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A STUDY ON GARCH(p, q) PROCESS

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Abstract. We consider the generalized autoregressive model with conditional heteroscedasticity process(GARCH). It is proved that if $\sum_{i=1}^{q} \alpha_i \sigma^2 + \sum_{i=1}^{p} \beta_i < 1$, then there exists a unique invariant initial distribution for the Markov process emdedding the given GARCH process. Geometric ergodicity, functional central limit theorems, and a law of large numbers are also studied.

1. Introduction

Let $\eta_n, n \in \mathbf{Z}$ denote a real-valued discrete time stochastic process. The generalized autoregressive conditional heteroscedasticity process (GARCH(p, q)) is given by $Y_n = \eta_n \sqrt{V}_n$ with $V_n = \delta + \sum_{i=1}^{q} \alpha_i Y_{n-i}^2 + \sum_{i=1}^{p} \beta_i V_{n-i}$, where $p \geq 0$, $q > 0$, $\delta > 0$, $\alpha_i \geq 0$, $i = 1, 2, \cdots, q$ and $\beta_i \geq 0$, $i = 1, 2, \cdots, p$.

For $p = 0$ the process reduces to the autoregressive conditional heteroscedasticity process (ARCH(q)). ARCH model is introduced by Engle [9] and is extended to GARCH by Bollerslev [5], allowing for more flexible lag structure. In the ARCH(q) process the conditional variance is specified as a linear function of the past sample variances only, whereas the GARCH(p, q) process allows lagged conditional variances to enter as well. ARCH/GARCH model has been proved useful in modelling economic phenomena such as foreign exchange rate, interest rate, inflation rate, etc.

For statistical analysis on these models, stationarity, ergodicity and various asymptotic properties are of great importance. A number of theoretical results and their applications for ARCH/GARCH models can be found in, for example, Engle [9], Weiss [27], Bera and Higgins [3], Guégan and Diebolt [11], Lu [19], Borkovec [6] for ARCH models

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and Bollerslev [5], Nelson [21], Bougerol and Picard [7], Li and Li [16], Rudolph [23], Ling [17], Lee and Kim [15] for GARCH processes.

Bollerslev [5] shows that $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1$ is a necessary and sufficient condition for second order stationarity of GARCH(p,q) model. Nelson [21] gave sufficient conditions for strict stationarity and ergodicity of GARCH$(1,1)$ model. Bougerol and Picard [7] proved that the negative Lyapounov exponent of some random matrices is necessary and sufficient for strict stationarity and ergodicity of GARCH(p,q) process. Central limit theorems and the law of large numbers for GARCH(p,q) process are considered in Rudolph [23].

Our aim in this paper is to give sufficient conditions for strict stationarity, geometric ergodicity, a law of large numbers, and functional central limit theorems for the process. To do this, we first rephrase the given process as a properly defined associated Markov process, and study the Markov process and then derive the desired results from that of the Markov chain. (see, e.g., Tjøstheim [24], Tong [25], Lee [14]).

A short overview of the paper is as follows: in Section 2, we define the corresponding Markov chain to GARCH model and prove the existence of strict stationary process. Geometric ergodicity is given in Section 3. Section 4 presents a functional central limit theorem.

General terminologies and relevant results in Markov chain theory are referred to Meyn and Tweedie [20].

2. Strict stationarity

A sequence of univariate stochastic process $Y_n,\ n \in \mathbf{Z}$ is said to be a GARCH(p,q) process if it satisfies the equation $Y_n = \eta_n \sqrt{V_n}$ with

$$V_n = \delta + \sum_{i=1}^{p} \beta_i V_{n-i} + \sum_{i=1}^{q} \alpha_i Y_{n-i}^2, \quad n \in \mathbf{Z}, \tag{2.1}$$

where $\alpha_i \geq 0,\ \ i = 1, 2, \cdots, q$ and $\beta_i \geq 0,\ \ i = 1, 2, \cdots, p,\ \eta_n,\ n \in \mathbf{Z}$ are independent and identically distributed (i.i.d.) random variables with mean $E(\eta_n) = 0$ and variance σ^2. Assume that $\delta > 0$, $p \geq 0$ and $q > 0$. If $p = 0$, then the process is said to be an ARCH(q) process.

There are various ways to represent the GARCH process as a Markov chain (see, e.g., Bougerol and Picard [7], Ling [17], Liu *et al.* [18]).

For GARCH(p,q) process given in (2.1), define a $(p+q-1)\times(p+q-1)$ matrix A_n by

$$A_n = \begin{bmatrix} \tau_n, & \beta_p, & \alpha, & \alpha_q \\ I_{p-1}, & 0, & 0, & 0 \\ \xi_n, & 0, & 0, & 0 \\ 0, & 0, & I_{q-2}, & 0 \end{bmatrix}, \tag{2.2}$$

where

$$\tau_n = (\beta_1 + \alpha_1\eta_n^2, \beta_2, \cdots, \beta_{p-1}) \in R^{p-1},$$
$$\xi_n = (\eta_n^2, 0, 0, \cdots, 0) \in R^{p-1},$$
$$\alpha = (\alpha_2, \alpha_3, \cdots, \alpha_{q-1}) \in R^{q-2},$$

and I_{p-1} and I_{q-2} are the identity matrices of size $p-1$ and $q-2$, respectively. Then $\{A_n\}$ are i.i.d. random matrices. We will always assume that $p,q \geq 2$, by adding some α_i or β_i equal to zero if needed. Now let

$$B = (\delta, 0, 0, \cdots, 0)^t \in R^{p+q-1},$$

and

$$X_n = (V_{n+1}, \cdots, V_{n-p+2}, Y_n^2, \cdots, Y_{n-q+2}^2)^t. \tag{2.3}$$

Then Y_n is a solution of (2.1) if and only if X_n is a solution of

$$X_{n+1} = A_{n+1}X_n + B, \qquad n \in \mathbf{Z}.$$

Since A_k, $k \geq n+1$ are independent of X_n, $\{X_n : n \geq 0\}$ with arbitrarily specified random vector X_0 independent of $\{\eta_n : n \geq 1\}$ can be regarded as a Markov chain with its n-step transition probability function, say, $p^{(n)}(x, dy)$.

A Markov process with n-step transition probability function $p^{(n)}(x, dy)$ is said to be *φ-irreducible* with respect to a nontrivial σ- finite measure φ if $\sum_{n\geq 1} 2^{-n} p^{(n)}(x, A) > 0$ for every x and every A with $\varphi(A) > 0$.

Let $\|\cdot\|$ denote any norm on R^{p+q-1}, and define a subordinated matrix norm on the set of $(p+q-1)\times(p+q-1)$ matrices by

$$\|G\| = \sup\{\frac{\|Gx\|}{\|x\|} : x \in R^{p+q-1}, x \neq 0\}.$$

Let $\rho(G)$ be the spectral radius of the matrix G, i.e.

$$\rho(G) = \max\{|\lambda| : \lambda \text{ is an eigenvalue of } G\}.$$

For matrices $A = (a_{ij})$ and $B = (b_{ij})$, $A \geq B$ means $a_{ij} \geq b_{ij}$ for any i, j.

Following lemma is due to Kesten and Spitzer [12].

LEMMA 2.1. *Let $B_1, B_2, \cdots$ be a sequence of i.i.d. nonnegative matrices. Then*

$$\lim \frac{1}{n} \log \|B_1 B_2 \cdots B_n\| \leq \log \rho(E(B_1)) \quad a.s.$$

LEMMA 2.2. *Let A_1 be the matrix defined in (2.2). Then $\rho(E(A_1)) < 1$ if and only if $\sum_{i=1}^{q} \alpha_i \sigma^2 + \sum_{i=1}^{p} \beta_i < 1$.*

PROOF. This result can be derived from the fact that for $a_i \geq 0$, $(i = 1, \cdots, n)$, all roots of the equation $x^n - a_1 x^{n-1} - \cdots - a_n = 0$ lie inside the unit circle if and only if $a_1 + \cdots + a_n < 1$.

THEOREM 2.1. *Suppose that $\sum_{i=1}^{q} \alpha_i \sigma^2 + \sum_{i=1}^{p} \beta_i < 1$. Then the following holds.*

(1) $p^{(n)}(x, dy)$ converges weakly to a unique invariant probability $\pi(dy)$ and Markov chain $\{X_n : n \geq 0\}$ in (2.3) with $X_0 \simeq \pi$ is strict stationary and ergodic. Here $X_0 \simeq \pi$ implies that the distribution of X_0 is π.

(2) For every x,

$$1/n \sum_{k=1}^{n} f(X_k(x)) \to \int f(y) \pi(dy), \quad a.s.$$

for all bounded continuous real-valued functions on R^{p+q-1}.

PROOF. (1) First note that $\{A_n\}$ is a sequence of i.i.d. random matrices whose entries are all nonnegative. From assumptions and the above lemmas, there exist $m > 0$ and $r > 0$ such that $\|A_m \cdots A_1\| < r < 1$ a.s. Let $\{X_n(x) : n \geq 0\}$ denote $\{X_n : n \geq 0\}$ in (2.3) if $X_0 = x, x \in R_+^{p+q-1}$ where $R_+^{p+q-1} = \{x = (x_1, \cdots, x_{p+q-1}) | x_i \geq 0, 1 \leq i \leq p+q-1\}$. Then we have that

$$\|X_{nm}(x) - X_{nm}(y)\| \leq r^n \|x - y\|, \tag{2.4}$$

from which together with $E(\|x_0 - A_1 x_0\|) < \infty$ for some x_0 in R_+^{p+q-1}, we obtain the weak convergence of $p^{(n)}(x, dy)$ to some probability measure, say, $\pi(dy)$. Actually, π is the distribution of $\lim_{n\to\infty} X_n(x)$ which is independent of x and since A_n is i.i.d., X_n with $X_0 \simeq \pi$ is a strictly stationary and ergodic Markov chain. (see, for example, Elton [8], Lee [13], Benda [2] etc.).

(2) Since X_n with $X_0 \simeq \pi$ is stationary and ergodic, by ergodic theorem, we have

$$1/n \sum_{k=0}^{n} f(X_n) \to E[f(X_0)], \quad a.s.$$

Also $\|A_n \cdots A_1 x - A_n \cdots A_1 X_0\| \le \|A_n \cdots A_1\| \|x - X_0\| \to 0$ *a.s.* as $n \to \infty$, and the conclusion follows.

3. Geometric ergodicity

Recall that a $\varphi-$ irreducible Markov process is said to be geometrically (*Harris*) *ergodic* if there exists a probability measure π and $\rho < 1$ such that

$$\rho^{-n} \sup_B |p^{(n)}(x, B) - \pi(B)| \to 0 \quad as\ n \to \infty. \tag{3.1}$$

When the given process has an irreducible Markovian structure, the following result due to Tweedie [26] gives a sufficient condition for geometric ergodicity of the process.

THEOREM 3.1. *Suppose that* $\{X_n\}$ *is irreducible aperiodic Markov chain with state space* R_+^{p+q-1}. *If there exist, for some compact set* C, *a nonnegative measurable function* v *on* R_+^{p+q-1}, $\gamma > 0$ *and* $0 < r < 1$ *satisfying*

$$E[v(X_{n+1})|X_n = x] \le rv(x) - \gamma, \quad x \in C^c \tag{3.2}$$

and

$$\sup_{x \in C} E[v(X_{n+1})|X_n = x] < \infty, \tag{3.3}$$

then $\{X_n\}$ *is geometrically ergodic.*

THEOREM 3.2. *Suppose that* $\{X_n : n \ge 0\}$ *in* (2.3) *is* ϕ*-irreducible aperiodic. If* $\sum_{i=1}^q \alpha_i \sigma^2 + \sum_{i=1}^p \beta_i < 1$, $\{X_n\}$ *is geometrically ergodic.*

PROOF. To obtain the geometric ergodicity of $\{X_n\}$, it remains to find the proper test function $v : R_+^{p+q-1} \to R_+$ under which the assumptions of Theorem 3.1 hold. The basic idea to define the test function v given later is similar to that of Ling [17].

Let $A = E(A_1)$ and I be the identity matrix. Since $\rho(A) < 1$, $I - A^t$ is invertible and $(I - A^t)^{-1} = I + \sum_{n=1}^{\infty} (A^t)^n$. Note that every entry of A is nonnegative. Let $x = (x_1, x_2, \cdots, x_{p+q-1})^t > 0$ if and only if $x_i > 0$ for all $i = 1, 2, \cdots, p+q-1$. Now choose $L = (l_1, l_2, \cdots, l_{p+q-1})^t > 0$ so that $M = (m_1, m_2, \cdots, m_{p+q-1})^t = (I - A^t)^{-1} L > 0$, and then define a function $v : R_+^{p+q-1} \to R_+$ by

$$v(x) := 1 + x^t M. \tag{3.4}$$

Then we have that

$$\begin{aligned} E[v(X_{n+1}) \mid X_n = x] &= E[1 + (A_{n+1}x + B)^t M] \\ &= 1 + x^t A^t M + B^t M \\ &= 1 + x^t M - x^t (I - A^t) M + B^t M \\ &= v(x)(1 - \frac{x^t (I - A^t) M}{1 + x^t M}) + B^t M. \end{aligned}$$

Let $l := \min\{l_1, l_2, \cdots, l_{p+q-1}\}$ and $m := \max\{m_1, m_2, \cdots, m_{p+q-1}\}$. Then

$$1 > \frac{x^t (I - A^t) M}{1 + x^t M} = \frac{x^t L}{1 + x^t M} \geq \frac{l \sum x_i}{1 + m \sum x_i}.$$

Since $M - L = A^t M \geq 0$, $m \geq l$. Choose any r and k so that $0 < r < \frac{l}{m} \leq 1$ and $k > \frac{r}{l - rm}$. Then for any $x \notin C_k = \{x \in R_+^{p+q-1} | \sum x_i \leq k\}$,

$$E[v(X_{n+1}) \mid X_n = x] \leq v(x)(1 - r) + B^t M.$$

Since $v(x)$ is increasing as $\sum x_i$ is increasing, for any $\gamma > 0$, we may choose r', $0 < r' < r < 1$, and $k' > k$ so that for any $x \in C_{k'}^c$,

$$E[v(X_{n+1}) \mid X_n = x] \leq v(x)(1 - r) + B^t M \leq v(x)(1 - r') - \gamma. \tag{3.5}$$

Clearly

$$\sup_{x \in C_{k'}} E[v(X_{n+1}) | X_n = x] < \infty. \tag{3.6}$$

By (3.5) and (3.6), the assumptions of Theorem 3.1 hold and geometric ergodicity of $\{X_n\}$ follows.

REMARK. Unfortunately, it is very awkward problem to show the irreducibility of GARCH(p, q) model with $q \geq 1$.

4. Functional central limit theorems

In this section, we consider the limiting distribution of the following stochastic processes: For each positive integer n, fixed $f \in L^2(R_+^{p+q-1}, \pi)$, define

$$F_n(t) = \frac{1}{\sqrt{n}} [\sum_{k=0}^{[nt]} (f(X_k) - \bar{f}) + (t - \frac{[nt]}{n})(f(X_{[nt]+1}) - \bar{f})], \quad t \geq 0. \tag{4.1}$$

Here $\bar{f} = \int f d\pi$. We say that the functional central limit theorem holds for $f \in L^2(R_+^{p+q-1}, \pi)$ if the sequence of stochastic process $F_n(t)$ in (4.1) converges in distribution to a Brownian motion.

A real-valued function f on R_+^{p+q-1} is said to be a Lipschitzian function if $|f(x)-f(y)| \leq K\|x-y\|$, for some $K>0$ and for all $x,y \in R_+^{p+q-1}$. Under the assumption $\sum_{i=1}^q \alpha_i\sigma^2 + \sum_{i=1}^p \beta_i < 1$, $\int x^2\pi(dx) < \infty$ holds and hence every Lipschitzian function is in $L^2(\pi)$.

Let $\|\cdot\|_2$ denote the L_2 norm on $L^2(\pi)$.

THEOREM 4.1. *Suppose* $\sum_{i=1}^q \alpha_i\sigma^2 + \sum_{i=1}^p \beta_i < 1$. *(1) If the distribution of* X_0 *is* π, *every Lipschitzian function* f *holds the functional central limit theorem. (2) If* $X_0 \equiv x$, $x \in R_+^{p+q-1}$, *every Lipschitzian function* f *holds the functional central limit theorem.*

PROOF. (1) Let P be the transition operator on $L^2(\pi)$ such that

$$(Pf)(x) = \int f(y)p(x,dy), \quad f \in L^2(\pi).$$

From inequality (2.4) and $\int x^2 d\pi < \infty$, we have

$$\begin{aligned}
& \sum_{n=0}^{\infty} \|P(f-\bar{f})\|_2 \\
\leq\ & \sum_{n=0}^{\infty} \Big(\int [\int E|f(X_n(x)) - f(X_n(y))|\pi(dy)]^2 \pi(dx) \Big)^{1/2} \\
\leq\ & \sum_{n=0}^{\infty} K^2 r^{[n/m]} E \\
& \times (\|A_{n-[n/m]m} \cdots A_1\|) \Big(\int\int |x-y|^2 \pi(dx)\pi(dy) \Big)^{1/2} \\
<\ & \infty. \qquad (4.2)
\end{aligned}$$

From this together with Lemma 3.3 and Theorem 3.1 in Bhattacharya and Lee [4], the conclusion follows.

(2) Let $F_n(\cdot)$ and $F_n^x(\cdot)$ be the processes defined by (4.1) with $X_0 \simeq \pi$ and $X_0 \equiv x$, respectively. For Lipschitzian function f, we have that

$$\begin{aligned}
E(\max_{0\leq t\leq 1} |F_n^x(t) - F_n(t)|) &\leq K n^{-1/2} \sum_{k=0}^{n} E\|X_k(x) - X_k\| \\
&\to 0 \text{ as } n \to \infty. \qquad (4.3)
\end{aligned}$$

Therefore $F_n^x(t)$ and $F_n(t)$ have the same limit.

REMARK. Benda [2] show that if $E\|A_1\|^2 < 1$, FCLT holds for every Lipschitzian f. In Lee and Kim [15], it is proved that $\|E(A_1^t A_1)\| < 1$ is sufficient under which FCLT holds for every Lipschitzian function. Note that $\|E(A_1^t A_1)\| \leq E\|A_1\|^2$ and $\rho(E(A_1)) \leq \|E(A_1)\| \leq E\|A_1\|$, by Jensen's inequality, and $(E\|A_1\|)^2 \leq E\|A_1\|^2$ by Cauchy-Schwarz inequality.

Recall that geometrically ergodic Markov process is absolutely regular and absolute regularity implies the strong mixing. From this fact, limit theorems for X_n can be obtained by applying various limit theorems for absolutely regular process and/or strong mixing process (see, Pham [22]).

If we assume that X_n is irreducible and aperiodic in addition to the assumption $\sum_{i=1}^{q} \alpha_i \sigma^2 + \sum_{i=1}^{p} \beta_i < 1$, then we identify a broad class of functions f for which FCLT holds.

THEOREM 4.2. *Suppose the assumptions in Theorem 3.2 hold. Then every function f with $f^2 \leq v_0(x)$ holds the functional central limit theorem, where $v_0(x) = v(x) + c$ for some constant $c > 0$ and $v(x)$ given in* (3.4). *In particular, every bounded measurable function f holds the functional central limit theorem.*

PROOF. From (3.5) and (3.6), we can easily obtain that

$$E[v(X_{n+1})|X_n = x] \leq rv(x) + bI_C, \tag{4.4}$$

for some constants b, $0 < r < 1$ and compact set C. Here I_C is the indicator function of C. Then Theorem 4.1 in Glynn and Meyn [10] ensures that if $f^2 \leq v_0$, then f is in the range of $P - I$ where P is the transition operator on $L^2(\pi)$ and I is an identity operator and hence the functional central limit theorem holds for f. If f is bounded measurable with $|f| < c_0$ for some $c_0 < \infty$, then we have $f^2 \leq v_0$, by taking $c = c_0^2$ in $v_0(x)$, and therefore the functional central limit theorem holds for such f.

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