

## Robust Control for Rotational Inverted Pendulums Using Output Feedback Sliding Mode Controller and Disturbance Observer

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This paper presents a system modeling, controller design and implementation for a rotational inverted pendulum system (RIPS), which is an under-actuated system and has the problem of unattainable velocity state. Two control strategies are applied to the RIPS. One is a sliding mode control method using the parameterization of both the hyperplane and the compensator for output feedback. The other is the disturbance observer which estimates disturbance and some modeling errors of RIPS with less computational effort. Some simulations and various kinds of experiments are performed in order to verify that the proposed controller has the ability to control RIPS whose velocity is assumed to be unavailable. The results of the simulations and experiments show that the proposed control system has superior performance for disturbance rejection and regulation at certain initial conditions as well as the robustness to model uncertainties.

**Key Words :** Output Feedback Sliding Mode Controller, Disturbance Observer, Rotational Inverted Pendulum

### 1. Introduction

Rotational inverted pendulum system (RIPS) is a typical under-actuated system, which has fewer inputs than the degree of freedom. The under-actuated system is very useful to low the cost of automatic controlled systems or robot systems which are required to minimize the number of actuators to make less power, smaller mass, and lower cost. Furthermore, direct-drive manipulators that have some failed actuators in the inaccessible space may be an under-actuated system. It is also important to overcome these kind of troubles. Earlier studies of under-actuated

systems had mainly been focused on single or double flexible inverted pendulums. Recently there are many active studies for the paralleled or rotational inverted pendulums, as bench mark test for applying the various kind of control laws, which have similarity with rocket launchers, traveling robots, attitude control of the artificial satellites, etc.

Sliding mode control method is suggested for the robustness to have invariance properties not to be affected by the matched uncertainties and possibilities to have reduced order motion, which is apparently independent on the control. In this control method, it is important to select the so-called sliding surfaces to provide an appropriate and stable sliding motions (Son et al., 1998). Many workers have designed controller by assuming full states to be available in selecting control laws. But in practice not all states are available. In order to overcome this difficulty, an observer can be used to generate estimates of the

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unavailable internal states or to select a sliding surface with only output information. (Edward and Spurgeon, 1995) proposed output feedback sliding mode control method by using linear control structures employed (Ryan and Corless, 1987). In order to allocate arbitrary pole placement for sliding mode dynamics, the Kimura-Davidson condition should be satisfied (Bag et al., 1997). For overcoming this difficulty Edward and Spurgeon proposed the compensator and switching function matrix to be parameterized.

In this paper, output feedback sliding mode control with disturbance observer (OFSMC/DO) is proposed for good performance and robustness of a RIPS. A disturbance observer, which estimates the external disturbance and some modeling errors of RIPS, is effective to compensate the effects of the unmatched uncertainties of the control system and is required less computational effort. Furthermore this controller contains the reduced order observer to attenuate the measurement noises. And system parameters are estimated by the signal compression method and experimentations. In order to show the effectiveness of the proposed control method, simulation and experiment are performed and results are compared with those of simulation and experiment of the full-state feedback LQ control and output feedback sliding mode control without disturbance observer (OFSMC) systems. The results of simulation and experiment show that the proposed control system has good regulation performance in the severe initial positions and good robustness to arbitrary disturbances.

### 2. System Modeling and Parameter Estimations

Figure 1 shows the schematic diagram of the RIPS. The RIPS consists of a vertical pendulum that is  $2l$  long, hinged at the end of the horizontal radial arm of length,  $R$ , which is directly connected to the shaft of a DC motor. Torque,  $f_\tau$ , of the DC motor is the control input of the system, and the pendulum angle,  $\phi$ , and the horizontal arm angle,  $\theta$ , as measured by encoders are the outputs. Only viscous friction is considered

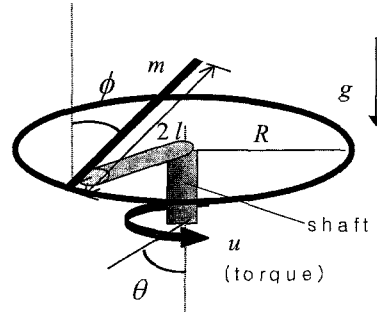


Fig. 1 Schematic diagram of the rotational inverted pendulum

in the model and all other frictions are treated as external disturbances, which are considered through a disturbance observer. Then, the equation of motion for the RIPS can be derived by the Lagrangian method as follows :

$$\begin{bmatrix} J_{arm} + m(R^2 + l^2 \sin^2 \phi) & mL R \cos \phi \\ mL R \cos \phi & J_{pen} + ml^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} c + ml^2 \sin 2\phi \dot{\phi} & -mLR \sin \phi \dot{\phi} \\ -ml^2 \sin \phi \cos \phi \dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ -mgl \sin \phi \end{bmatrix} = \begin{bmatrix} f_\tau \\ 0 \end{bmatrix} \quad (1)$$

where  $m$ ,  $c$ ,  $J_{arm}$  and  $J_{pen}$  are the mass of the pendulum, the damping coefficient, the inertias of the arm and the pendulum, respectively. The coefficient matrices of the  $\ddot{\theta}$ ,  $\ddot{\phi}$ ,  $\dot{\theta}$ ,  $\dot{\phi}$ , which contain the nonlinear coupled term with each states, can be linearized near the upright position. And, the state equation form of (1) is

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{FH}{F^2 - EG} & \frac{cG}{F^2 - EG} & 0 \\ 0 & \frac{EH}{F^2 - EG} & -\frac{cF}{F^2 - EG} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{G}{F^2 - EG} \\ \frac{F}{F^2 - EG} \end{bmatrix}$$

$$E = J_{arm} + mR^2, \quad F = mL R, \quad G = J_{pen} + ml^2, \quad H = -mgl,$$

$$x(t) = [\theta \ \phi \ \dot{\theta} \ \dot{\phi}]^T$$

Table 1 shows the system parameters estimated by the signal compression method and measured by experiments.

**Table 1** System parameters of the plant

Parameter	$J_{arm}$	$J_{pen}$	$m$	$R$	$l$	$c$
Value	0.083	0.0015	0.12	0.29	0.16	0.126
Unit	kgm <sup>2</sup>	kgm <sup>2</sup>	kg	m	m	Nms/rad

### 3. Control System Design of the Rotational Inverted Pendulum

#### 3.1 Output feedback sliding mode controller design

The procedure for designing a sliding mode control system is composed of two phases. The first phase consists of selecting a sliding surface or a switching surface in order to assure a desired sliding motion. The second phase consists of selecting a control law that will force the trajectories of system states onto the sliding surface and keep them on the surface.

##### 3.1.1 The selection of sliding surface

To select the sliding surface by using the output information and to determine the parameters of the reduced order observer and controller, a linear time-invariant model with some uncertainties is considered as follows :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f(t, x, u) \\ y(t) = Cx(t) \end{cases} \quad (3)$$

where  $x \in \mathbb{R}^n$ ,  $U \in \mathbb{R}^m$ ,  $y \in \mathbb{R}^p$  with  $m \leq p \leq n$  and  $f(\cdot)$  is an unknown function, which represents model uncertainties in the system that satisfy the matching condition. It is assumed that the transformation,  $T$ , of coordinates on the matrices  $A$ ,  $B$  and  $C$  exists such that

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, C = [0 \ T] \quad (4)$$

where

$$A_{11} = \left[ \begin{array}{cc|c} A_{11}^0 & A_{12}^0 & A_{12}^m \\ \hline 0 & A_{22}^0 & \\ \hline 0 & A_{21}^0 & A_{22}^m \end{array} \right] \quad (5)$$

and,  $A_{11} \in \mathbb{R}^{(n-m) \times (n-m)}$ ,  $B_2 \in \mathbb{R}^{m \times m}$ ,  $T \in \mathbb{R}^{p \times p}$ ,  $A_{11}^0 \in \mathbb{R}^{r \times r}$ ,  $A_{22}^0 \in \mathbb{R}^{(n-p-r) \times (n-p-r)}$ ,  $A_{21}^0 \in \mathbb{R}^{(p-m) \times (n-p-r)}$ .

In order to use the output information in the selection of the sliding surface and the corresponding control law, the switching surface is supposed to have the form  $s(t) = FCx(t)$ . And let us change  $F$  to be the canonical form of the premise, that is,

$$S = \{ x \in \mathbb{R}^n ; FCx(t) = 0 \} = [F_1 C_1 + F_2] x(t) \quad (6)$$

where

$$C_1 = [0_{(p-m) \times (n-p)} I_{(p-m)}], \\ F \in \mathbb{R}^{m \times p}, F_1 \in \mathbb{R}^{m \times (p-m)}, F_2 \in \mathbb{R}^{(m \times m)}$$

Then, the reduced order sliding motion is governed by a free motion with system matrix

$$A_{11}^s = A_{11} - A_{12} F_2^{-1} F_1 C_1 \quad (7)$$

Define  $G = F_2^{-1} F_1$ , then the sliding surface design problem is equivalent to an output feedback stabilization problem for the system  $(A_{11}, A_{12}, C_1)$ . If the original system has invariant zeros,  $A_{12}$  and  $A_{12}^m$  in (4) and (5) are partitioned as  $[A_{121} \ A_{122}]^T$  and,  $[A_{121}^m \ A_{122}^m]^T$  respectively and the new sub-system  $(\tilde{A}_{11}, A_{122}, \tilde{C}_1)$  can be considered as

$$\tilde{A}_{11} = \begin{bmatrix} A_{22}^0 & A_{122}^m \\ A_{21}^0 & A_{22}^m \end{bmatrix}, \tilde{C}_1 = [0_{(p-m) \times (n-p-r)} I_{(p-m)}] \quad (8)$$

As a result it can be shown (Lee and Aoshima, 1986) that the spectrum of  $A_{11}^s$  contains the invariant zeros of  $(A, B, C)$  and in particular

$$\lambda(A_{11} - A_{12}GC_1) = \lambda(A_{11}^0) \cup \lambda(\tilde{A}_{11} - A_{122}G\tilde{C}_1) \quad (9)$$

If the pair  $(A, B)$  is completely controllable then the pair  $(\tilde{A}_{11}, A_{122})$  is completely controllable and the pair  $(\tilde{A}_{11}, \tilde{C}_1)$  is completely observable by construction and thus if the triple  $(\tilde{A}_{11}, A_{122}, \tilde{C}_1)$  satisfies the Kimura-Davison conditions, output feedback pole placement methods can be used to place poles appropriately. However, if the Kimura-Davison conditions are not satisfied by the nominal triple, the system can be augmented by an appropriately dimensioned dynamic compensator, which dynamics is as follows :

$$\dot{x}_c(t) = Hx_c(t) + Dy(t) \tag{10}$$

where the matrices  $H \in \mathbb{R}^{(q \times q)}$  and  $D \in \mathbb{R}^{q \times p}$  are to be determined. Define a new sliding surface in the augmented states space, formed from the plant and compensator state spaces, as

$$S_c = \{ (x, x_c) \in \mathbb{R}^{n+q}; F_c x_c + FCx = 0 \} \tag{11}$$

where  $F_c \in \mathbb{R}^{m \times q}$  and  $F \in \mathbb{R}^{m \times p}$ . Define  $D_1 \in \mathbb{R}^{q \times (p-m)}$ ,  $D_2 \in \mathbb{R}^{q \times m}$  as

$$[D_1 \ D_2] = DT \tag{12}$$

If the states of the uncertain system are partitioned as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{matrix} \updownarrow n-m \\ \updownarrow m \end{matrix} \tag{13}$$

then the compensator can be written as

$$\dot{x}_c(t) = Hx_c(t) + D_1 C_1 x_1(t) + D_2 x_2(t) \tag{14}$$

And, consider the system

$$\begin{cases} \dot{\hat{x}}(t) = \tilde{A}_{11} \hat{x}(t) + A_{122} \tilde{u}(t) \\ \tilde{y}(t) = \tilde{C}_1 \hat{x}(t) \end{cases} \tag{15}$$

where  $\tilde{u}$  and  $\tilde{y}$  are the (fictitious) inputs and outputs, respectively. After passing through several phases, a reduced order Luenburger observer for the system (15) and the state feedback law using the observer states and the outputs are given by

$$\begin{cases} \dot{z}(t) = Hz(t) + D_1 \tilde{y}(t) + D_2 \tilde{u}(t) \\ \tilde{u}(t) = -K_c z(t) - K_y \tilde{y}(t) \end{cases} \tag{16}$$

where

$$H = A_{22}^0 + LA_{21}^0, \quad D_1 = A_{122}^m + LA_{22}^m - (A_{22}^0 + LA_{21}^0)L, \\ D_2 = A_{1221} + LA_{1222}, \quad K_c = K_1, \quad K_y = K_2 - K_1L,$$

$$K = \begin{bmatrix} n-r-p & p-m \\ \tilde{K}_1 & \tilde{K}_2 \end{bmatrix}$$

and  $L \in \mathbb{R}^{(n-p-r) \times (p-m)}$  is any gain matrix so that  $A_{22}^0 + LA_{21}^0$  is stable,  $K$  is stable and partition the state feedback matrix for the controllable pair  $(\tilde{A}_{11}, A_{122})$  so that  $(\tilde{A}_{11} - A_{122}K)$  is stable. And,  $A_{1221}$  and  $A_{1222}$  are the sub-matrices of  $A_{122}$  as follows :

$$A_{122} = \begin{bmatrix} A_{1221} \\ A_{1222} \end{bmatrix} \begin{matrix} \updownarrow n-p-r \\ \updownarrow p-m \end{matrix} \tag{17}$$

Then the closed system from (15) in conjunction with (16) is as follows :

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} - A_{122}K_y \tilde{C}_1 & -A_{122}K_c \\ (D_1 - D_2K_y) \tilde{C}_1 & H - D_2K_c \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ z(t) \end{bmatrix} \tag{18}$$

### 3.1.2 The selection of control law

Assume that there are  $r$  (stable) invariant zeros and partition the state vector  $x_1$  from (13) so that

$$x_1 = \begin{bmatrix} x_r \\ x_{11} \\ x_{12} \end{bmatrix} \begin{matrix} \updownarrow r \\ \updownarrow n-p-r \\ \updownarrow p-m \end{matrix} \tag{19}$$

and define a new dynamical system by

$$\begin{aligned} \dot{z}_r(t) &= A_{11}^0 z_r(t) + A_{12}^0 x_c(t) \\ &+ (A_{121}^m - A_{12}^0 L) x_{12}(t) + A_{121} x_2(t) \end{aligned} \tag{20}$$

and augment (14) with (20) to form a new compensator.

$$\dot{\hat{x}}_c(t) = \begin{bmatrix} A_{11}^0 & A_{12}^0 \\ 0 & H \end{bmatrix} \hat{x}_c + \begin{bmatrix} (A_{121}^m - A_{12}^0) & A_{121} \\ D_1 & D_2 \end{bmatrix} T^T y(t) \tag{21}$$

Also, the sliding surface  $S_c$  can be written as

$$S_c = \{ \hat{x} \in \mathbb{R}^n : S \hat{x} = 0 \} \tag{22}$$

where  $S = F_2 [0_{m \times r} \ K_c \ K_y \ I_m]$ ,  $\hat{x} = [z_r^T \ x_c^T \ x_{12}^T \ x_2^T]^T$ , and a linear feedback component

$$u_1(t) = -[A^{-1} S \hat{A} - A^{-1} \Phi S] \hat{x}(t) \tag{23}$$

where  $A = SB$ ,  $\Phi \in \mathbb{R}^{m \times m}$ , is stable design matrix, and

$$\hat{A} = \begin{bmatrix} A_{11}^0 & A_{12}^0 & A_{121}^m - A_{12}^0 L & A_{121} \\ 0 & H & D_1 & D_2 \\ 0 & A_{21}^0 & A_{21}^m - A_{21}^0 L & A_{1222} \\ A_{211} & A_{212} & A_{213} - A_{212} L & A_{22} \end{bmatrix}$$

The sliding mode control law is then

$$u(t) = u_1(t) + u_n(t) \tag{24}$$

where

$$u_n(t) = \begin{cases} \rho A^{-1} \text{sgn}(Ps(t)) & \text{for } s(t) \neq 0 \\ 0 & \text{for } s(t) = 0 \end{cases} \tag{25}$$

$P$  is the unique positive definite solution to the Lyapunov equation

$$P \Phi + \Phi^T P = -I \tag{26}$$

$\rho$  is defined as follows : (Young, 1993)

$$\rho = \frac{\kappa \|A\| \|u_1(t)\| + \|A\| \Phi(y) + \gamma_2}{1 - \kappa \chi(A)} \tag{27}$$

where  $\gamma_2$  is a small positive constant,  $\kappa$  and  $\kappa\chi(\Lambda)$  are small positive real numbers which satisfy the following conditions

$$\kappa < \sqrt{\lambda_{\min}(\mathbf{B}^T\mathbf{B})}, \kappa\chi(\Lambda) \|\mathbf{B}^{-1}\| < 1 \quad (28)$$

And, the  $\text{sgn}(\cdot)$  in Eq. (25) is replaced by the sigmoid-like function  $v_\delta(s)$  to suppress the chattering of the control input.

$$v_\delta(s) = \frac{Ps(t)}{\|Ps(t)\| + \delta} \quad (29)$$

where  $\delta$  is a small positive constant.

### 3.2 Disturbance observer design

The basic concept of the disturbance observer is to reconstruct external disturbances by the inverse model and to cancel external disturbances by reconstructed ones in the feedback. Figure 2 shows the control system structure with the disturbance observer. In Fig. 2,  $u$ ,  $d$ ,  $y$ ,  $\hat{d}$  are control input, disturbance, output and estimated disturbance respectively. The system output  $y(s)$  in Fig. 2 is represented as follows :

$$y(s) = G_{dy}d(s) + G_{uy}u(s) + G_{\xi y}\xi(s) \quad (30)$$

where

$$G_{dy}(s) = \frac{PP_n(1-Q)}{Q(P-P_n) + P_n}$$

$$G_{uy}(s) = \frac{PP_n}{Q(P-P_n) + P_n}$$

$$G_{\xi y}(s) = \frac{PQ}{Q(P-P_n) + P_n}$$

One of the most important things in the design of the disturbance observer is to select the  $Q(s)$ , which has an effect on the robustness of the system and the performance of disturbance rejection. If  $Q(s) \approx 1$ , the disturbance observer can make the whole system to be robust because of eliminating low frequency external torques and differences between the nominal plant and the real one, and if  $Q(s) \approx 0$ , there is no noise effects and the observer effects on the system does not nearly exist. Therefore, to eliminate the modeling error and external torque in the low frequency range and to eliminate sensor noises in the high

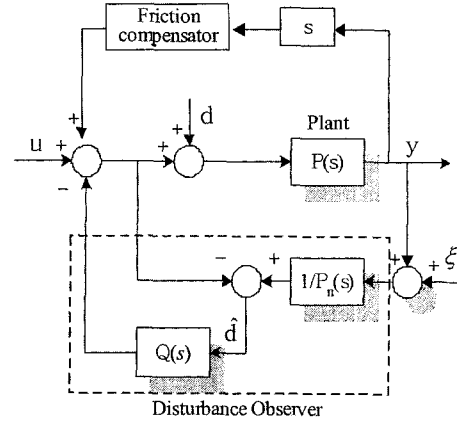


Fig. 2 Structure of the disturbance observer

frequency range the shape of the frequency response is  $Q(s) \approx 1$  at the low frequency range and  $Q(s) \approx 0$  at the high frequency range.

In this paper the form of the  $Q(s)$  which was proposed (Umeno et al., 1993) has constructed 2 relative degree of freedom as follows :

$$Q(s) = \frac{1 + \sum_{k=1}^{N-2} a_k (s\tau)^k}{1 + \sum_{k=1}^N a_k (s\tau)^k} \quad (31)$$

where  $N$  is the order of the low pass filter  $Q(s)$ ,  $a_k$  is a tuning parameter and  $\tau$  is the inverse of cut-off frequency, which is appropriately selected by considering the frequency ranges of the plant dynamics, disturbances and sensor noises. And to avoid the phase lag phenomenon in the higher order filter design, a modified 3rd-order filter is selected.

$$Q(s) = \frac{3\beta^2s + \alpha\beta^3}{s^3 + 3\alpha\beta s^2 + 3\beta^2s + \alpha\beta^3} \quad (32)$$

The parameters and  $\alpha$  are  $\beta$  used to improve the compensation of phase lag and performance of the robust stability, selected as  $\alpha=0.3$  and  $\beta=46$  in this paper.

In designing of the control system with disturbance observer, feedback control is generally carried out at the same time to compensate the performance of command tracking. Especially, since the disturbance observer is not proper in the large model uncertainties, the modeling error should be minimized to have good performance of the

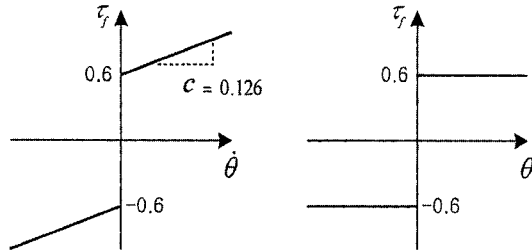


Fig. 3 Estimated coulomb and viscous friction

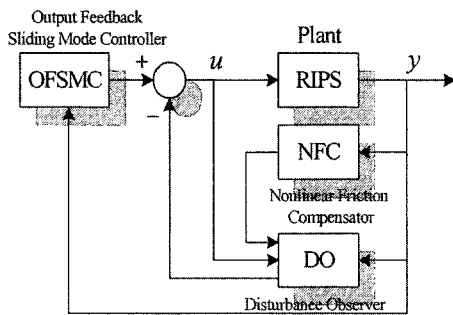


Fig. 4 Output feedback sliding mode controller with the disturbance observer

disturbance observer. So the friction compensator is added in this system, to consider Coulomb friction torque in the joint between the horizontal arm shaft and the DC motor. Figure 3 shows the friction model which is considered in this paper. The  $c$  is a coefficient of the viscous friction. Figure 4 shows the structure of the output feedback sliding mode controller with a disturbance observer.

#### 4. Simulation and Experiment

In order to show the performance and robustness of the OFSMC/DO system for the RIPS, it is compared with the LQ control and OFSMC systems through computer simulation and experiment. Figure 5 shows the schematic diagram of the experimental setup. And Figure 6 shows the photograph of the experimental setup. In this experiment, a low-pass filter with cut-off frequency of 100 rad/sec and damping ratio of 0.7 is adopted to eliminate sensor noises. The sampling time is chosen 1 msec to avoid aliasing problems in the discretization process.

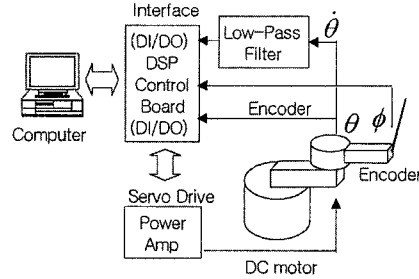


Fig. 5 Schematic diagram of the experimental setup

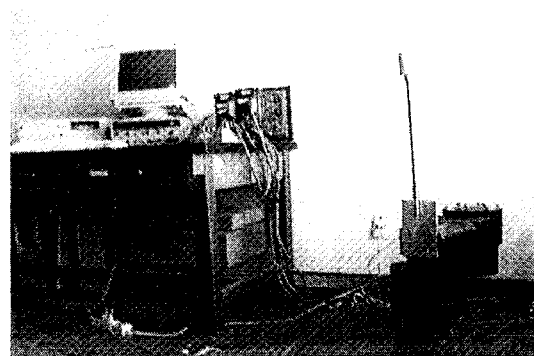


Fig. 6 Photograph of the experimental setup

#### 4.1 Simulation results

Figure 7 shows the comparison of the regulation performance of the RIPS with initial position of 0.37 rad. The LQ control system has the smallest overshoot in the pendulum position and poor regulation performance of the horizontal arm position. The OFSMC system has also a big overshoot and poor regulation. And, the OFSMC/DO system has the biggest overshoot and shortest settling time in the pendulum position and outstanding regulation performance of the horizontal arm. Figure 8 shows the response of control systems for the tappings, the magnitude of tappings considered are 0.05 rad near 2 sec and 0.1 rad near 3.5 sec. The LQ control system has a good performance of pendulum position regulating, but the performance of horizontal position is bad because the LQ control system is designed as a SISO system. And the OFSMC systems with/without DO system have good transient responses for the pendulum position control. However, in the case of the performance of horizontal position control, the OFMC system with DO is more robust regulation performance than the OFMC

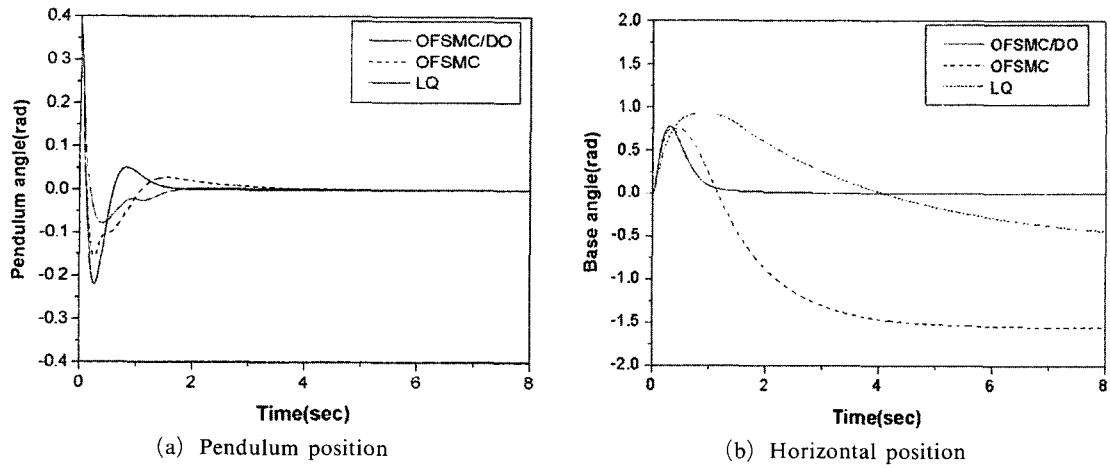


Fig. 7 Simulation result with  $\phi(0) = 0.37$

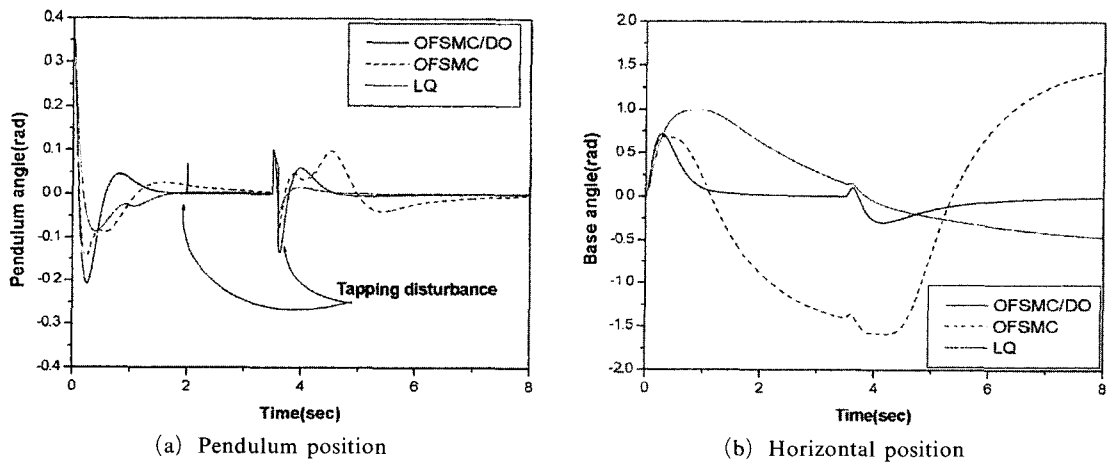


Fig. 8 Simulation result with tapping

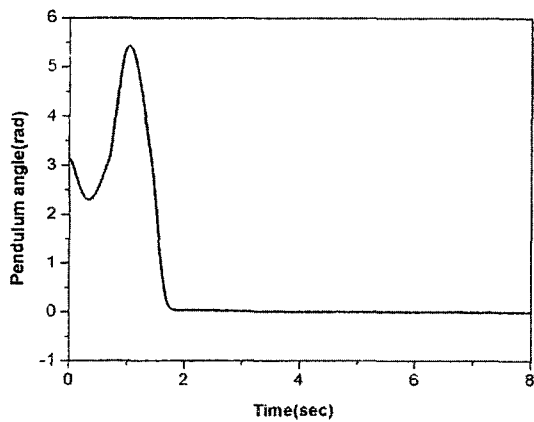


Fig. 9 Simulation result of the pendulum position for swing-up control

system without DO. Also, the swing-up motion is performed and the OFSMC/DO system is used for swing-up. Fig. 9 shows the time response from the bottom position to the top position of the pendulum.

#### 4.2 Experimental results

Figure 10 shows the time responses of the pendulum and horizontal arm with initial position of 0.37 rad for the OFSMC/DO, OFSMC, and LQ control systems. Experimental results are some difference with simulation results, because there is an uncertainty in real system that is not considered during designing control systems. The

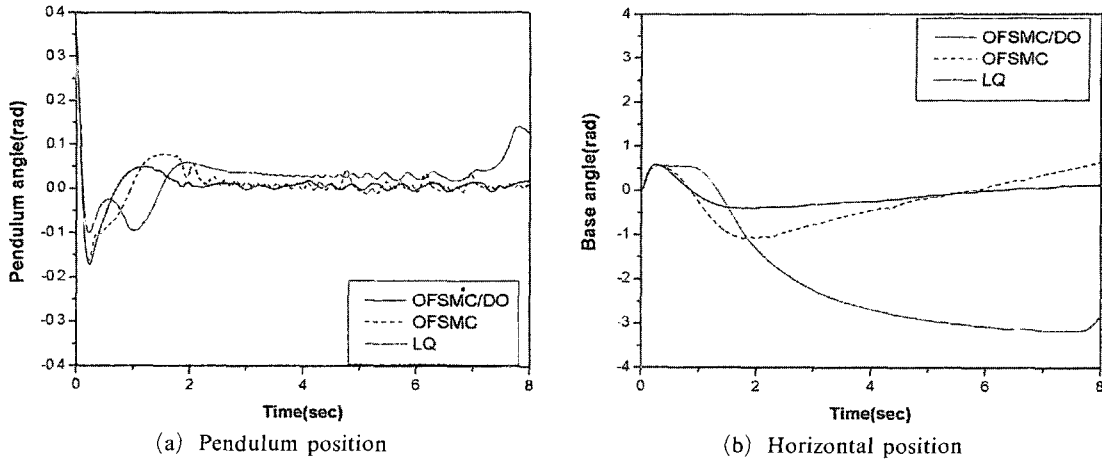


Fig. 10 Experimental result with  $\phi(0) = 0.37$

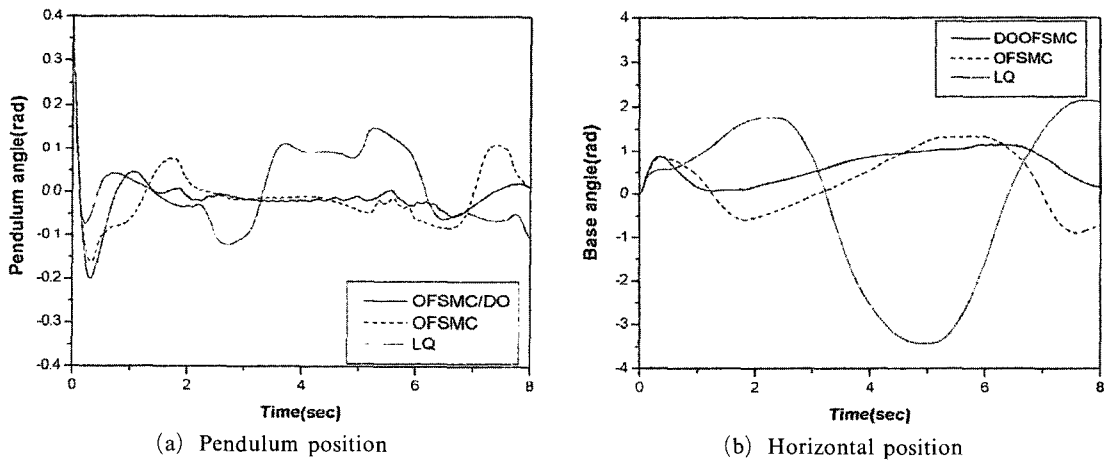


Fig. 11 Experimental result with slushing effect

LQ control system has the worst response due to model uncertainties such as actuator dynamics and nonlinear effects of the plant, and the pendulum is sometimes fallen down. In the response of the horizontal arm, the arm is shaken continuously. In the case of the OFSMC with/without DO system has more good regulation performance than LQ system.

To show the robust performance of those control systems, the tip with 60g water which is for making a disturbance is attached to the end of the pendulum. As shown in Fig. 11, the OFSMC/DO system also has more good performance of the regulations than the performance of LQ and OFMC even when the surface of the water is

slopping from side to side. However, the position of the pendulum and horizontal arm are continuously shaken for all control systems. Figure 12 shows the time response for the tapping disturbances, which apply the plant in the steady states, every 2 sec, two times. The OFSMC/DO system has better performance than the OFSMC system and the response of the LQ control system is omitted because of too bad performance. Figure 13 shows the result of the swing-up control using the proposed control method. This result shows that the proposed control method has ability to implement swing-up system which starts from stable position to maintain the unstable inverted position. From these experimental



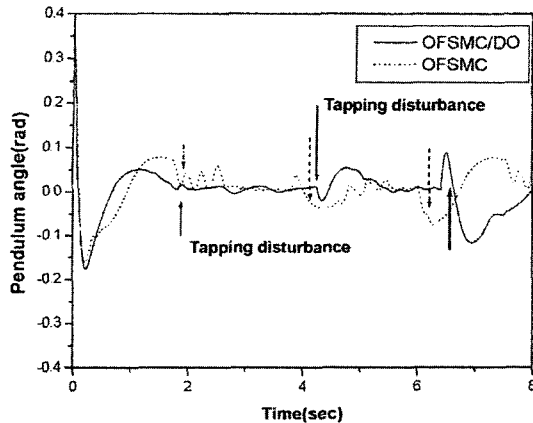


Fig. 12 Experimental result of the pendulum position with tapping disturbance

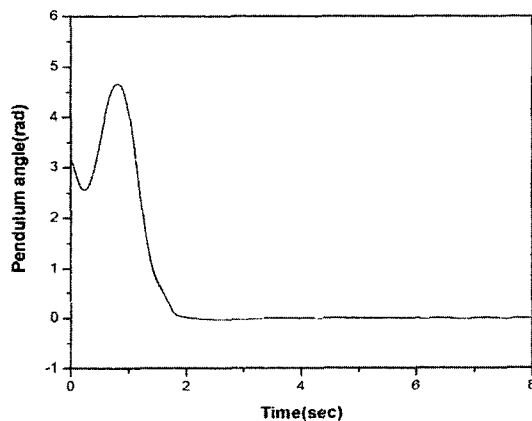


Fig. 13 Experimental result for swing-up control

results, it is found that the OFSMC/DO system has ability to overcome the limit of the control system to be constructed on the base of the linear model and robustness under the arbitrary initial position and initial velocity conditions.

## 5. Conclusion

The dynamic equations of the RIPS are modeled by the signal compression method and the OFSMC/DO method is proposed and applied to a RIPS which is an under-actuated system and has the problem of unattainable velocity state. The OFSMC/DO method is a sliding mode control method using the parameterization of both the hyperplane and the compensator for output

feedback with disturbance observer which estimates the disturbance and some model uncertainties. The results of simulation and experiment show that the OFSMC/DO system is able to stabilize and has better performance and robustness compared with the OFSMC and LQ control systems for under-actuated systems such as RIPS and robots. Especially, the experimental results show that the OFSMC/DO system is very useful in real environmental problems such as analog sensor noises and external disturbances.

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