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Formulation of New Hyperbolic Time-shift Covariant Time-frequency Symbols and Its Applications

Byeong-Gwan Iem*
*Department of Control and Instrumentation Engineering Kangnung National University
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Abstract

We propose new time-frequency (TF) tools for analyzing linear time-varying (LTV) systems and nonstationary random processes showing hyperbolic TF structure. Obtained through hyperbolic warping the narrowband Weyl symbol (WS) and spreading function (SF) in frequency, the new TF tools are useful for analyzing LTV systems and random processes characterized by hyperbolic time shifts. This new TF symbol, called the hyperbolic WS, satisfies the hyperbolic time-shift covariance and scale covariance properties, and is useful in wideband signal analysis. Using the new, hyperbolic time-shift covariant WS and 2-D TF kernels, we provide a formulation for the hyperbolic time-shift covariant TF symbols, which are 2-D smoothed versions of the hyperbolic WS. We also propose a new interpretation of linear signal transformations as weighted superposition of hyperbolic time shifted and scale changed versions of the signal. Application examples in signal analysis and detection demonstrate the advantages of our new results.

Keywords: *Time-frequency symbol, Weyl symbol, Hyperbolic time-shift covariance, Spreading function, Quadratic time-frequency representations*

I. Introduction

Quadratic time-frequency representations (QTFR) have been used for analyzing deterministic signals showing time-varying characteristics[1,2]. Some prominent examples are the Wigner distribution, the Spectrogram, Choi-Williams distribution and so on[1,2]. These QTFRs are appropriate analysis tools for signals with linear time- frequency distributions. While the QTFRs are useful in analyzing and processing deterministic signal, time-f requency (TF) symbols have been widely used in the analysis of linear time-varying (LTV) systems and non-stationary random processes[3-5]. Some prominent examples in TF symbols are the Weyl symbol (WS), the Levin symbol, the Page symbol and so on[3,5,6]. These TF symbols are proper tools for analyzing LTV systems and non-stationary random processes with linear TF characteristics[3-6]. Since in nature there may exist random processes/systems showing various (non-linear) characteristics in the TF plane, to analyze such processes/systems, we need different TF tools which match non-linear structures of the processes/systems. Thus, there have been abundant TF symbols like TF shift covariant symbols, time-shift and scale covariant symbols, exponential frequency shift covari

Corresponding author: Byeong-Gwan Iem (ibg@kangnung.ac.kr)
Department of Control and Instrumentation Engineering Kangnung National University, 210-702, 120 Gangnenug Daehangno, Gangneung, Gangwon Province, Korea

ant symbols, exponential time-shift covariant symbols and so on[6,7]. Here, "covariance" means whatever happens in a process happens in a TF symbol. Based on common covariance properties satisfied by TF symbols, classes of TF symbols have been formulated[6]. That is, TF symbol members in a certain class satisfy common properties which can be utilized for analyzing random processes and linear systems showing specific TF characteristics. For example, the conventional narrowband Weyl symbol, the Kohn-Nirenberg symbol, the Levin symbol, the Page symbol, and the α-generalized Weyl symbol in the TF shift covariant class satisfy the constant TF shift covariance property, and they are proper analysis tools for random processes and LTV systems with linear TF distribution[6,7].

In this paper, we propose a new class of TF symbols satisfying hyperbolic time-shift covariance and scale covariance properties. We also study its relation with existing TF symbols. The structure of this paper is as follows. In Section 2, we provide the definitions and properties of narrowband and wideband Weyl symbols. In chapter 3, we propose the new hyperbolic time-shift covariant symbol class, its examples and properties, and its relation to the hyperbolic frequency-shift TF symbol class. In Section 4, we provide application examples in analysis and detection problems to show the benefit of the new hyperbolic time-shift covariant TF symbol. In Section 5, we conclude this paper.

II. Conventional Time-frequency Symbols

2.1. Narrowband Weyl Correspondence

The conventional narrowband WS and its 2-dimensional (2-D) Fourier transform, the spreading function (SF), are defined, respectively, as[3]

$$WS_{\mathcal{L}}(t,f) = \int_{-\infty}^{\infty} K_{\mathcal{L}}(t+\frac{\tau}{2}, t-\frac{\tau}{2})\, e^{-j2\pi\tau f} d\tau \quad (1)$$

$$= \int_{-\infty}^{\infty} \Gamma_{\mathcal{F}\mathcal{L}\mathcal{F}^{-1}}(f+\frac{\nu}{2}, f-\frac{\nu}{2})\, e^{j2\pi t\nu} d\nu \quad (2)$$

$$SF_{\mathcal{L}}(\tau,\nu) = \int_{-\infty}^{\infty} K_{\mathcal{L}}(t+\frac{\tau}{2}, t-\frac{\tau}{2})\, e^{-j2\pi t\nu} dt \quad (3)$$

for an operator $\mathcal{L}$ on $L_2(\mathbf{R})$ with the operator kernel $K_{\mathcal{L}}(t, \tau)$ [8]. With $\mathcal{F}\mathcal{L}\mathcal{F}^{-1}$, we obtain the unitarily equivalent frequency domain operator of $\mathcal{L}$ and its bi-frequency kernel $\Gamma_{\mathcal{F}\mathcal{L}\mathcal{F}^{-1}}(f, \nu)$. When the operator $\mathcal{L}$ and $\mathcal{F}\mathcal{L}\mathcal{F}^{-1}$ are applied to a signal, they can be written as $(\mathcal{L}x)(t) = \int K_{\mathcal{L}}(t, \tau)\, x(\tau)\, d\tau$ and $(\mathcal{F}\mathcal{L}\mathcal{F}^{-1} X)(f) = \int \Gamma_{\mathcal{F}\mathcal{L}\mathcal{F}^{-1}}(f, \nu) X(\nu)\, d\nu$, respectively. Here, $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the Fourier and inverse Fourier transform operators, respectively. When the operator $\mathcal{L}$ is the autocorrelation operator R_X of a non-stationary random process x(t) with the kernel $K_{R_X}(t, \tau) = E[x(t)x^*(\tau)]$ and the expectation operator $E[\cdot]$, the WS can be interpreted as the time-varying spectrum of the process. It is an analogy of the Wiener-Khinchine theorem in the stationary case. Also, when the operator $\mathcal{L}$ and its kernel $K_{\mathcal{L}}(t, \tau)$ are an LTV system and its time-varying impulse response, the WS can be considered as the transfer function of the LTV system. It is an extended concept of the conventional relationship between the impulse response and the transfer function of a linear time-invariant (LTI) system. The mapping between the operator $\mathcal{L}$ and the Weyl symbol is called the Weyl correspondence[3,5].

The 1-D inner product of the operator input x(t) and output $(\mathcal{L}\, x)(t)$ can be expressed as the 2-D inner product of the Wigner distribution (WD)[1,2] of the operator input and the WS of the operator,

$$\int_{-\infty}^{\infty} (\mathcal{L}\, x)(t) x^*(t) dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} WS_{\mathcal{L}}(t,f)\; WD_X(t,f)\, dt\, df \quad (4)$$

and the relationship in (4) is called the quadratic form of x(t)[5]. Here, $WD_X(t,f) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2}) x^*(t-\frac{\tau}{2}) e^{-j2\pi\tau f} d\tau$ is the Wigner distribution[1,2] of the process x(t). The quadratic form provides a definition of a TF concentration measure[5], and is useful in TF detection[9,10] and analysis[11] applications. In the sense of the quadratic form (4), we say that the WS is *associated* with the WD.

The WS in (1) and (2) preserves constant time shifts, constants frequency shifts, and scale changes on a random process[3,4]

$$Y(f)=X(f)e^{-j2\pi\tau f} \Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(t-\tau,f) \quad (5)$$

$$Y(f)=X(f-\nu) \Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(t,f-\nu) \quad (6)$$

$$Y(f)=\frac{1}{\sqrt{|a|}}X(\frac{f}{a}) \Rightarrow WS_{R_Y}(t,f)=WS_{R_X}(at,\frac{f}{a}) \quad (7)$$

where $\mathbf{R}_X$ and $\mathbf{R}_Y$ are the autocorrelation operators of X(f) and Y(f), respectively, with the bi-frequency operator kernel $\Gamma_{R_X}(f,\nu)=E[X(f)X^*(\nu)]$. The WS also satisfies the unitarity property defined as

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} WS_{\mathcal{L}}(t,f)\, WS_{\mathcal{V}}(t,f)\,dt\,df=\sum_{m,n}\varepsilon_m\gamma_n\left|\int h_n(t)\,g_m(t)\,dt\right|^2 \quad (8)$$

where ε_m and $g_m(t)$ are eigenvalues and eigenfunctions, respectively, of the kernel of the operator $\mathcal{L}$, and γ_n and $h_n(t)$ are similarly defined for the operator $\mathcal{V}$ on $L_2(\mathbf{R})$. Using (1) and the kernel expansion $K_{\mathcal{L}}(t,\tau)=\sum_n \varepsilon_n g_n(t)\, g_n^*(\tau)$, one can express the WS of $\mathcal{L}$ as a weighted summation of the Wigner distribution of the eigenfunctions of $\mathcal{L}$, i.e.

$$WS_{\mathcal{L}}(t,f)=\sum_n \varepsilon_n WD_{g_n}(t,f).$$

The SF in (3) provides an important interpretation of a time varying system output as a weighted superposition of time-shifted and frequency-shifted versions of the input signal x(t), where the weight is the SF[5], i.e. $(\mathcal{L}\,x)(t) = \iint SF_{\mathcal{L}}(\tau,\nu)\, e^{-j\pi\tau\nu}\, x(t-\tau)\, e^{j2\pi\nu t}\, d\tau\, d\nu$. Thus, the SF provides the amount of time shifts and frequency lags produced by the LTV system. This is comparable to the conventional interpretation of the (convolution) output of a linear time-invariant (LTI) system as a weighted superposition of time-shifted versions of the input signal. The weight is the impulse response of the LTI system and shows the amount of time shifts produced by the LTI system. The support region of the SF has been used to define under-spread random process[4], a useful concept in detection applications[9].

2.2. Wideband Weyl Correspondence

The wideband version of the WS, called the P_0WS, is defined as[6,7,12]

$$P_0WS_{\mathcal{B}}(t,f)=f\int_{-\infty}^{\infty}\Gamma_{\mathcal{B}}(f\lambda(\alpha)e^{\alpha/2}, f\lambda(\alpha)e^{-\alpha/2})e^{j2\pi tf\alpha}\,d\alpha, \quad f>0$$

where $\mathcal{B}$ is a frequency domain operator acting on a signal X(f), i.e. $(\mathcal{B}\,X)(f)=\int \Gamma_{\mathcal{B}}(f,\nu)X(\nu)\,d\nu$. Here, $\Gamma_{\mathcal{B}}(f,\nu)$ is the bi-frequency kernel of $\mathcal{B}$ defined on $L_2(R^+)$ and $\lambda(\alpha)=(\alpha/2)/(\sinh(\alpha/2))$. The mapping between $\mathcal{B}$ and P_0WS is called the wideband or affine Weyl correspondence [6,12]. The P_0WS is *associated* with the unitary Bertrand P_0-distribution[13], and the 1-D inner product of the operator input and output can be written as the 2-D inner product of the P_0WS of the operator and the P_0-distribution of the input

$$\int_0^{\infty}(\mathcal{B}\,X)(f)\,X^*(f)\,df=\int_0^{\infty}\int_{-\infty}^{\infty}P_0WS_{\mathcal{B}}(t,f)\,P_{0X}^*(t,f)\,dt\,df.$$

The P_0WS satisfies the time-shift covariance in (5), the scale covariance in (7), and the hyperbolic time-shift covariance properties, i.e.

$$Y(f)=X(f)e^{-j2\pi\varsigma\ln f} \Rightarrow P_0WS_{R_Y}(t,f)=P_0WS_{R_X}(t-\frac{\varsigma}{f},f).$$

The wideband version of the spreading function is defined as[6,12]

$$WSF_{\mathcal{B}}(t,f)=\int_0^{\infty}f\,\Gamma_{\mathcal{B}}(f\lambda(\alpha)e^{\alpha/2}, f\lambda(\alpha)e^{-\alpha/2})\,\lambda(\alpha)\,e^{j2\pi tf}\,df.$$

The WSF can be used to interpret the system output as the weighted superposition of time-shifted and scale changed versions of the input, i.e.

$$(\mathcal{B}\,X)(f)=\iint WSF_{\mathcal{B}}(\tau,\alpha)e^{j2\pi\tau_1(\alpha)}\frac{1}{\sqrt{e^{\alpha}}}X(\frac{f}{e^{\alpha}})\,d\tau\,d\alpha$$

where $\tau_1(\alpha)=e^{-\alpha/2}/\lambda(\alpha)$.

When LTV systems produce dispersive (non-constant) time shifts or dispersive frequency shifts, the conventional narrowband/wideband WS and SF are no longer adequate to analyze LTV systems whose nonstationary characteristics are not matched to simple time and frequency shifts. Thus, in this paper, we propose new TF symbols and spreading functions as tools for analyzing systems which produce dispersive (especially hyperbolic) time shifts on the signal. These new TF symbols are important since they can be interpreted as time-varying transfer functions for LTV systems producing hyperbolic time shifts. We derive such TF symbol and spreading function by hyperbolically warping the conventional narrowband WS and SF, respectively. We provide a new TF formulation of the quadratic form in (4) for linear systems with TF characteristics matched to a hyperbolic warping. Special examples will be given to demonstrate how the hyperbolic time-shift covariant TF symbol and spreading function greatly simplify when matched to the system. Analysis and detection application examples demonstrate the importance of these new TF techniques.

III. Hyperbolic Time-shift Covariant Symbols

3.1. Hyperbolic Weyl Symbol and Spreading Function

If a system imposes hyperbolic time shifts and scale changes on the input signal, new WS and new SF are needed for analysis. The TF geometry of these new WS and SF should reflect the hyperbolic system changes on the input signal. Thus, for an operator $\mathcal{B}$ on $L_2(\mathrm{R}^+)$ with kernel $\Gamma_B(f, \nu)$, we define the hyperbolic WS (HWS) and SF (HSF), respectively, as

$$\mathrm{HWS}_{\mathcal{B}}(t,f) = f\int_{-\infty}^{\infty} \Gamma_B(f\,e^{\beta/2}, fe^{-\beta/2})e^{-j2\pi tf\beta}\,d\beta, \quad f>0 \qquad (9)$$

$$\mathrm{HSF}_{\mathcal{B}}(\varsigma,\beta) = \int_0^{\infty} \Gamma_B(f\,e^{\beta/2}, fe^{-\beta/2})e^{-j2\pi\varsigma\ln f}\,df.$$

The relation between the HWS and the HSF is given as

$$\mathrm{HSF}_{\mathcal{B}}(\varsigma,\beta) = \mathcal{F}^{-1}{}_{\gamma\to\beta}\left[\mathcal{P}_{f\to\varsigma}\left[\mathrm{HWS}_{\mathcal{B}}(\tfrac{\gamma}{f}, f)\right]\right]$$

where $\mathcal{P}_{f\to\varsigma}\{X(f)\}=\int X(f)\,e^{j2\pi\varsigma\ln(f)}/f\,df,\ f>0$, is a version of the Mellin transform[13,14]. The HWS and HSF can be obtained from the narrowband WS and SF defined in (2) and (3) through axis warping as

$$\mathrm{HWS}_{\mathcal{B}}(t,f) = \mathrm{WS}_{\mathcal{W}_h\mathcal{B}\mathcal{W}_h^{-1}}(\tfrac{tf}{f_r},\ f_r\ln\tfrac{f}{f_r}),\quad f>0 \qquad (10)$$

$$\mathrm{HSF}_{\mathcal{B}}(\varsigma,\beta) = \mathrm{SF}_{\mathcal{W}_h\mathcal{B}\mathcal{W}_h^{-1}}(\tfrac{\varsigma}{f_r},\ f_r\,\beta) \qquad (11)$$

where f_r is a positive reference frequency. Here, $\mathcal{W}_h$ is an axis warping operator defined as $(\mathcal{W}_h X)(f)=\sqrt{e^{f/f_r}}\,X(f_r e^{f/f_r})$ and $(\mathcal{W}_h^{-1}\mathcal{W}_h X)(f)=X(f)$[14]. The equation (10) shows that the hyperbolic WS in (9) can be obtained from the conventional WS in (2) by first unitarily warping the operator $\mathcal{B}$ and then transforming the TF axes. For the HSF in (11), the axes are simply scaled since they show only relative TF lags, not absolute TF locations.

The HWS preserves hyperbolic time shifts and scale changes on a random process x(t), i.e.

$$Y(f)=X(f)e^{-j2\pi\varsigma\ln f} \Rightarrow \mathrm{HWS}_{R_Y}(t,f)=\mathrm{HWS}_{R_X}(t-\tfrac{\varsigma}{f}, f) \qquad (12)$$

$$Y(f)=\frac{1}{\sqrt{|a|}}X(\tfrac{f}{a}) \Rightarrow \mathrm{HWS}_{R_Y}(t,f)=\mathrm{HWS}_{R_X}(at,\tfrac{f}{a}). \qquad (13)$$

The HWS satisfies the unitarity property in (8)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{HWS}_{\mathcal{B}}(t,f)\,\mathrm{HWS}_{\mathcal{D}}(t,f)\,dt\,df = \sum_{m,n}\lambda_n\mu_m\left|\int U_n(f)E_m(f)\,df\right|^2$$

where λ_n and $U_n(f)$ are the eigenvalues and eigenfunctions of the kernel of the operator $\mathcal{B}$ and μ_m and $E_m(f)$ are the eigenvalues and eigenfunctions of the kernel of the operator $\mathcal{D}$, respectively. The quadratic form in (4) can now be written in terms of the HWS and $Q_X(t,f)$, the Altes-Marinovic Q-distribution[14],

$$\int_0^{\infty}(\mathcal{B}X)(f)X^*(f)\,df = \int_{-\infty}^{\infty}\int_0^{\infty}\mathrm{HWS}_{\mathcal{B}}(t,f)\,Q_X(t,f)\,dt\,df.$$

This new form of the quadratic form may be useful in detection applications of non-stationary processes and systems with hyperbolic TF characteristics. These formulations are important as they provide a new interpretation of these system outputs as weighted superposition of hyperbolic time-shifted and scale changed versions of the input signal, i.e.

$$(\mathcal{B}X)(f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{HSF}_{\mathcal{B}}(\varsigma,\beta)\, e^{j\pi\varsigma\beta}\, e^{-j2\pi\varsigma\ln(f)}\, X\!\left(\frac{f}{e^{\beta}}\right)\frac{d\beta\, d\varsigma}{\sqrt{e^{\beta}}}.$$

Thus, $\mathrm{HSF}_{\mathcal{B}}$ weighs the relative importance of hyperbolic time shifts and scale changes caused by a linear system. In Section 4, we provide application examples to demonstrate the importance of the HWS.

3.2. Hyperbolic Time-shift Covariant Symbols

The hyperbolic time-shift covariant symbols (HTS) can be defined in terms of the 2-D kernel $\Theta^{(H)}_{\mathrm{HTS}}(t,f)$ and the hyperbolic Weyl symbol HWS in (9) as

$$\mathrm{HTS}^{(H)}_{\mathcal{B}}(t,f)=\int_{0}^{\infty}\int_{-\infty}^{\infty} \Theta^{(H)}_{\mathrm{HTS}}\!\left(ft-f't',\ln\!\left(\frac{f}{f'}\right)\right)\mathrm{HWS}_{\mathcal{B}}(t',f')\, dt'\, df',\ f>0. \tag{14}$$

Here, We use the superscript (H) to indicate the hyperbolic version of TF symbols. Any TF symbol in this formulation satisfies the hyperbolic time shift covariance property in (12) and the scale covariance property in (13). We can show that if the 2-D kernel $|\Theta^{(H)}_{\mathrm{HTS}}(t,f)|=1$, the HTS satisfies the unitarity property as in

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{HTS}^{(H)}_{\mathcal{B}}(t,f)\ \mathrm{HTS}^{(H)}_{\mathcal{D}}(t,f)\, dt\, df=\sum_{m,n}\lambda_n\mu_m\left|\int U_n(f)E_m(f)\, df\right|^2.$$

By specifying the 2-D kernel in (14), we can obtain special examples of the HTS. For examples, if $\Theta^{(H)}_{\mathrm{HTS}}(t,f)=\delta(t)\delta(f)$, the HTS simplifies to the hyperbolic Weyl symbol HWS in (9) and if $\Theta^{(H)}_{\mathrm{HTS}}(t,f)=\int \delta(b+\ln\lambda(\beta))\, e^{j2\pi t\beta}\, d\beta$, the HTS becomes the wideband P_0-Weyl symbol with $\lambda(\beta)$ defined in Section 2.2.

3.3. Relationship with the Hyperbolic Frequency-shift Covariant Symbols

The hyperbolic frequency-shift covariant TF symbols have been introduced in[6,7]. The hyperbolic version of the WS satisfying the hyperbolic frequency-shift covariance is defined as

$$\hat{\mathrm{HWS}}_{\mathcal{N}}(t,f)=t\int_{-\infty}^{\infty} K_{\mathcal{N}}(t\,e^{\varsigma/2},te^{-\varsigma/2})\,e^{-j2\pi tf\varsigma}\, d\varsigma\ ,\quad t>0. \tag{15}$$

Here, the operator $\mathcal{N}$ on $L_2(\mathbf{R}^+)$ is defined in time with its kernel $K_{\mathcal{N}}(t,f)$[6,7]. If we compare the new hyperbolic time-shift covariant WS in (9) proposed in Section 3.1 and the hyperbolic frequency-shift covariant WS in (15), we can see that they have the duality relationship each other. The hyperbolic time-shift covariant WS is obtained from the warping relationship in (10) using the frequency domain formulation of the WS in (2) and the hyperbolic axis warping operator in frequency . On the other hand, the hyperbolic frequency shift covariant WS in (15) can be obtained using the time domain formulation of the WS in (1) and the hyperbolic axis warping operator in time [6,7]. The narrowband WS in (1) is the only TF symbol which is equal to its dual frequency formulation in (2).

IV. Application Examples

4.1. Analysis Application

In order to demonstrate the importance of the new hyperbolic time-shift covariant TF symbol , we analyze a hyperbolic random process $X(f)=\alpha_i X_i(f)$. Here, α_i are uncorrelated, zero-mean random weights and $X_i(f)=e^{j2\pi C_i \ln(f)}, f>0, i=1,2,3$ are hyperbolic FM, deterministic signals. Note that each signal term $X_i(f)$ has hyperbolic group delay, C_i/f. One can show that the hyperbolic WS in (9) of the correlation operator $\mathbf{R_X}$ with kernel $\Gamma_{R_X}(f,\nu)=\mathbf{E}[X(f)X^*(\nu)]$ simplifies to

$$\mathrm{HWS}_{\mathbf{R_X}}(t,f)=\sum_{i=1}^{3}\mathrm{E}[|\alpha_i|^2]\,\delta\!\left(t-\frac{C_i}{f}\right),\quad f>0. \tag{16}$$

Figure 1 shows the contour plots of (a) the conventional WS versus (b) the HWS of $\mathbf{R_X}$ of a windowed $X(f)$. Both show time-varying spectra with hyperbolic TF characteristics. The advantage of the HWS in (16), is that it is ideally localized along the three group delay curves $t=C_i/f$ in the TF plane. The disadvantage of the conventional WS

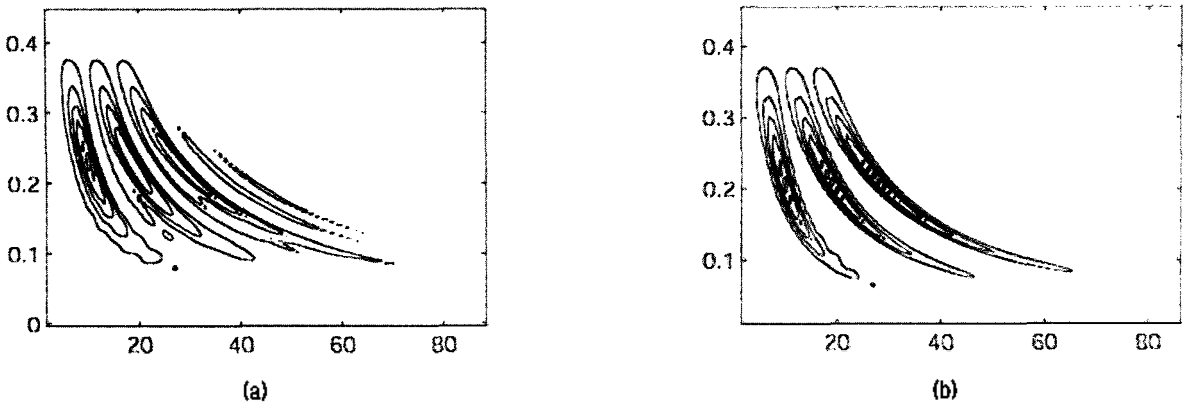

Figure 1. a) Weyl symbol, WSRx(t, f), and b) hyperbolic WS, HWSRx(t, f) of a hyperbolic process X(f). The horizontal axis is for time and the vertical for normalized frequency.

is that it produces spurious components along hyperbolae since it does not match the intrinsic hyperbolic TF characteristics.

4.2. Detection Application

Next, we consider the detection of a known deterministic signal s(t) with hyperbolic TF characteristics in nonstationary Gaussian random noise n(t). Assume that the noise has the correlation function $R_n(t, \tau)$ whose support region area is less than unity in the hyperbolic SF domain. Here, the support region of a hyperbolic SF, $HSF_{Rn}(\zeta, \beta)$, of the noise process n(t) is the region in (ζ, β) where $HSF_{Rn}(\zeta, \beta) \neq 0$. The test statistic of the optimal likelihood ratio detector is **Re**{ $<\mathbf{R}_n^{-1}\ x,\ s>$ } where **Rn** is the correlation operator and $x(t) = s(t) + n(t)$ is the received signal. The inner product is defined as $<x, y> = \int x(t)y^*(t)dt$ and **Re**{a} is the real part of a.

Using the hyperbolic version of the quadratic form, one obtains

$$\mathrm{Re}\{<R_n^{-1}x,s>\} = \int_{-\infty}^{\infty}\int_0^{\infty} HWS_{R_n^{-1}}(t,f)\cdot \mathrm{Re}\{Q_{XS}(t,f)\}\,dt\,df$$

where $Q_{XS}(t,f)$ is the cross Q-distribution of $x(t)$ and $s(t)$. Similar to the conventional underspread operator approximations in [6,7], we show that if the hyperbolic SFs of two operator $\mathcal{Y}$ and $\mathcal{S}$ are confined in a small area (jointly underspread), then the hyperbolic WS of the composite operator $\mathcal{YS}$ can be approximated as the product of the hyperbolic WS of each operator, i.e.

$$HWS_{\mathcal{YS}}(t,f) \approx HWS_{\mathcal{Y}}(t,f)\cdot HWS_{\mathcal{S}}(t,f).$$

For the two correlation operators R_n and R_n^{-1}, we show that

$$HWS_{R_n R_n^{-1}}(t,f) \approx HWS_{R_n}(t,f)\cdot HWS_{R_n^{-1}}(t,f) \approx 1.$$

This simplifies the TF test statistic for detecting a deterministic signal

$$\mathrm{Re}\{<R_n^{-1}x,s>\} \approx \int_{-\infty}^{\infty}\int_0^{\infty} \mathrm{Re}\{Q_{XS}(t,f)\}/HWS_{R_n}(t,f)\,dt\,df.$$

V. Conclusions

The conventional WS and SF are most useful for systems producing constant time shifts and frequency shifts on the signal. The WS are time-frequency representations that can be interpreted as time-varying spectra for random processes. In this paper, using warping techniques, we proposed the new hyperbolic time-shift covariant WS and SF. By applying 2-D TF kernel to the new hyperbolic time-shift covariant WS, we also derived the hyperbolic time-shift TF symbols. By selecting a specific kernel, we can obtain a certain hyperbolic TF symbol. We also showed the duality relationship between the newly obtained hyperbolic time-shift covariant WS and the hyperbolic frequency-shift covariant WS. The analysis and detection application examples confirm the importance and usefulness of the new hyperbolic time-shift covariant TF symbols.

References

1. F. Hlawatsch and G. F. Boudreaux-Bartels, "Linear and quadratic time-frequency signal representations," *IEEE Signal Proc. Magazine*, **9**, 21-67, April 1992.
2. L. Cohen, "Time-frequency distributions," *Proceedings of the IEEE*, **77**, 941-981, July 1989.
3. W. Kozek, "On the generalized Weyl correspondence and its application to time-frequency analysis of linear time-varying systems," *IEEE-SP Int. Symp. on TFTS*, Canada, 167-170, Oct. 1992.
4. W. Kozek, F. Hlawatsch, H. Kirchauer and U. Trautwein, "Correlative TF analysis and classification of nonstationary processes," *IEEE-SP Int. Symp. on TFTS, Phil.*, PA, 417-420, Oct. 1994.
5. R. G. Shenoy and T. W. Parks, "The Weyl correspondence and TF analysis," *IEEE Trans. on Sig. Proc.*, 318-331, Feb. 1994.
6. B. Iem, "Generalizations of the Weyl symbol and the spreading function via TF warpings: Theory and application," Ph.D dissertation, Univ. of RI, Kingston, RI, 1998.
7. B. Iem, A. Papandreou-Suppappola and G. F. Boudreaux-Bartels, "Wideband Weyl symbols for dispersive time-varying processing of systems and random signals," *IEEE Trans. on Signal Proc.*, 1077-1090, May 2002.
8. I. Gohberg and S. Goldberg, *Basic Operator Theory*, (Birkhauser, Boston, MA, 1980)
9. G. Matz and F. Hlawatsch, "Time-frequency formulation and design of optimal detectors," *Int. Symp. on TFTS*, Paris, 213-216, June 1996.
10. A. M. Sayeed and D. L. Jones, "Optimal quadratic detection and estimation using generalized joint signal representations," *IEEE Trans. on Signal Proc.*, 3031-3043, Dec. 1996.
11. G. Matz, F. Hlawatsch, and W. Kozek, "Generalized evolutionary spectral analysis and the Weyl spectrum of nonstationary random processes," *IEEE Trans. on Signal Proc.*, 1520-1534, June 1997.
12. R. G. Shenoy and T. W. Parks, "Wideband ambiguity functions and affine Wigner distributions," *Proc. EURASIP*, **41**, 339-363, 1995.
13. P. Bertrand and J. Bertrand, "TF representation of broad band signals," *Proc. IEEE ICASSP*, NY, 2196-2199, April 1988.
14. A. Papandreou, F. Hlawatsch and G. F. Boudreaux-Bartels, "The hyperbolic class of QTFRs, Part I," *IEEE Trans. on Signal Proc.*, 3425-3444, Dec. 1993.

[Profile]

• Byeong-Gwan Iem

Byeong-Gwan Iem received his B. S. and M. S. degrees in Electronics engineering from Yonsei University, Seoul, Korea in 1988 and 1990, respectively. He received his Ph. D degree in electrical engineering from the university of Rhode Island in 1998. From 1991 to 1993, he was with Dacom corporation, Seoul, where he was responsible for planning its public data network and analyzing its network traffic data. From 1999 to 2002, he worked in Samsung Electronics, where he was involved in research and development of the third generation wireless communication system. His research interests are in signal processing and its applications to communications.