Feedback Linearization Control of the Looper System in Hot Strip Mills

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This paper studies on the linearization of a looper system in hot strip mills, that plays an important role in regulating a strip tension or a strip width. Nonlinear dynamic equations of the looper system are analytically linearized by a static feedback linearization algorithm with a compensator. The proposed linear model of the looper is validated by a comparison with a linear model using Taylor's series. It is shown that the linear model by static feedback well describes nonlinearities of the looper system than one using Taylor's series. Furthermore, it is shown from the design of an ILQ controller that the linear model by static feedback is very useful in designing a linear controller of the looper system.

Key Words : Hot Strip Mills, Looper, Linearization by Taylor Series, Linearization by Static Feedback, ILQ (Inverse Linear Quadratic optimal control)

1. Introduction

Using a linearization technique by static feedback, this paper studies on the development of a mathematical model for a looper control system in hot strip mills. The looper located in each stands of hot strip mills regulates a strip tension and a strip flow, and is irregularly varied by an unbalanced speed of main work rolls. In particular, since a strip width is mainly influenced by a strip tension, the looper plays an important role in controlling a strip width. Thus, it is very important to develop a high performing looper control system. Many researchers have proposed various control techniques derived from linear and nonlinear algorithms (Kim and Hwang, 2002).

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Most control algorithms applied to the actual looper system are PID or non-interference control (Anbe et al., 1996 ; Imanari et al., 1997 ; Hesketh et al., 1998). However, it is well known that these possess many technical defects in view of control performances. Recently, various studies on high class control algorithms such as LQ/LQG/H^∞ optimal control, nonlinear control, adaptive, neural network and artificial intelligence are accomplished (Price, 1973 ; Hearns et al., 1996 ; Hearns and Grimble, 1997 ; Asano et al., 2000). Among these control algorithms, it is well known from experimental results that linear optimal control algorithms (LQ, H^∞) are feasible to control the looper (Asano et al., 2000). The control performance of each linear controller, however, mainly depends on the accuracy of a linear model. Thus, the goal of this paper is to develop a new mathematical model of the looper system for linear control.

When the looper system is strictly described by a mathematical model, it would contain strong nonlinearities. A mathematical model for linear control is generally obtained from the approximation of Taylor series. The linear model is useful only within a small interval at the neighborhood of an operating point, and it does not well describe the nonlinearity of the system dynamics. Furthermore, it is very difficult to validate approximation errors of the model, and to adjust system gains. This paper develops a linear model of the looper system using a linearization technique by static feedback to solve these problems.

A basic idea is that nonlinear terms in dynamics of the looper are eliminated by new control inputs obtained from the state feedback of a nonlinear system. There are many studies performed on a feedback linearization (Jakubzyk and Respondek, 1980 ; Isidori and Ruberti, 1984 ; Falb and Wolovich, 1967). This paper shows a methodology with a linear model for the looper using the results of Falb and Wolovich.

The contents of this paper are as follows : In Section 2, the nonlinear dynamics of the looper system is introduced, and a linear model is constructed by a static feedback linearization algorithm as described in Section 3. In Section 4, the validation of the linear model is accomplished by computer simulation using MATLAB 6.1. The linear model is characterized then by the comparison with a nonlinear model, and usefulness of the obtained model is validated by applying the feedback linearization model for ILQ control. Finally, the conclusions are briefly described.

2. Dynamics of a Looper System

We consider a looper system given in Fig. 1. Its dynamic equations are composed of dynamic

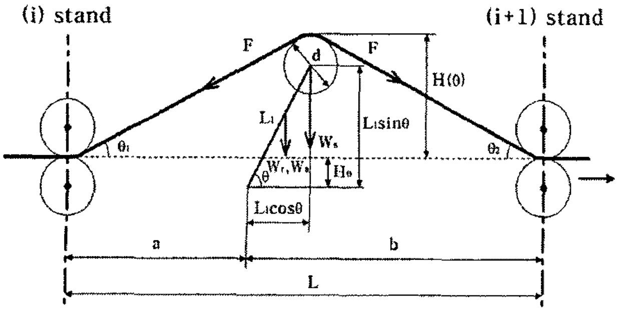

Fig. 1 A schematic diagram of the looper system

equations of the looper and the strip tension, respectively (Price, 1973).

2.1 Dynamic equation of a looper

From Newton's second law, the dynamic equation of the looper is given as follows :

$$J_L \frac{d^2\theta}{dt^2} = T_{lm} - T_l + w_\theta(t) \tag{1}$$

where T_{lm} and T_l denote a drive motor torque and a total load torque of the looper, respectively. $w_\theta(t)$ is the modeling error. T_l is given by

$$T_l = T_f + T_{sw} + T_{lw} + T_d \tag{2}$$

where T_f, T_{sw}, T_{lw} and T_d are load torques induced from a strip tension, a strip weight, a weight of a looper roll/arm, and the friction force between the looper roll and the strip, respectively. In Eq. (2), unmodelled dynamics such as a strip bending force etc. are included in the modeling errors $w_\theta(t)$. Note that the details are referred to the reference (Kim and Hwang, 2002).

2.2 Dynamic equation of a strip tension

From Hook's law, a strip tension (τ) within an elastic limit is described as in Eq. (3).

$$\frac{d\tau}{dt} = F_1\left[F_3(\theta)\frac{d\theta}{dt} - F_4(\tau) - v_e(t) - w_\tau(t)\right] \tag{3}$$

In Eq. (3), $F_1(=E/L)$ denotes the ratio of a strip Young's modulus (E) to a strip length ($\xi(t)+L$). $F_3(\theta)$, $F_4(\tau)$ and $v_e(t)$ are represented as the difference between the geometric strip length and the actual strip length, the effect factor for a strip speed, and the difference between strip speeds going out from the backward stand and coming to the forward stand, respectively. $w_\tau(t)$ denotes the unmodelled dynamics.

Note that $\xi(t)$ is the variation of a strip length caused by the difference between a strip speed going out from the backward stand and the strip speed coming to the forward stand. As a rule, $\xi(t)$ is ignored since it is very small in comparison with the distance (L) between two stands. While the Young's modulus (E) depends on material characteristics, it is assumed as a constant.

The details about the parameters are referred to the reference (Kim and Hwang, 2002).

2.3 Dynamic equations of actuators

In hot strip mills, control inputs are given as a motor torque (T_{lm}) of the looper and a speed difference of the strip (v_e). They are respectively controlled by LCC (Looper Current Controller) and ASR (Automatic Speed Regulator) in actual plant. LCC and ASR are regarded as actuators in this paper, and represented by the first-order delay systems. Thus, dynamics equations of LCC and ASR are given by Eq. (4).

$$\begin{aligned} \frac{dT_{lm}(t)}{dt} &= -k_m T_{lm}(t) + k_m u_1(t) \\ \frac{dv_e(t)}{dt} &= -k_a v_e(t) + k_a u_2(t) \end{aligned} \tag{4}$$

In Eq. (4), $u_2(t)$ and denote the inputs to LCC and ASR. Whereas k_m and k_a are the reciprocals of the time constants of the LCC and ASR, respectively.

Figure 2 shows the block diagram of the looper system, where a gear ratio of the looper drive motor, G_r, a cross-sectional area of strip, A_s, and a second inertia moment of the looper, J_L, are also shown.

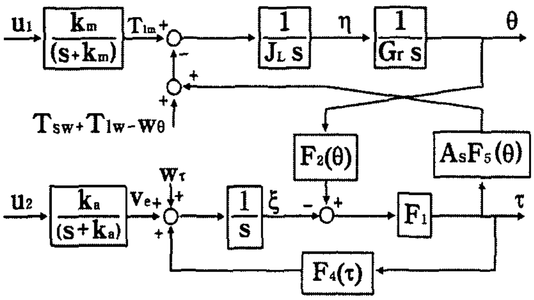

Fig. 2 Block diagram of the looper system

3. Linearization of the Looper System by Static Feedback Algorithm

3.1 Basic idea of a feedback linearization

Consider a nonlinear system given as

$$\begin{aligned} &\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i = f(x) + g(x) u, \\ &x \in R^n,\ u \in R^m,\ f(0) = 0 \end{aligned} \tag{5}$$

Assuming that the system given in Eq. (5) is controllable, it can be represented as a linear system of Eq. (6) by introducing a new state variable (z) and a control input (v) as

$$\begin{aligned} &\dot{z} = \hat{A} z + \sum_{i=1}^{m} B_i(x) v_i = \hat{A} z + \hat{B} v \\ &z \in R^n,\ v \in R^m \end{aligned} \tag{6}$$

Provided a coordinate transformation, $z = S(x)$, and a feedback $u = \alpha(x) + \beta(x) v$ exist, the nonlinear system of Eq. (5) can be linearizable by feedback (Lee, 2001). Fig. 3 represents the block diagram of the static feedback linearization system.

Note that the linear system obtained by static feedback can be transformed into a linear model

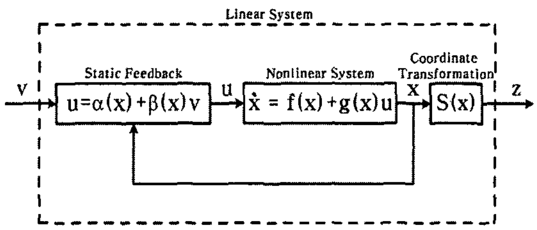

Fig. 3 Block diagram of a static feedback linearization

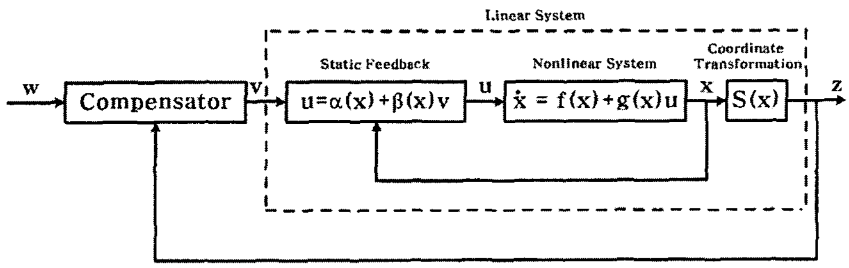

Fig. 4 Block diagram of a static feedback linearization with a compensator

with a Brunovsky canonical form. Then, the linear model becomes unstable since it has all zero eigenvalues. Thus, a new compensator should be added to the system of Fig. 3 as shown in Fig. 4. The linear model becomes stable conseguently.

Note that feedback linearization is achieved from the elimination of nonlinear terms in a new control input obtained from the feedback of state variables in the nonlinear system. The details are referred as the reference (Lee, 2001).

3.2 Feedback linearization of the looper system

In Eqs. (1), (3) and (4), we define state and input-output variables as follows:

$$\begin{aligned} \boldsymbol{x}&=[\theta\ \eta\ \tau\ T_{tm}\ v_e]^T \\ \boldsymbol{y}&=[\theta\ \tau]^T,\ \boldsymbol{u}=[u_1\ u_2]^T \\ \boldsymbol{w}&=[w_\theta\ w_\tau]^T \end{aligned} \tag{7}$$

Then, a state-space equation of the looper is given

$$\frac{d}{dt}\boldsymbol{x}=\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}\boldsymbol{x}+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_m & 0 \\ 0 & k_a \end{bmatrix}\boldsymbol{u}+\begin{bmatrix} 0 & 0 \\ 1/J_L & 0 \\ 0 & -F_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\boldsymbol{w}, \tag{8}$$

$$\boldsymbol{y}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}\boldsymbol{x}$$

where,

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}=\begin{bmatrix} \eta(t)/G_r \\ [T_{tm}-T_f-T_{sw}-T_{tw}-T_d]/J_L \\ F_1[F_3(\theta)\dot{\theta}-F_4(\tau)-v_e(t)] \\ -k_m T_{tm} \\ -k_a v_e \end{bmatrix}$$

Thus, a linear model of the looper system is obtained from a feedback linearization algorithm as follows:

[STEP 1] Since Kronecker's indices of the nonlinear system of Eq. (8) are $X_1=3$ and $X_2=2$ respectively, the nonlinear model can be linearizable by feedback. It is satisfied that

(i) $\dim(\Delta_{X_{\max}-1})=5$
(ii) $\Delta_i,\ (0\leq i\leq X_{\max}-2)$

are involutive distributions in which the dimensions are constant, and denotes an invariant distribution.

Note that the condition (i) guarantees controllability of the system under a state feedback or a coordinate transformation. The condition (ii) is a necessary and sufficient condition for the existence of scalar seed functions $h_i(\boldsymbol{x})\ (1\leq i\leq m)$ such that

$$L_g L_f^l h_i(\boldsymbol{x})=0,\ 1\leq i\leq m,\ 0\leq l\leq x_i-2 \tag{9}$$

where m is 2 as the number of inputs and L denotes Lie derivative.

[STEP 2] Seed functions $h_i(\boldsymbol{x})$ satisfying Eq. (9) are respectively determined as follows:

$$h_1(\boldsymbol{x})=\theta=z_1^1,\ h_2(\boldsymbol{x})=\tau=z_1^2$$

[STEP 3] Using the seed functions given in STEP 2, a coordinate transformation $z=S(\boldsymbol{x})$ is obtained by Eq. (10).

$$z=\begin{bmatrix} z^1 \\ z^2 \end{bmatrix}=\begin{bmatrix} z_1^1 \\ z_2^1 \\ z_3^1 \\ z_1^2 \\ z_2^2 \end{bmatrix}=\begin{bmatrix} h_1(\boldsymbol{x}) \\ L_f h_1(\boldsymbol{x}) \\ L_f^2 h_1(\boldsymbol{x}) \\ h_2(\boldsymbol{x}) \\ L_f h_2(\boldsymbol{x}) \end{bmatrix}=\begin{bmatrix} \theta \\ \frac{1}{G_r}\eta \\ \frac{1}{G_r}f_3 \\ \tau \\ f_2 \end{bmatrix} \tag{10}$$

[STEP 4] A linearization feedback

$$\boldsymbol{u}=\alpha(\boldsymbol{x})+\beta(\boldsymbol{x})\,\boldsymbol{v}$$

is given from Eq. (11) as follows:

$$\boldsymbol{v}=\begin{bmatrix} L_f^{X_1}h_1(\boldsymbol{x}) \\ \vdots \\ L_f^{X_m}h_m(\boldsymbol{x}) \end{bmatrix}+\begin{bmatrix} L_g L_f^{X_1-1}h_1(\boldsymbol{x}) \\ \vdots \\ L_g L_f^{X_m-1}h_m(\boldsymbol{x}) \end{bmatrix}\boldsymbol{u} \tag{11}$$

Then, $\alpha(x)$ and $\beta(x)$ are given by

$$\alpha(x)=-\begin{bmatrix} L_g L_f^{X_1-1}h_1(\boldsymbol{x}) \\ \vdots \\ L_g L_f^{X_m-1}h_m(\boldsymbol{x}) \end{bmatrix}^{-1}\begin{bmatrix} L_f^{X_1}h_1(\boldsymbol{x}) \\ \vdots \\ L_f^{X_m}h_m(\boldsymbol{x}) \end{bmatrix}$$

$$\beta(x)=\begin{bmatrix} L_g L_f^{X_1-1}h_1(\boldsymbol{x}) \\ \vdots \\ L_g L_f^{X_m-1}h_m(\boldsymbol{x}) \end{bmatrix}$$

Thus, the linearization feedback for Eq. (8) is obtained by Eq. (12) as follows:

$$u=\alpha(x)+\beta(x)v \tag{12}$$

$$\alpha(x)=\begin{bmatrix} -\frac{k_m K_4}{G_r}(K_1f_1+K_2f_2+K_3f_3+K_4f_4+K_5f_5) \\ -\frac{Q_5}{K_a}(Q_1f_1+Q_2f_2+Q_3f_3+Q_4f_4+Q_5f_5) \end{bmatrix}$$

$$\beta(x)=\begin{bmatrix} \frac{G_r}{k_m} & 0 \\ 0 & \frac{1}{k_aQ_5} \end{bmatrix}$$

where K_i and Q_i, $i=1, 2, 3, 4, 5$ are given by

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \end{bmatrix}=\begin{bmatrix} \frac{\partial f_3}{\partial x_1} \\ \frac{\partial f_3}{\partial x_2} \\ \frac{\partial f_3}{\partial x_3} \\ \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_3}{\partial x_5} \end{bmatrix}=\begin{bmatrix} -\frac{1}{J_L}\left\{A_s\tau\frac{\partial F_5(\theta)}{\partial\theta}+\frac{\partial T_{sw}(\theta)}{\partial\theta}+\frac{\partial T_{lw}(\theta)}{\partial\theta}\right\} \\ -\frac{1}{J_L}A_sF_5(\theta) \\ -\frac{c_f}{J_L} \\ \frac{1}{J_L} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix}=\begin{bmatrix} \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_2}{\partial x_5} \end{bmatrix}=\begin{bmatrix} F_1\eta\frac{\partial F_3(\theta)}{\partial\theta} \\ -F_1\frac{\partial F_4(\tau)}{\partial\tau} \\ F_1\cdot F_3(\theta) \\ 0 \\ -F_1 \end{bmatrix}$$

[**STEP 5**] By applying an input u of Eq. (12) to the nonlinear system of Eq. (8), the feedback linearization model of the looper is obtained by

$$\begin{aligned} \dot{z}&=\hat{A}z+\hat{B}v \\ y&=Cz \end{aligned} \tag{13}$$

where, the system coefficients $\hat{A}$, $\hat{B}$ and are given as follows :

$$\hat{A}=\begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix},\ \hat{B}=\begin{bmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&0 \\ 0&1 \end{bmatrix}, \tag{14}$$

$$C=\begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

[**STEP 6**] Since all the eigenvalues of $\hat{A}$ in Eq. (14) are zero, the linear model is unstable. Thus, a compensator is added to the linearized system of Eq. (13) as shown in Fig. 4. Using constant coefficients C_l^i, Eq. (12) is reconstructed as follows :

$$\begin{aligned} &u_i=\alpha(x)+\beta(x)(-C_l^i z_{X_i}^i+v_i) \\ &1\le i\le m,\ 0\le l\le X_i-1 \end{aligned} \tag{15}$$

where coefficients C_l^i are determined by a trial-and-error method (Lee, 2001). Thus, $\hat{A}$ in Eq. (14) must be replaced by A in Eq. (16).

$$A=\begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ -C_0^1&-C_1^1&-C_2^1&0&0 \\ 0&0&0&0&1 \\ 0&0&0&-C_0^2&-C_1^2 \end{bmatrix} \tag{16}$$

4. Model Validation by Computer Simulation

The dynamic characteristics and the usefulness of the linear model obtained in Section 3 is evaluated. First, the feedback linearization model is compared with the nonlinear model. Second, the usefulness is evaluated by analysing the performances of an ILQ control system developed from the feedback linearization model.

4.1 Validation of a feedback linearization model

In this section, the feedback linearization model is compared with the nonlinear model performing computer simulation. The looper system is considered as a looper located between 5 and 6 stands. The operating points of a looper angle and a strip tension are 18° and 7.1 MPa, respectively. Details of each parameters are also given in Table 1.

Figure 5 represents outputs of a nonlinear model and two linearization models obtained using Taylor's series and static feedback. It is clearly shown from the Fig. 5 that the linear model by Taylor's linearization model does not correspond with the nonlinear model. However, the feedback linearization model matches well

Table 1 Parameter values of a looper system

parameter	value	parameter	value
L_1[mm]	612.4	w[mm]	1300
H_0[mm]	184	h[mm]	3.13
d[mm]	184	Wr[kg]	225
a[mm]	2185.2	Wa[kg]	1655
b[mm]	3314.8	E[kg/mm^2]	7.8×10^{-6}

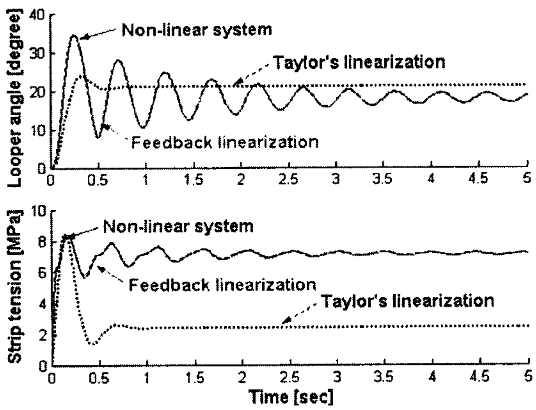

Fig. 5 Outputs of linear models by Talor series and static feedback

with the nonlinear model. When LCC input T_{lm} and ASR input v_e are varied within the range of $\pm40\%$ at each operating point value, Figs. 6 and 7 represent the outputs of the feedback linearization model and the nonlinear model, respectively. Figures 6 and 7 show that the feedback linearization model has a good correspondence with the nonlinear model.

Note that the nonlinear dynamics of the looper system is characterized as follows: Fig. 6 shows that strip tension and looper angle also increase as the LCC input (T_{lm}) increases. However, strip tension is not affected by the input (v_e) of ASR. And the chattering phenomenon of the looper appeared accordingly as the looper angle decreases. Figure 7 shows that the strip tension and looper angle also decrease as the speed difference between the two main work rolls increases, which is opposite to the results shown in Figs. 6 and 7. In particular, it is shown that strip tension is strongly influenced by the speed difference of main work rolls.

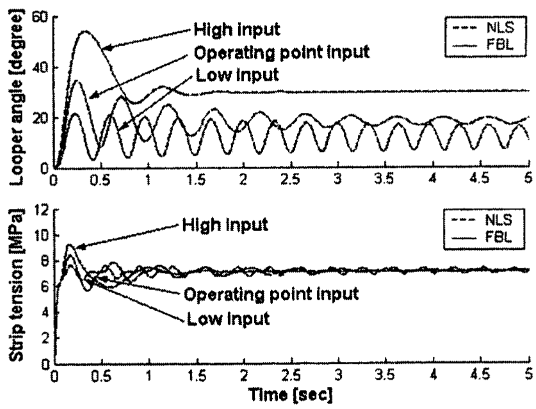

Fig. 6 Outputs according to variations of LCC input T_{lm}

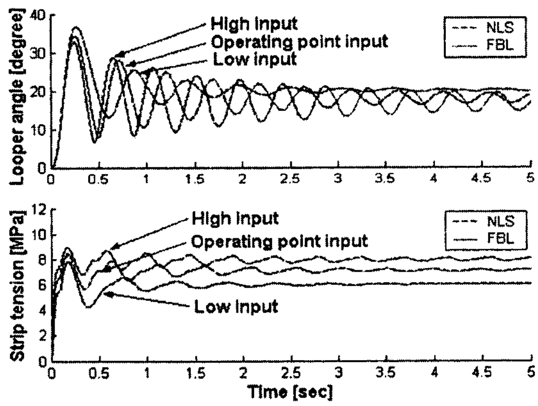

Fig. 7 Outputs according to variations of ASR input v_e

As a result, it is noted that the strip tension is more sensitive to the speed difference between two main work rolls than the motor torque of the looper.

4.2 Usefulness of the feedback linearization model

An ILQ looper control system using the feedback linearization model is designed. And the usefulness for linear control of the looper is evaluated. The details on the design algorithms of an ILQ controller are referred to the reference (Kim and Hwang, 2002).

Figure 8 shows the outputs of an ILQ looper control system. Solid line(-) and dotted line(--) represent outputs of control systems based on

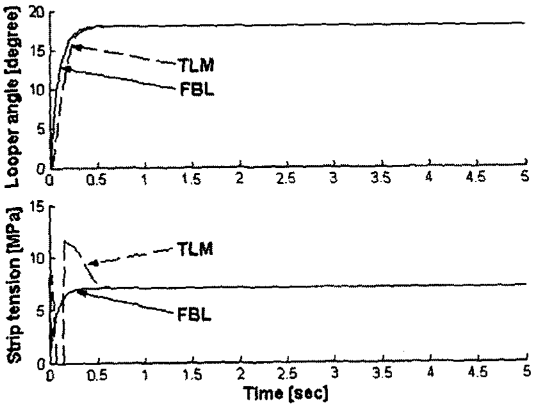

Fig. 8 Outputs of an ILQ control system based on linear models by Taylor series and static feedback

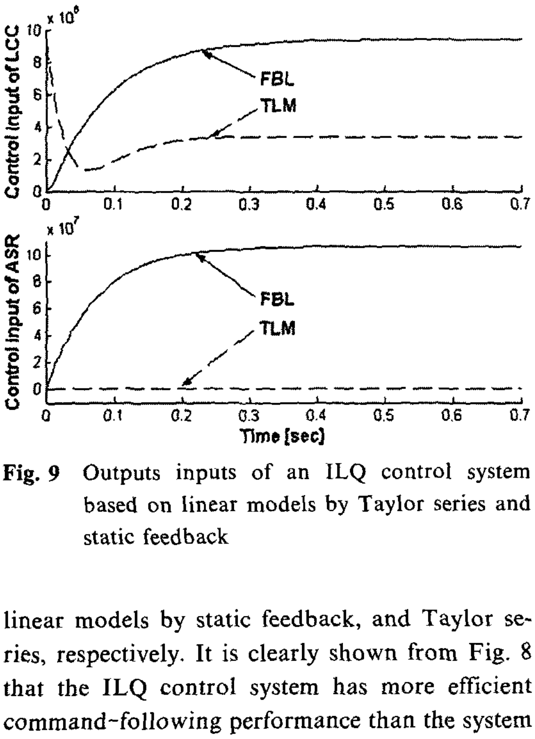

Fig. 9 Outputs inputs of an ILQ control system based on linear models by Taylor series and static feedback

linear models by static feedback, and Taylor series, respectively. It is clearly shown from Fig. 8 that the ILQ control system has more efficient command-following performance than the system based on Taylor linearization model. Note that the strip tension is controlled very efficiently in the model by static feedback and Taylor series.

Figure 9 shows the control input of an ILQ control system. Figure 9 shows that control input of the ILQ control system using the feedback linearization model is larger than that based on Taylor linearization model. This fact indicates that the control input depends on the performance of the ILQ control system. In other words, control input can be adjustable according to the design specifications of the ILQ control system. Control input has nothing to do with a mathematical model, in general. In results, linear controller of the looper in hot strip mills developed from this study is more efficient than that using Taylor series.

5. Conclusions

This paper proposed a new linear model of the looper system using static feedback linearization technique. The results show that the linear model using static feedback well approximates nonlinearties of the looper. Futhermore, the model is very efficient for designing a linear controller of the looper system. However, hardware load can be excessive because the states of the nonlinear system must be continuously measured. A future research to solve this hardware problem is strongly suggested.

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