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SOME NEW ASYMMETRIC ORTHOGONAL ARRAYS

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ABSTRACT

In this paper we make use of the parity check matrices of the codes based on inverting construction Y_1 to construct a number of new asymmetric orthogonal arrays with higher strength and higher number of levels using the method of construction of asymmetric orthogonal arrays given by Suen *et al.* (2001).

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1. INTRODUCTION

Factorial experiments with mixed levels are often encountered in practice because the choice of factor levels may vary with the nature of the factor. Asymmetrical orthogonal arrays defined by Rao (1973) are commonly used for planning such experiments. They have been used extensively by Taguchi (1987) and his colleagues in industrial experiments for quality improvement. Their use in agricultural experiments has also been widespread. Asymmetrical orthogonal arrays play a crucial role in experimental design as universally optimal fractions of asymmetric factorials. See Cheng (1980) and Mukerjee (1982).

A large number of techniques are known for constructing orthogonal arrays based on Galois field, finite geometry, difference schemes, Hadamard matrices, mutually orthogonal latin squares and error correcting codes. For an excellent review on these methods, see Hedayat *et al.* (1999), Dey and Mukerjee (1999) and Wu and Hamada (2000).

The literature on the construction of asymmetric orthogonal arrays of strength higher than 2 is scanty. Cock and Stufken (2000) give a method for constructing asymmetric orthogonal arrays of strength 2 with a large number of 2-level

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factors using Hadamard matrices. Suen *et al.* (2001) gave a general method of constructing asymmetric orthogonal arrays of arbitrary strength and obtained several new families of tight asymmetric orthogonal arrays of strength 3 and some new families of asymmetric orthogonal arrays of strength 4.

An orthogonal array OA(N, n, $q_1 \times q_2 \times \cdots \times q_n$, g) is an $N \times n$ matrix with symbols in the i^{th} column from a finite set of q_i (≥ 2) symbols, $1 \leq i \leq n$, such that in every $N \times g$ submatrix, all possible combinations of symbols appear equally often as a row. In particular, if $q_1 = \cdots = q_n = q$, say, then we get asymmetric orthogonal array which will be denoted by OA(N, n, q, g).

Getting motivated from the statement "Find a good way to use error-correcting codes to construct asymmetric orthogonal arrays" mentioned in Hedayat *et al.* (1999), in this paper we make use of the parity check matrices of the codes based on inverting construction Y_1 to construct a number of new asymmetric orthogonal arrays with higher strength and higher number of levels using the method of construction of asymmetric orthogonal arrays given by Suen *et al.* (2001).

We construct asymmetric orthogonal arrays from the codes obtained using inverting construction Y_1. These asymmetric orthogonal arrays have strength greater than 2.

Section 2 contains a brief introduction of coding theory. In Section 3, asymmetric orthogonal arrays are constructed using codes based on inverting construction Y_1 given by Edel and Bierbrauer (1998).

2. CODING THEORY

A linear $[n, k, d]_q$ code C over GF(q), q prime or prime power, of length n, dimension k and minimum distance d is a k-dimensional subspace of n-dimensional vector space $V_n(q)$. The elements of C are called codewords. The minimum distance d of the code is the smallest number of positions in which two different codewords of C differ. Equivalently, d is the smallest number of nonzero symbols in any nonzero codeword of C. A linear code may be concisely specified by giving a $k \times n$ generator matrix $\mathbf{G}$ whose rows form a basis for the code. The generator matrix can always be expressed in the form

$$\mathbf{G} = [\mathbf{I}_k \ \ \mathbf{A}]$$

where $\mathbf{A}$ is a $k \times (n-k)$ matrix with elements from GF(q). The dual code $C^{\perp}$ with parameters $[n, n-k, d^{\perp}]_q$ of an $[n, k, d]_q$ code C is of dimension $n-k$ and its generator matrix $\mathbf{H}$ can always be written in the form

$$\mathbf{H} = [-\mathbf{A}^t \;\; \mathbf{I}_{n-k}].$$

$\mathbf{H}$ is called as the parity check matrix of code C. Any $d-1$ columns of the parity check matrix, are linearly independent. For more details, see MacWilliams and Sloane (1977).

3. ASYMMETRIC ORTHOGONAL ARRAYS

Most of the methods presently known for constructing asymmetric orthogonal array apply only to arrays of strength 2. However, relatively less work on the construction of asymmetric orthogonal arrays of strength greater than two is available. For a review on these methods, see Dey and Mukerjee (1998). Suen *et al.* (2001) gave a general method of constructing asymmetric orthogonal arrays of arbitrary strength and obtained several new families of tight asymmetric orthogonal arrays of strength 3 and some new families of asymmetric orthogonal arrays of strength 4. They used the concept of full column rank that is linear independence of the columns of $\mathbf{C}$ matrix, which they constructed using the properties of finite fields. The method suggested by them is useful for us in connecting coding theory and asymmetric orthogonal arrays. The proposed construction is based on the method given by them and using the property of dual distance of parity check matrix of linear code.

Construction method suggested by Suen *et al.* (2001) considers an OA(N, n, $q_1 \times q_2 \times \cdots \times q_n$, g) whose columns are called as factors denoted by $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$. Also, consider Galois field GF(q), of order q, where q is a prime or prime power.

For the factor $\mathbf{F}_i$ $(1 \leq i \leq n)$, define u_i columns, say $\mathbf{p}_{i_1}, \ldots, \mathbf{p}_{i_{u_i}}$, each of order $r \times 1$ with elements from GF(q). Thus, for the n factors, we have in all $\sum u_i$ columns. Let $\boldsymbol{\xi}$ be a $q^r \times r$ matrix whose rows are all possible r-tuples over GF(q). Their result is stated here in the form of a theorem as follows:

THEOREM 3.1. *Consider an $r \times \sum u_i$ matrix $\mathbf{C} = [\mathbf{A}_1 : \mathbf{A}_2 : \cdots : \mathbf{A}_n]$, where $\mathbf{A}_i = [\mathbf{p}_{i_1} \cdots \mathbf{p}_{i_{u_i}}]$, $1 \leq i \leq n$, such that for every choice of g matrices $\mathbf{A}_{i_1}, \ldots, \mathbf{A}_{i_g}$ from $\mathbf{A}_1, \ldots, \mathbf{A}_n$, the $r \times \sum u_{i_j}$ matrix $[\mathbf{A}_{i_1} \ldots \mathbf{A}_{i_g}]$ has full column rank over* GF(q). *Then an* OA(q^r, n, $(q^{u_1}) \times \cdots \times (q^{u_n})$, g) *can be constructed. For this theorem to hold, it is necessary that $r \geq \sum u_{i_j}$ for each choice of g indices $i_1, \ldots, i_g$ from $1, \ldots, n$.*

Suen *et al.* (2001) presented methods for choosing $\mathbf{C}$ to satisfy the conditions of Theorem 3.1 and generated orthogonal arrays of strength three or four. We do not construct these **C** matrices; instead, we use the parity check matrices (having the property that any $d-1$ columns are linearly independent) of the linear codes (in coding theory) based on inverting construction Y_1 given by Edel and Bierbrauer (1998). Theorem 3.1 is then used to construct asymmetric orthogonal arrays of higher strength and with higher number of levels.

Edel and Bierbrauer (1998) gave a computer based method for extending linear codes, which is inverse of construction Y_1. As a result they obtained codes with record breaking parameters. Construction Y_1 and inverting construction Y_1 are explained in Appendix. We illustrate the method of construction of asymmetric orthogonal array with the help of an example as follows:

EXAMPLE 3.1. Let us consider the parity check matrix of $[19, 11, 7]_7$ code given by Edel and Bierbrauer (1998):

$$\mathbf{H} = \begin{pmatrix} 1&0&0&0&0&0&0&3&5&2&4&1&1&0&0&4&5&3&3 \\ 0&1&0&0&0&0&0&1&3&1&4&1&0&3&0&3&5&3&1 \\ 0&0&1&0&0&0&0&2&5&1&4&0&3&1&0&3&3&4&2 \\ 0&0&0&1&0&0&0&1&5&1&0&3&1&2&0&1&3&0&5 \\ 0&0&0&0&1&0&0&1&5&0&5&1&2&1&0&1&0&3&5 \\ 0&0&0&0&0&1&0&1&0&3&4&2&1&1&0&0&1&4&3 \\ 0&0&0&0&0&0&1&0&1&1&1&1&1&1&0&0&0&0&6 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1 \end{pmatrix}.$$

To construct an $\mathrm{OA}(7^8, 16, (7^2)^3 \times 7^{13}, 3)$, we choose the following matrices corresponding to the factors of the array:

$$\mathbf{A}_1 = \begin{pmatrix} 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \end{pmatrix}, \quad \mathbf{A}_2 = \begin{pmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \end{pmatrix}, \quad \mathbf{A}_3 = \begin{pmatrix} 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \end{pmatrix} \quad \text{and } \mathbf{A}_i, \ \text{for } 4 \le i \le 16. \tag{3.1}$$

$\mathbf{A}_i$'s are the $(i+3)^{th}$ column of the matrix **H** for $4 \le i \le 16$. The rank condition of Theorem 3.1 is always satisfied for $g = 3$ by the above matrix **H**. This can also be shown with above choices of $\mathbf{A}_i$ matrices corresponding to the 16 factors.

(i) Let $i, j, k \in \{4, 5, \ldots, 16\}$; $i \neq j \neq k$. For this choice of the indices i, j, k, the matrix $[\mathbf{A}_i \ \mathbf{A}_j \ \mathbf{A}_k]$ will always have rank 3 since minimum distance of the associated code is 7 which implies that any 6 columns of the parity check matrix of the associated code are linearly independent.

(ii) Let $i = 1$ and $j, k \in \{4, 5, \dots, 16\}$; $i \neq j \neq k$. For this choice of indices i, j, k, the matrix $[\mathbf{A}_i\ \mathbf{A}_j\ \mathbf{A}_k]$ will always have rank 4 since any 6 or fewer columns of $\mathbf{H}$ are linearly independent.

(iii) Let $i = 1$, $j = 2$ and $k \in \{4, 5, \dots, 16\}$; $i \neq j \neq k$. For this choice of indices i, j, k, the matrix $[\mathbf{A}_i\ \mathbf{A}_j\ \mathbf{A}_k]$ will always have rank 5 since any 6 or fewer columns of $\mathbf{H}$ are linearly independent.

(iv) Let $i = 1$, $j = 2$ and $k = 3$. For this choice of indices i, j, k, the matrix $[\mathbf{A}_i\ \mathbf{A}_j\ \mathbf{A}_k]$ will always have rank 6 since any 6 or fewer columns of $\mathbf{H}$ are linearly independent.

Thus, in each case, the rank condition of Theorem 3.1 is satisfied and the desired array can be constructed.

Compute $\boldsymbol{\xi}\mathbf{H}$ where $\boldsymbol{\xi}$ is a $7^8 \times 8$ matrix whose rows are all possible 8-tuples over GF(7). Next, replace the 49 combinations $(00), (01), \dots, (66)$ in the first two columns by 49 distinct symbols, $0, 1, 2, 3, \dots, 48$, respectively. Repeat this for the next two sets of two columns. Then, we get an $\mathrm{OA}(7^8, 16, (7^2)^3 \times 7^{13}, 3)$.

Similarly, taking $\mathbf{A}_1$ and $\mathbf{A}_2$ as in (3.1) and $\mathbf{A}_i$'s, as the $(i+2)^{th}$ column of the matrix $\mathbf{H}$ for $3 \leq i \leq 17$, we can construct an $\mathrm{OA}(7^8, 17, (7^2)^2 \times 7^{15}, 4)$. The rank condition of Theorem 3.1 is always satisfied for $g = 4$. Again, replacing the first two columns and then next two columns by 49 distinct symbols, we get an $\mathrm{OA}(7^8, 17, (7^2)^2 \times 7^{15}, 4)$.

We can construct an $\mathrm{OA}(7^8, 18, (7^2) \times 7^{17}, 5)$ by taking $\mathbf{A}_1$ as in (3.1) and $\mathbf{A}_i$'s, as the $(i+1)^{th}$ column of the matrix $\mathbf{H}$ for $2 \leq i \leq 18$. For every choice of 5 matrices, the rank condition of Theorem 3.1 is satisfied for $g = 5$. Hence we get an $\mathrm{OA}(7^8, 18, (7^2) \times 7^{17}, 5)$.

To construct an $\mathrm{OA}(7^8, 16, (7^3) \times (7^2) \times 7^{14}, 3)$, we choose the following matrices corresponding to the factors of the array:

$$\mathbf{A}_1 = \begin{pmatrix} 1&0&0\\ 0&1&0\\ 0&0&1\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0 \end{pmatrix}, \quad \mathbf{A}_2 = \begin{pmatrix} 0&0\\ 0&0\\ 0&0\\ 1&0\\ 0&1\\ 0&0\\ 0&0\\ 0&0 \end{pmatrix} \quad \text{and } \mathbf{A}_i, \text{ for } 3 \leq i \leq 16\,.$$

$\mathbf{A}_i$'s are the $(i+3)^{th}$ column of the matrix $\mathbf{H}$ for $3 \leq i \leq 16$. It can be easily seen that for every choice of 3 matrices the rank condition of Theorem 3.1 is satisfied by the matrix $[\mathbf{A}_j\ \mathbf{A}_k\ \mathbf{A}_\ell]$, $j \neq k \neq \ell \neq j$, $1 \leq j, k, \ell \leq 16$. Hence on computing $\boldsymbol{\xi}\mathbf{H}$, where $\boldsymbol{\xi}$ is a $7^8 \times 8$ matrix whose rows are all possible 8-tuples over GF(7) and replacing the 7^3 combinations under the first three columns by the 343 distinct symbols $0, 1, 2, \ldots, 342$, respectively, we get an $\mathrm{OA}(7^8, 16, (7^3) \times (7^2) \times 7^{14}, 3)$.

We can construct many asymmetric orthogonal arrays with higher strength. The asymmetric orthogonal arrays generated by using the codes based on the inverting construction Y_1 are given in Table 1.

TABLE 1

Code	*Asymmetric orthogonal arrays*	*Code*	*Asymmetric orthogonal arrays*
$[127, 106, 7]_2$	$\mathrm{OA}(2^{21}, 124, (2^2)^3 \times 2^{121}, 3)$ $\mathrm{OA}(2^{21}, 125, (2^2)^2 \times 2^{123}, 4)$ $\mathrm{OA}(2^{21}, 126, (2^2) \times 2^{125}, 5)$ $\mathrm{OA}(2^{21}, 125, (2^3) \times 2^{124}, 4)$	$[155, 133, 7]_2$	$\mathrm{OA}(2^{22}, 152, (2^2)^3 \times 2^{149}, 3)$ $\mathrm{OA}(2^{22}, 153, (2^2)^2 \times 2^{151}, 4)$ $\mathrm{OA}(2^{22}, 154, (2^2) \times 2^{153}, 5)$ $\mathrm{OA}(2^{22}, 153, (2^3) \times 2^{152}, 4)$
$[162, 139, 7]_2$	$\mathrm{OA}(2^{23}, 159, (2^2)^3 \times 2^{156}, 3)$ $\mathrm{OA}(2^{23}, 160, (2^2)^2 \times 2^{158}, 4)$ $\mathrm{OA}(2^{23}, 161, (2^2) \times 2^{160}, 5)$ $\mathrm{OA}(2^{23}, 160, (2^3) \times 2^{159}, 4)$	$[45, 24, 9]_2$	$\mathrm{OA}(2^{21}, 41, (2^2)^4 \times 2^{37}, 4)$ $\mathrm{OA}(2^{21}, 42, (2^2)^3 \times 2^{39}, 5)$ $\mathrm{OA}(2^{21}, 43, (2^2)^2 \times 2^{41}, 6)$ $\mathrm{OA}(2^{21}, 44, (2^2) \times 2^{43}, 7)$ $\mathrm{OA}(2^{21}, 41, (2^3)^2 \times 2^{39}, 4)$ $\mathrm{OA}(2^{21}, 41, (2^3) \times (2^2)^2 \times 2^{38}, 4)$ $\mathrm{OA}(2^{21}, 43, (2^3) \times 2^{42}, 6)$
$[49, 27, 9]_2$	$\mathrm{OA}(2^{22}, 45, (2^2)^4 \times 2^{41}, 4)$ $\mathrm{OA}(2^{22}, 46, (2^2)^3 \times 2^{43}, 5)$ $\mathrm{OA}(2^{22}, 47, (2^2)^2 \times 2^{45}, 6)$ $\mathrm{OA}(2^{22}, 48, (2^2) \times 2^{47}, 7)$ $\mathrm{OA}(2^{22}, 45, (2^3)^2 \times 2^{43}, 4)$ $\mathrm{OA}(2^{22}, 45, (2^3) \times (2^2)^2 \times 2^{42}, 4)$ $\mathrm{OA}(2^{22}, 47, (2^3) \times 2^{46}, 6)$	$[63, 39, 9]_2$	$\mathrm{OA}(2^{24}, 59, (2^2)^4 \times 2^{55}, 4)$ $\mathrm{OA}(2^{24}, 60, (2^2)^3 \times 2^{57}, 5)$ $\mathrm{OA}(2^{24}, 61, (2^2)^2 \times 2^{59}, 6)$ $\mathrm{OA}(2^{24}, 62, (2^2) \times 2^{61}, 7)$ $\mathrm{OA}(2^{24}, 59, (2^3)^2 \times 2^{57}, 4)$ $\mathrm{OA}(2^{24}, 59, (2^3) \times (2^2)^2 \times 2^{56}, 4)$ $\mathrm{OA}(2^{24}, 61, (2^3) \times 2^{60}, 6)$
$[72, 47, 9]_2$	$\mathrm{OA}(2^{25}, 68, (2^2)^4 \times 2^{64}, 4)$ $\mathrm{OA}(2^{25}, 69, (2^2)^3 \times 2^{66}, 5)$ $\mathrm{OA}(2^{25}, 70, (2^2)^2 \times 2^{68}, 6)$ $\mathrm{OA}(2^{25}, 71, (2^2) \times 2^{70}, 7)$ $\mathrm{OA}(2^{25}, 68, (2^3)^2 \times 2^{66}, 4)$ $\mathrm{OA}(2^{25}, 68, (2^3) \times (2^2)^2 \times 2^{65}, 4)$ $\mathrm{OA}(2^{25}, 70, (2^3) \times 2^{69}, 6)$	$[77, 51, 9]_2$	$\mathrm{OA}(2^{26}, 73, (2^2)^4 \times 2^{69}, 4)$ $\mathrm{OA}(2^{26}, 74, (2^2)^3 \times 2^{71}, 5)$ $\mathrm{OA}(2^{26}, 75, (2^2)^2 \times 2^{73}, 6)$ $\mathrm{OA}(2^{26}, 76, (2^2) \times 2^{75}, 7)$ $\mathrm{OA}(2^{26}, 73, (2^3)^2 \times 2^{71}, 4)$ $\mathrm{OA}(2^{26}, 73, (2^3) \times (2^2)^2 \times 2^{70}, 4)$ $\mathrm{OA}(2^{26}, 75, (2^3) \times 2^{74}, 6)$
$[85, 74, 6]_3$	$\mathrm{OA}(3^{11}, 83, (3^2)^2 \times 3^{81}, 3)$ $\mathrm{OA}(3^{11}, 84, (3^2) \times 3^{83}, 4)$ $\mathrm{OA}(3^{11}, 83, (3^3) \times 3^{82}, 3)$	$[95, 83, 6]_3$	$\mathrm{OA}(3^{12}, 93, (3^2)^2 \times 3^{91}, 3)$ $\mathrm{OA}(3^{12}, 94, (3^2) \times 3^{93}, 4)$ $\mathrm{OA}(3^{12}, 93, (3^3) \times 3^{92}, 3)$
$[103, 90, 6]_3$	$\mathrm{OA}(3^{13}, 101, (3^2)^2 \times 3^{99}, 3)$ $\mathrm{OA}(3^{13}, 102, (3^2) \times 3^{101}, 4)$ $\mathrm{OA}(3^{13}, 101, (3^3) \times 3^{100}, 3)$	$[22, 12, 7]_3$	$\mathrm{OA}(3^{10}, 19, (3^2)^3 \times 3^{16}, 3)$ $\mathrm{OA}(3^{10}, 20, (3^2)^2 \times 3^{18}, 4)$ $\mathrm{OA}(3^{10}, 21, (3^2) \times 3^{20}, 5)$ $\mathrm{OA}(3^{10}, 20, (3^3) \times 3^{19}, 4)$ $\mathrm{OA}(3^{10}, 19, (3^3) \times (3^2) \times 3^{17}, 3)$

(continued)

TABLE 1 (*continued*)

Code	*Asymmetric orthogonal arrays*	*Code*	*Asymmetric orthogonal arrays*
$[27,16,7]_3$	$OA(3^{11},24,(3^2)^3\times 3^{21},3)$ $OA(3^{11},25,(3^2)^2\times 3^{23},4)$ $OA(3^{11},26,(3^2)\times 3^{25},5)$ $OA(3^{11},25,(3^3)\times 3^{24},4)$ $OA(3^{11},24,(3^3)\times(3^2)\times 3^{22},3)$	$[34,22,7]_3$	$OA(3^{12},31,(3^2)^3\times 3^{28},3)$ $OA(3^{12},32,(3^2)^2\times 3^{30},4)$ $OA(3^{12},33,(3^2)\times 3^{32},5)$ $OA(3^{12},32,(3^3)\times 3^{31},4)$ $OA(3^{12},31,(3^3)\times(3^2)\times 3^{29},3)$
$[92,76,7]_3$	$OA(3^{16},89,(3^2)^3\times 3^{86},3)$ $OA(3^{16},90,(3^2)^2\times 3^{88},4)$ $OA(3^{16},91,(3^2)\times 3^{90},5)$ $OA(3^{16},90,(3^3)\times 3^{89},4)$ $OA(3^{16},89,(3^3)\times(3^2)\times 3^{87},3)$	$[42,29,7]_3$	$OA(3^{13},39,(3^2)^3\times 3^{36},3)$ $OA(3^{13},40,(3^2)^2\times 3^{38},4)$ $OA(3^{13},41,(3^2)\times 3^{40},5)$ $OA(3^{13},40,(3^3)\times 3^{39},4)$ $OA(3^{13},39,(3^3)\times(3^2)\times 3^{37},3)$
$[108,91,7]_3$	$OA(3^{17},105,(3^2)^3\times 3^{102},3)$ $OA(3^{17},106,(3^2)^2\times 3^{104},4)$ $OA(3^{17},107,(3^2)\times 3^{106},5)$ $OA(3^{17},106,(3^3)\times 3^{105},4)$ $OA(3^{17},105,(3^3)\times(3^2)\times 3^{103},3)$	$[53,39,7]_3$	$OA(3^{14},50,(3^2)^3\times 3^{47},3)$ $OA(3^{14},51,(3^2)^2\times 3^{49},4)$ $OA(3^{14},52,(3^2)\times 3^{51},5)$ $OA(3^{14},51,(3^3)\times 3^{50},4)$ $OA(3^{14},50,(3^3)\times(3^2)\times 3^{48},3)$
$[29,16,8]_3$	$OA(3^{13},26,(3^2)^3\times 3^{23},4)$ $OA(3^{13},27,(3^2)^2\times 3^{25},5)$ $OA(3^{13},28,(3^2)\times 3^{27},6)$ $OA(3^{13},27,(3^3)\times 3^{26},5)$ $OA(3^{13},25,(3^3)\times(3^2)^2\times 3^{22},3)$ $OA(3^{13},26,(3^3)\times(3^2)\times 3^{24},4)$	$[85,70,7]_3$	$OA(3^{15},82,(3^2)^3\times 3^{79},3)$ $OA(3^{15},83,(3^2)^2\times 3^{81},4)$ $OA(3^{15},84,(3^2)\times 3^{83},5)$ $OA(3^{15},83,(3^3)\times 3^{82},4)$ $OA(3^{15},82,(3^3)\times(3^2)\times 3^{80},3)$
$[35,21,8]_3$	$OA(3^{14},32,(3^2)^3\times 3^{29},4)$ $OA(3^{14},33,(3^2)^2\times 3^{31},5)$ $OA(3^{14},34,(3^2)\times 3^{33},6)$ $OA(3^{14},33,(3^3)\times 3^{32},5)$ $OA(3^{14},30,(3^3)^2\times(3^2)\times 3^{27},3)$ $OA(3^{14},30,(3^3)\times(3^2)^2\times 3^{27},4)$	$[24,12,9]_3$	$OA(3^{12},20,(3^2)^4\times 3^{16},4)$ $OA(3^{12},21,(3^2)^3\times 3^{18},5)$ $OA(3^{12},22,(3^2)^2\times 3^{20},6)$ $OA(3^{12},23,(3^2)\times 3^{22},7)$ $OA(3^{12},20,(3^3)^2\times 3^{18},4)$ $OA(3^{12},20,(3^3)^2\times(3^2)\times 3^{17},3)$ $OA(3^{12},20,(3^3)\times(3^2)^2\times 3^{17},4)$ $OA(3^{12},22,(3^3)\times 3^{21},6)$
$[19,10,7]_4$	$OA(4^9,16,(4^2)^3\times 4^{13},3)$ $OA(4^9,17,(4^2)^2\times 4^{15},4)$ $OA(4^9,18,(4^2)\times 4^{17},5)$ $OA(4^9,17,(4^3)\times 4^{16},4)$ $OA(4^9,16,(4^3)\times(4^2)\times 4^{14},3)$	$[65,57,5]_4$	$OA(4^8,63,(4^2)^2\times 4^{61},2)$ $OA(4^8,64,(4^2)\times 4^{63},3)$ $OA(4^8,63,(4^3)\times 4^{62},2)$
$[26,16,7]_4$	$OA(4^{10},23,(4^2)^3\times 4^{20},3)$ $OA(4^{10},24,(4^2)^2\times 4^{22},4)$ $OA(4^{10},25,(4^2)\times 4^{24},5)$ $OA(4^{10},24,(4^3)\times 4^{23},4)$ $OA(4^{10},23,(4^3)\times(4^2)\times 4^{21},3)$	$[145,135,5]_4$	$OA(4^{10},143,(4^2)^2\times 4^{141},2)$ $OA(4^{10},144,(4^2)\times 4^{143},3)$ $OA(4^{10},143,(4^3)\times 4^{142},2)$

(*continued*)

TABLE 1 (*continued*)

Code	Asymmetric orthogonal arrays	Code	Asymmetric orthogonal arrays
$[20, 13, 6]_4$	$OA(4^7, 18, (4^2)^2 \times 4^{16}, 3)$	$[87, 78, 5]_4$	$OA(4^9, 85, (4^2)^2 \times 4^{83}, 2)$
	$OA(4^7, 19, (4^2) \times 4^{18}, 4)$		$OA(4^9, 86, (4^2) \times 4^{85}, 3)$
	$OA(4^7, 18, (4^3) \times 4^{17}, 3)$		$OA(4^9, 85, (4^3) \times 4^{84}, 2)$
$[27, 19, 6]_4$	$OA(4^8, 25, (4^2)^2 \times 4^{23}, 3)$	$[81, 70, 6]_4$	$OA(4^{11}, 79, (4^2)^2 \times 4^{77}, 3)$
	$OA(4^8, 26, (4^2) \times 4^{25}, 4)$		$OA(4^{11}, 80, (4^2) \times 4^{79}, 4)$
	$OA(4^8, 25, (4^3) \times 4^{24}, 3)$		$OA(4^{11}, 79, (4^3) \times 4^{78}, 3)$
$[36, 27, 6]_4$	$OA(4^9, 34, (4^2)^2 \times 4^{32}, 3)$	$[106, 94, 6]_4$	$OA(4^{12}, 104, (4^2)^2 \times 4^{102}, 3)$
	$OA(4^9, 35, (4^2) \times 4^{34}, 4)$		$OA(4^{12}, 105, (4^2) \times 4^{104}, 4)$
	$OA(4^9, 34, (4^3) \times 4^{33}, 3)$		$OA(4^{12}, 104, (4^3) \times 4^{103}, 3)$

A complete list of asymmetric orthogonal arrays generated by using the codes given by Edel and Bierbrauer (1998) with higher number of levels is available with the authors.

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APPENDIX : CONSTRUCTION Y_1 AND INVERTING CONSTRUCTION Y_1

Let C be a q-ary linear code with parameters $[n,\ k,\ d]_q$. Let v be a codeword of the dual code $C^\perp$ of weight w. Then the subcode of C, which consists of the words having vanishing entry at the support of v has parameters $[n-w,\ k-w+1,\ d]$. Construction Y_1 was used to construct the Nordstrom-Robinson code from the Golay Code (see MacWilliams and Sloane, 1977). This operation can be inverted and is known as inverting construction Y_1 given by Edel and Bierbrauer (1998). Let a code C with parameters $[n,\ k,\ d]_q$ be given and let $\mathbf{H}$ be a parity check matrix of C. Let $\mathbf{H}^*$ be obtained by adding a row with entries 0 to $\mathbf{H}$. $\mathbf{H}^*$ is lengthened by adding ℓ columns such that the resulting matrix still has the property that any $d-1$ columns are linearly independent. The lengthened matrix is then the parity check matrix of a code $[n+l,\ k+l-1,\ d]_q$. Now the task is to find as many new columns as possible. The columns added to $\mathbf{H}^*$ will have nonzero entries in the last row. The codes with new parameters have been obtained by using this procedure by Edel and Bierbrauer (1998).

REFERENCES

CHENG, C. S. (1980). "Orthogonal arrays with variable numbers of symbols", *The Annals of Statistics*, **8**, 447-453.

COCK, D. D. AND STUFKEN, J. (2000). "On finding mixed orthogonal arrays of strength 2 with many 2-level factors", preprint.

DEY, A. AND MUKERJEE, R. (1998). "Techniques for constructing asymmetric orthogonal arrays", *Journal of Combinatorics, Information and System Sciences*, **23**, 351–366.

DEY, A. AND MUKERJEE, R. (1999). *Fractional Factorial Plans*, John Wiley, New York.

EDEL, Y. AND BIERBRAUER, J. (1998). "Inverting construction Y_1", *IEEE Transactions on Information Theory*, **44**, 1993.

HEDAYAT, A. S., SLOANE, N. J. A. AND STUFKEN, J. (1999). *Orthogonal Arrays : Theory and Applications*, Springer-Verlag, New York.

MACWILLIAMS AND SLOANE, N. J. A. (1977). *The Theory of Error-Correcting Codes*, North Holland, Amsterdam.

MUKERJEE, R. (1982). "Universal optimality of fractional factorial plans derivable through orthogonal arrays", *Calcutta Statistical Association Bulletin*, **31**, 63–68.

RAO, C. R. (1973). "Some combinatorial problems of arrays and applications to design of experiments", In *Survey of Combinatorial Theory* (J. N. Srivastava, ed.), 349–359, North Holland, Amsterdam.

SUEN, CHUNG-YI, DAS, A. AND DEY, A. (2001). "On the construction of asymmetric orthogonal arrays", *Statistica Sinica*, **11**, 241–260.

TAGUCHI, G. (1987). *System of Experimental Design : Engineering Methods to Optimize Quality and Minimize Costs*, Unipub, New York.

WU, C. F. J. AND HAMADA, M. (2000). *Experiments : Planning, Analysis and Parameter Design Optimization*, John Wiley, New York.