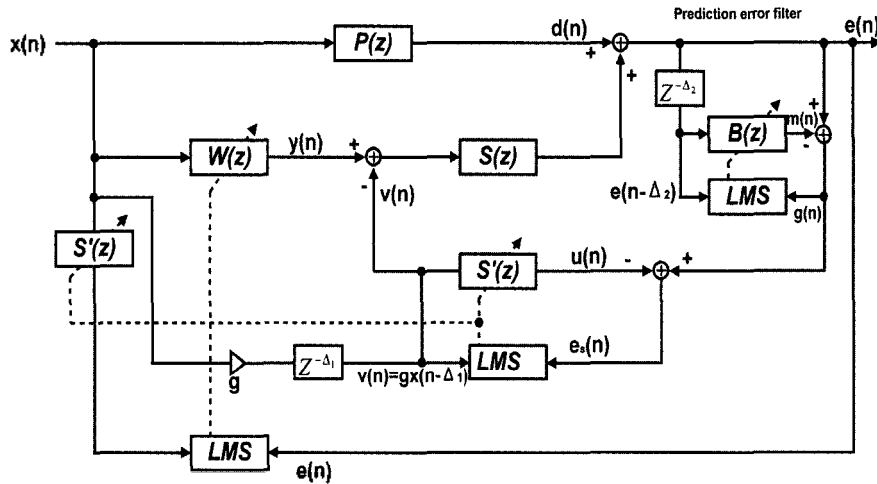


Figure 4. Modified Kuo model.

problem of having to use additive noise to eliminate noise.

Comparing the system in Fig. 4 with Fig. 2, an additive noise generator is used in the Kuo model; however, the reference signal is used as additive noise instead of using a additive noise generator in the proposed method, which is the primary difference between the two models. When a reference signal is used as additive noise, a new gain control parameter  $g$  is introduced to control the magnitude for improving the performance of the system. A function from the gain control parameter is added that is able to control the average power of the additive noise. We designed the generation of the additive noise that is uncorrelated with the reference signal using the delay value  $\Delta_1$ .

Therefore, additive noise  $v(n)$  is

$$v(n) = gx(n - \Delta_1) \quad (15)$$

where  $g$  is the parameter for controlling the magnitude of the reference signal, and  $\Delta_1$  is the delay value. The system would be bad if the additive noise had too large a value; therefore,  $g$  is used for generating an adaptable magnitude. The delay value  $\Delta_1$  is used in delaying the reference signal  $x(n)$  by  $\Delta_1$  in order to generate the signal uncorrelated with the signal of the adaptive filter and the random additive error signal  $v(n)$ .

The adaptive prediction error filter  $B(z)$  in Fig. 4 estimates the error signal component  $f(n)$  included in the

error signal as in the Kuo model and reduces effects of any errors.

Since the secondary path  $S(z)$  uses the FIR filter, order  $M$ , the measured error signal by error microphone is as follows;

$$\begin{aligned} e(n) &= d(n) + [y(n) + gx(n - \Delta_1)] * s(n) \\ &= d(n) + y(n) * s(n) + gx(n - \Delta_1) * s(n) \end{aligned} \quad (16)$$

where  $s(n)$  is the impulse response of the secondary path  $S(z)$  in time  $n$ .

To rewrite (16), it assumes that  $f(n)$  is  $d(n) + y(n) * s(n)$ , and  $v'(n)$  is  $gx(n - \Delta_1) * s(n)$ .

Thus

$$e(n) = f(n) + v'(n) \quad (17)$$

The prediction error filter output  $g(n)$  in the range of  $\Delta_1 \geq M$  is expressed as (18).

$$g(n) \approx v(n) * s(n) \quad (18)$$

where  $v(n) = gx(n - \Delta_1)$ .

The coefficient  $\hat{s}(n)$  of  $\hat{S}(z)$  updates the adaptive weights according to

$$\hat{s}(n+1) = \hat{s}(n) - \mu_3 v(n) e_s(n) \quad (19)$$

where  $\mu_3$  is the adaptation step size of the secondary path filter,  $e_s(n)$  is the error signal inserted into the adaptive filter.

## IV. Computer Simulations

Some computer simulations were conducted in order to evaluate performance of the proposed method. White noise was used as a reference signal of which the mean was zero and the average power was 1. The adaptive filter was modeled as a FIR filter of order 128. The primary path, the secondary path, and modeled secondary path used the FIR filter of order 64, and the prediction error filter was a FIR filter of order 25. Since three adaptive algorithms

were used, each step size was set as follows; the adaptive filter ( $W(z)$ ) was 0.006, modeled secondary path ( $S(z)$ ) was 0.002, and the error-prediction filter ( $B(z)$ ) was 0.0005, respectively. The delay for the prediction error filter ( $\Delta_2$ ) was set at 66.

Added noise in the proposed model generated the signal that was uncorrelated with the reference signal by delaying the value that could be obtained by multiplying the reference signal by the parameter  $g$  value.

Hence, estimation performance for secondary path and noise control performance was affected by the gain control parameter  $g$  value and the delay value  $\Delta_1$ . It is therefore important to obtain precisely the range and delay value of the gain control parameter  $g$  value. Residual error according to variation of the  $g$  value is/are shown in Fig. 5 for obtaining the range for system convergence. The magnitude of residual error is shown from results in the system by varying the process of repetition from 1 to 3000 in Fig. 3, using computer simulation.

Fig. 5 (a) shows the residual error from the system when the  $g$  value is set to 0.8, and (b) is when the  $g$  value is set to 1. Also, (c) is when  $g$  value is set to 1.05, and (d) is when  $g$  value is set to 1.1. Residual error show getting converge when  $g < 1$ , as shown in Fig. 5 (a) and (b). On the other hand, the residual error is diverged at 2200 when  $g > 1$ ; that is, 1.05 and 1.1. Hence, the  $g$  value should be set at less than 1, because residual error in the system diverge and cause the system to become unstable when the gain control parameter  $g$  value is more than 1.

$g$  value would be set as Eq. (20) to converge the system.

$$0 < g \leq 1 \quad (20)$$

However, Eq. (20) is the gain control parameter  $g$  value in the convergence range of residual error performance. The noise control performance and the secondary path estimation performance of the system were evaluated in the frequency domain in order to set the range of the  $g$  value with an accurate estimation. This was accomplished by decreasing the gain control parameter  $g$  value to less than 1 at a fixed rate. Fig. 6 shows the simulation results for determining optimum  $g$ . To set a range of  $g$ , we

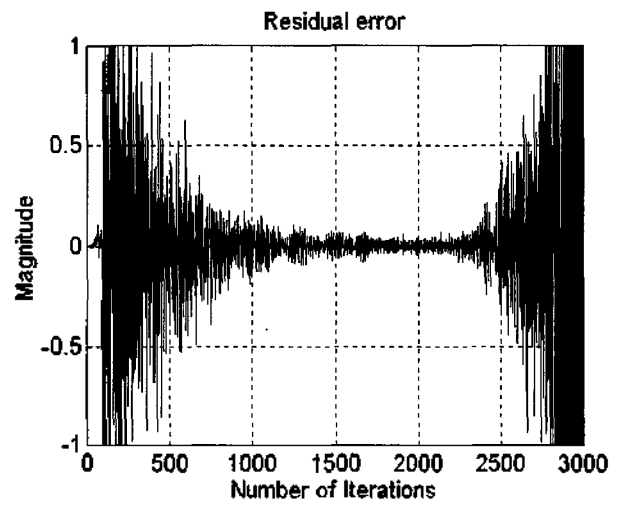
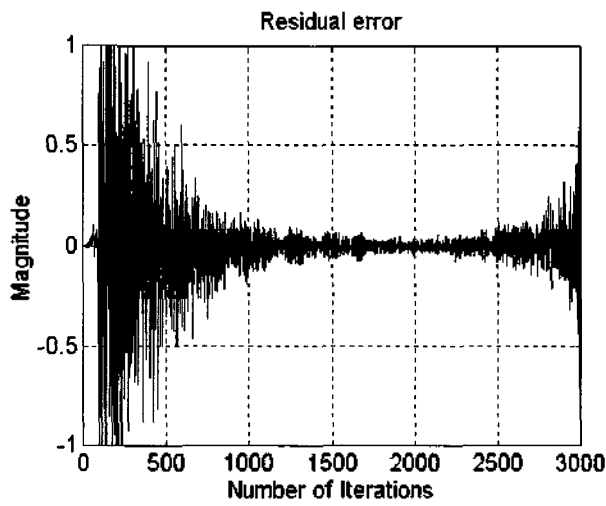
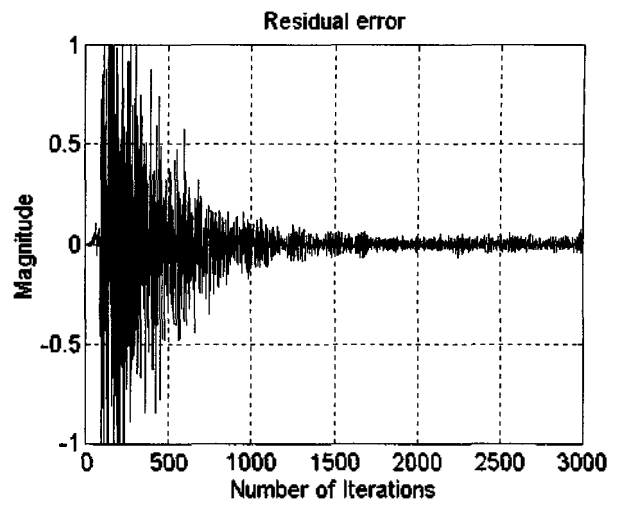
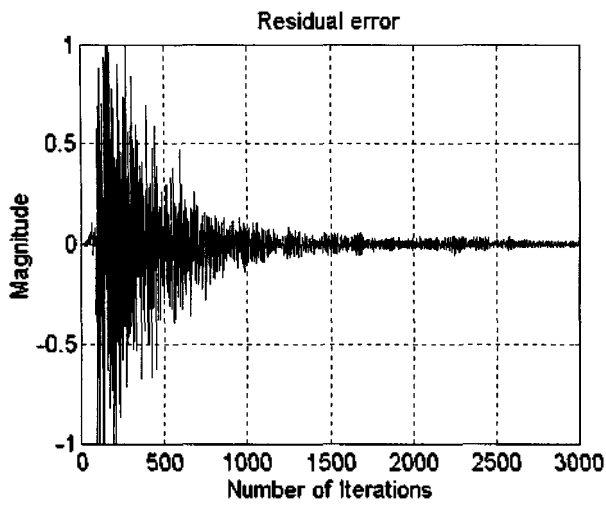


Figure 5. Residual error with  $g$ .

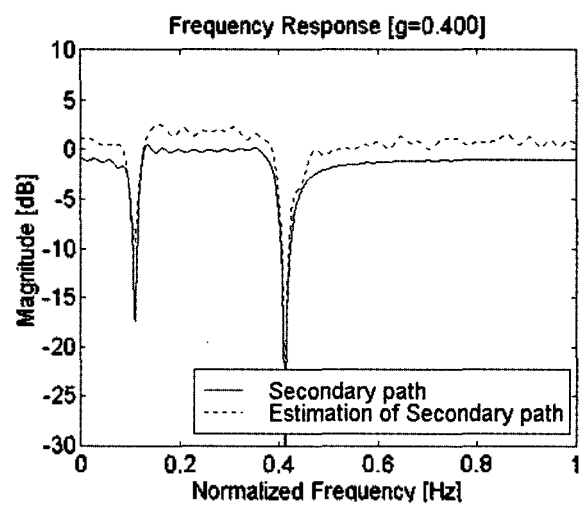
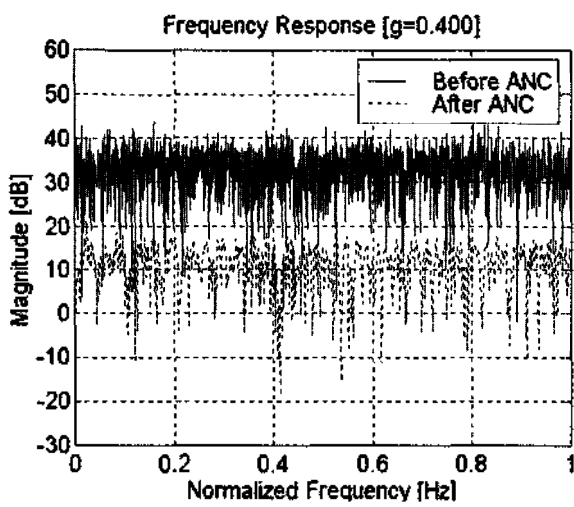


Figure 6. The frequency response of the proposed model.

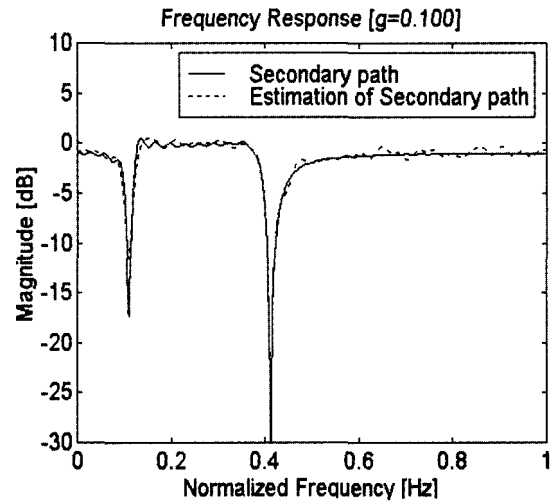
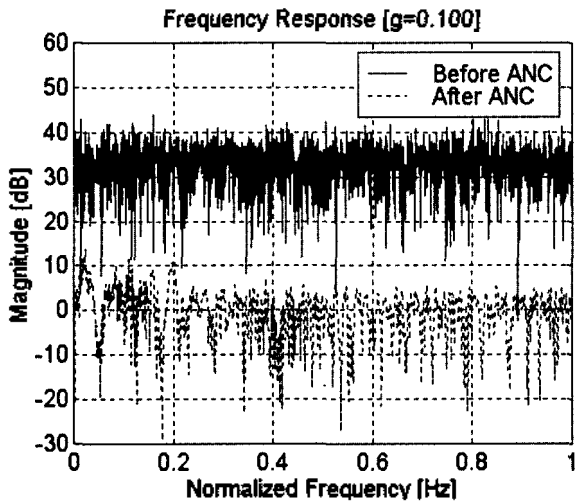
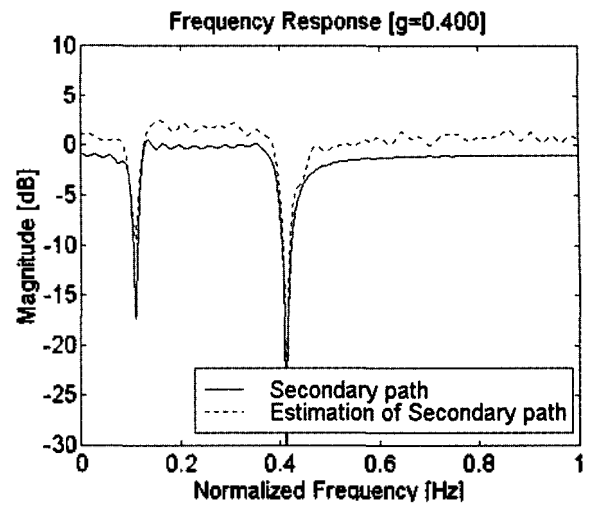
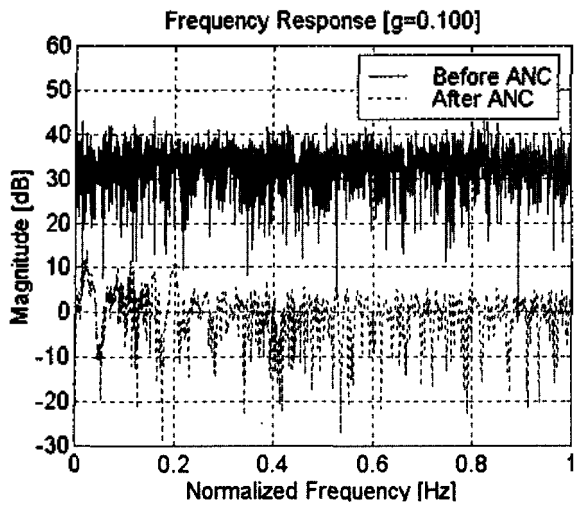


Figure 6. The frequency response of the proposed model. (Continue)

simulated noise control performance and modeling performance of the secondary path at regular intervals decreasing data from 1.

Fig. 6 shows the noise control performance and modeling performance in the secondary path, respectively. It is the result when  $g$  is 0.4, 0.1, and 0.03, respectively. Noise control performance and the secondary path modeling performance are improved as  $g$  decreases. The secondary path modeling performance corresponds to the value of the secondary path in the actual environment, in case  $g$  is smaller than 0.1.

From the results, we could determine that the secondary path modeling performance of the proposed model corresponds to the Kuo model when  $g \leq 0.1$ . Therefore,  $g$  should be set in the range of  $g \leq 0.1$  for precise secondary

path modeling performance.

We varied the delay value  $\Delta_1$  from 1 to 300 for setting the delay value  $\Delta_1$  in the proposed model. The difference between the modeled secondary path and the secondary path in the actual environment is shown in the frequency axes in Fig. 7.

We chose 92 for the delay value  $\Delta_1$  since the modeling error is minimum at 92 in Fig. 7.

Fig. 8 shows the estimated frequency response in the noise control performance and the secondary path modeling performance of the Kuo model and the proposed model. The simulation conditions below are the same as the above-stated simulation conditions. Since additive noise is used in the Kuo model, the magnitude and characteristics of additive noise used in the proposed



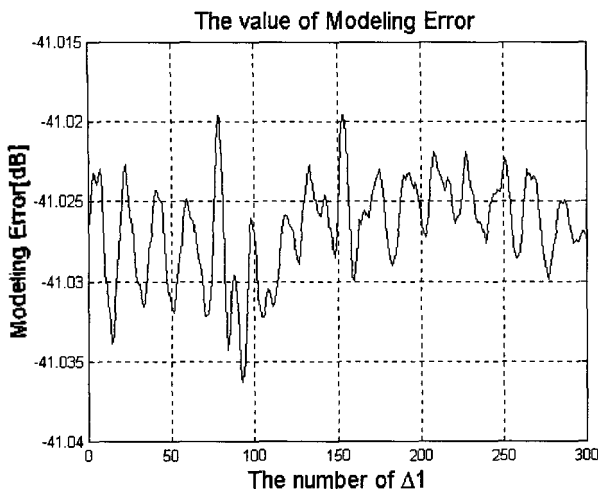


Figure 7. Modeling error with  $\Delta_1$ .

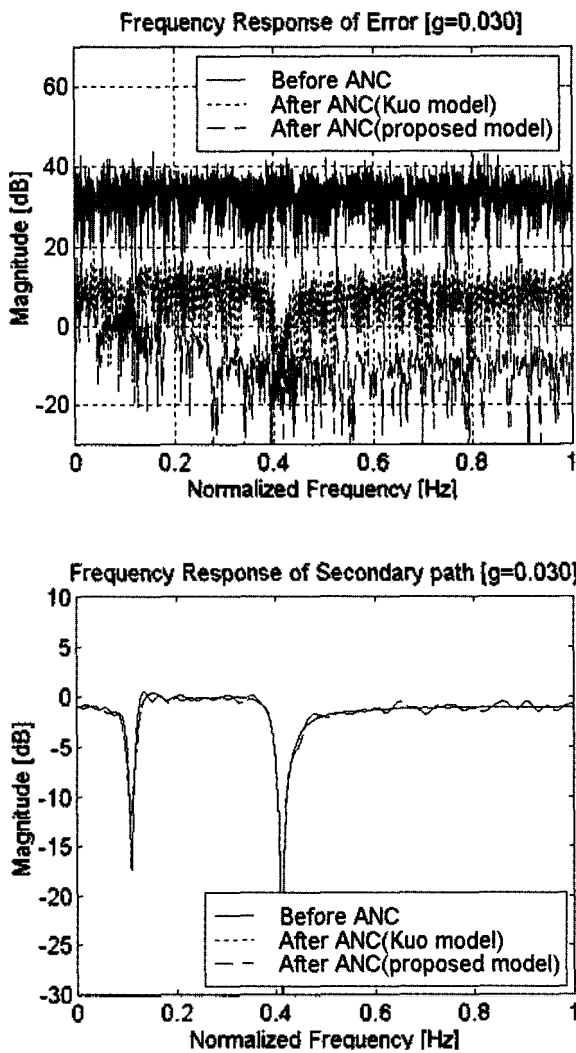


Figure 8. Frequency response of the Kuo model and the proposed model.

model are different from those in the Kuo model.

Therefore, white noise of which the mean is zero was used as a reference signal in order to make the magnitude of the two signals similar. Then, we conducted simulation using signals of the same magnitude. Simulation results showed that modeling performance of the secondary path of the proposed model corresponded to the modeling performance of the secondary path of the Kuo model, and the proposed model was improved by about 20dB in noise control performance.

## V. Conclusion

The Kuo model has improved the secondary path modeling performance by using prediction error filters on-line in the secondary path modeling methods. However, the Kuo model showed poor noise control performance in the system, and additive noise has to be used in order to eliminate noise.

In this paper, we have proposed a secondary path modeling method through a modified active noise control system using a reference signal as additive noise for improving problems of the existing Kuo model. The gain control parameter and delay for optimal performance of the proposed system were 0.03 and 92, respectively. According to the simulation results, the estimated performance of the secondary path was the same as that of the Kuo model, and noise control performance was improved by 20 dB, compared to that of the Kuo model. In the proposed model system was composed simply since it did not need to use an additive noise generator.

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### [Profile]

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