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OPTIMAL RELIABILITY DESIGN FOR THIN-WALLED BEAM OF VEHICLE STRUCTURE CONSIDERING VIBRATION

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ABSTRACT−In the deterministic optimization of a structural system, objective function, design constraints and design variables are treated in a nonstatistical fashion. However, such deterministic engineering optimization tends to promote the structural system with less reliability redundancy than obtained with conventional design procedures using the factor of safety. Consequently, deterministic optimized structures will usually have higher failure probabilities than unoptimized structures. Therefore, a balance must be developed between the satisfactions of the design requirements and the objectives of reducing manufacturing cost. This paper proposes the reliability-based design optimization (RBDO) technique, which enables the optimum design that considers confidence level for the vibration characteristics of structural system. Response surface method (RSM) is utilized to approximate the performance functions describing the system characteristics in the RBDO procedure. The proposed optimization technique is applied to the pillar section design considering natural frequencies of a vehicle structure.

KEY WORDS : Body in white (B.I.W.), First-order reliability method (FORM), Monte Carlo simulation (MCS), Reliability-based design optimization (RBDO), Response surface method (RSM), Thin-walled beam

1. INTRODUCTION

Conventional design optimization methodology (i.e., deterministic optimization) for an engineering system is based on a computational simulation that dose not allow any variation and uncertainty on mathematical modeling as well as physical quantity. In such deterministic optimization, engineers aim to reduce the production cost and improve quality without caring about the effects of uncertainties concerning the material properties, geometry, loading conditions, and boundary conditions. In this way, the resulting optimal configuration may present a lower confidence level and violate an engineering requirement.

Traditionally, to solve any variation and uncertainty in structural and mechanical design, the factor of safety has been widely introduced. The factor of safety provides a margin of safety to account for uncertainties such as errors in predicted loading of a part, variations in the material properties, and differences between the ideal model and actual material behavior. But, the design methods based on the factor of safety are not rational in the sense that the same factor of safety might imply different values of reliability in different situations. That is, the conventional design approach using the factor of safety is not adequate from a reliability standpoint. Hence, another design methodology that does consider the probabilistic nature of the design is needed so that system reliability can be calculated at the design stage. These problems can be solved by using the reliability-based design optimization (RBOD), which couples optimization technique and reliability problem. The RBDO is to design structural and mechanical systems that should be economic and reliable, by introducing the stochastic method in optimization procedure.

For the deterministic optimization, many efficient numerical methods have been developed and applied to various structural and mechanical systems. But, for the RBDO problems, the coupling between mechanical modeling, reliability analysis and optimization methods represent a very complex task and lead to very high computational time. To solve such difficult and complex problems, numerous endeavors have been made during last decade. In the literature, many developments have been realized in the RBDO fields (Belegundu, 1988; Chandu *et al.*, 1995).

In this paper, we present the RBDO technique considering the vibration characteristics for the body-in-white (B.I.W.) of a passenger vehicle. In the reliability-

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based design optimization of vehicle pillar section, panel thickness and scale vector are defined as design variables. To formulate explicitly the performance functions (i.e., the limit-state functions) for the section properties of the vehicle pillar and for the weight and the natural frequencies of the B.I.W, we utilize the response surface method (RSM), which is one of the design of experiments (DOE). The RBDO on each pillar section is performed by using the optimization program, DOT (Vanderplaats *et al.*, 1989), and the results of the RBDO and the deterministic optimization are compared.

2. STRUCTURAL OPTIMIZATION

2.1. Deterministic Optimization

The conventional deterministic optimization on the vehicle B.I.W., which is composed of complicated thin-walled panels, can be expressed as

$$\begin{aligned} &\text{Minimize :} && f(d) > f(x) \\ &\text{Subjecto to :} && g_j(x) \le 0, \quad j = 1,2,\ldots, J \\ & && h_k(x) = 0, \quad k = 1,2,\ldots, K \qquad (1) \\ & && (x_i)^L \le x_i \le (x_i)^U \end{aligned}$$

where, $f(d)$, $g_j(x)$, and $h_k(x)$ denote the objective function, the inequality constraint, and the equality constraint, respectively. Also, x_i denotes the i^{th} design variable, and $(x_i)^L$ and $(x_i)^U$ are the given lower and upper limit of each design variable (i.e., side constraints).

In this optimization, the mass of the B.I.W. is set up in the design objective, and the natural frequencies of the 1^{st} torsion mode and the 1^{st} bending mode are set up in the design constraints. Also, the panel thickness and the scale vector on each pillar section are set up in the design variables. Through the deterministic optimum design, the panel thickness, the scale vector, and the section properties for the vehicle pillar are determined.

2.2 Reliability-based Design Optimization

The first step in evaluating the reliability of a structural system is to decide on specific performance criteria and the relevant load and resistance parameters (i.e., basic variable, z), and the functional relationships among them corresponding to each performance criterion. Mathematically, this relationship or performance function can be described as

$$g = g(z) \qquad (2)$$

The failure surface or limit-state of interest can be defined as $g(z) = 0$. This is boundary between the safe and unsafe regions in the design parameter space. Using Equation (2), we find that failure occurs when $g(z) < 0$. Therefore, the probability of failure, P_f, is given by the integral (Thanedar *et al.*, 1992)

$$P_f = P\{g(z) < 0\} = \int_\Omega \ldots \int f_z(z) dz_1 \ldots dz_n \qquad (3)$$

in which $f_z(z)$ is the joint probability function for the basic random variable, z, and the integration is performed over the failure region, Ω.

However, in general case, the joint probability density function of random variable is practically impossible to obtain. Even if this information is available, evaluating the integral is very difficult. Consequently, for practical structures and performance criteria, it is difficult to compute the probability of failure precisely. So, one approach is to use analytical approximation of this integral that is simpler to compute. The performance function about the mean value can be approximated by using the 1^{st} order Taylor series expansion and can be expressed as

$$\begin{aligned} g(z) &= g(\mu) + \sum_{i=1}^{n} \left(\frac{\partial g}{\partial z_i}\right) \cdot (z_i - \mu_i) + H(z) \\ &= a_0 + \sum_{i=1}^{n} a_i \cdot z_i + H(z) \\ &= g_1(z) + H(z) \end{aligned} \qquad (4)$$

where, z_i and μ_i denote respectively the random variable and the mean value for the random variables. From Equation (4), the mean value and the variance of the performance function can be defined respectively as

$$\mu_g \approx a_0 + \sum_{i=1}^{n} a_i \cdot \mu_{g_i}, \quad \sigma_g^2 \approx \sum_{i=1}^{n} a_i^2 \cdot \sigma_{z_i}^2 \qquad (5)$$

The probability of failure depends on the ratio of the mean value of performance function, g, to its standard deviation. This ratio is commonly known as the safety index or reliability index and denoted as

$$\beta = \frac{g(\mu_z)}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial z_i}\right)^2 \cdot (\sigma_{z_i})^2}} = \frac{\mu_g}{\sigma_g} \qquad (6)$$

Figure 1 shows MPP (most probable point) and FORM that approximates structural reliability in the first-order function at the MPP.

In this study, we carry out the reliability-based design optimization for the thin-walled pillar section using together FORM and MVFOSM (mean value first-order second-moment) that performs effectively sensitivity analysis on the performance function. The reliability-based design optimization on the vehicle structure can be written as

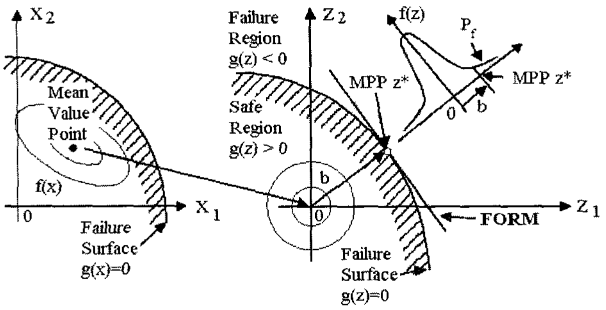

Figure 1. First-order reliability method.

Minimize : $f(d)$
Subject to :

$$P(g_{P_m}(d) \geq 0) \geq P_{sm}, \quad m = 1, 2, ..., M \tag{7}$$
$$g_{d_j}(\mu_{xr}; d_d, d_\mu) \geq 0, \quad j = 1, 2, ..., J$$
$$(d_i)^L \leq d_i \leq (d_i)^U \quad i = 1, 2, ..., I$$

where $P(\cdots)$ is the probability operator, $g_p(\cdots)$ is the performance function for probability constraint, $g_d(\cdots)$ is the performance function for deterministic constraint, μ_{xr} is the mean value of the first moment, d_d is the design variable on the deterministic system parameter, x_d, and d_μ is the design variable on the mean value, μ_{xr} of random system parameter.

2.3. Definition of Design Variable

In general, the pillars of the vehicle structure consist of inner panel, outer panel, and reinforcement panel. For the RBDO, thickness of the thin panels and scale vector are defined as design variables. The scale vector is introduced to consider the shape design of the cross section of the pillars, instead of using the nodal coordinates. With these design variables, section properties such as area, area moment of inertia, and torsion constant of the pillars are computed and used for finite element analysis. As shown in Figure 2, the thickness and the scale vector are used as the design parameters to formulate the approximate function for the section properties. By using the scale vector, it is possible to perform the shape optimal design with the reduced the number of design variables (Pyun *et al.*, 2000).

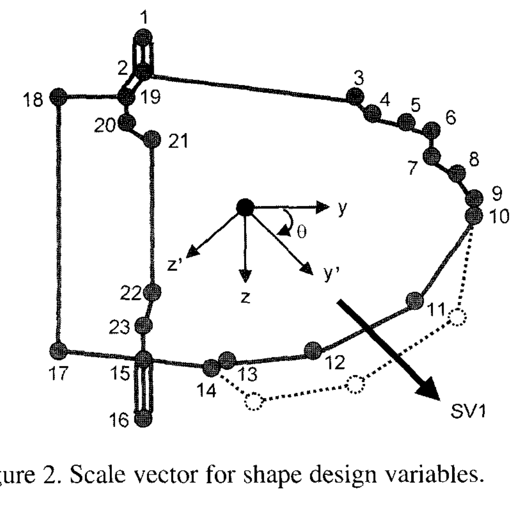

Figure 2. Scale vector for shape design variables.

In Figure 2, the coordinate value of the node 12 for the $y'z'$ coordinate system can be expressed as Equation (8).

$$\begin{pmatrix} y_{12}' \\ z_{12}' \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} y_{12} \\ z_{12} \end{pmatrix}_{init} = \begin{bmatrix} \cos\theta(y_{12})_{init} + \sin\theta(z_{12})_{init} \\ -\sin\theta(y_{12})_{init} + \cos\theta(z_{12})_{init} \end{bmatrix} \tag{8}$$

In this case, when the scale vector SV_1 is set up in the design variable, the new coordinate value of the node 12 for the yz coordinate system can be written as Equation (9).

$$\begin{pmatrix} y_{12} \\ z_{12} \end{pmatrix}_{new} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} y_{12}'SV_1 \\ z_{12}' \end{pmatrix} = \begin{bmatrix} \cos\theta y_{12}'SV_1 - \sin\theta z_{12}' \\ \sin\theta y_{12}'SV_1 + \cos\theta z_{12}' \end{bmatrix} \tag{9}$$

As shown in Equations (8)-(9), the new section shapes can be obtained when the rotation angle θ of the $y'z'$ coordinate system and the scale vector SV_1 are known.

3. RBDO PROCESS

To perform the stochastic optimum design considering the reliability of the vehicle structure, many sort of programs are needed. MSC/NASTRAN is used to perform the deterministic optimization and the vibration analysis. NESSUS and DOT are used to perform the reliability analysis and the RBDO. SECOPT (Lee, 1995) is used to calculate the section properties of the vehicle pillar. And also, the ModelCenter is used to integrate each program. Figure 3 shows the RBDO process for the thin-walled beam section of the vehicle pillar.

4. APPLICATION OF NUMERICAL OPTIMIZATION

4.1. Application Model

Figure 4 illustrates the B.I.W. finite element model of the passenger car used in this study. As shown in this figure, the vehicle structure consists of beam element, shell element, rigid bar element, and spring element. Figures 5 and 6 show the 1st torsion vibration mode and the 1st bending vibration mode of the B.I.W. Table 1 shows the

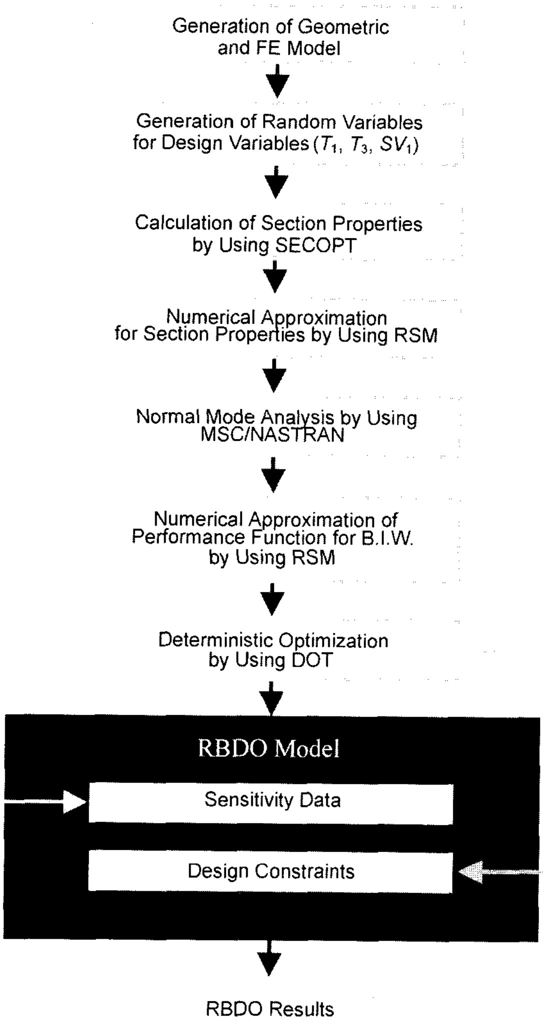

Figure 3. Process of RBDO.

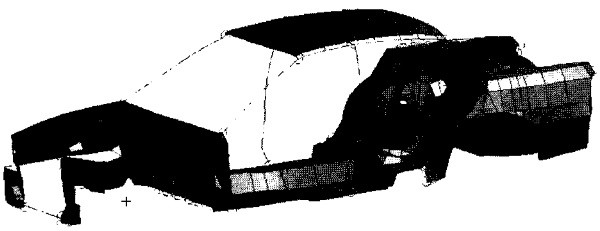

Figure 4. B.I.W. FE model of a passenger car.

mass and the natural frequencies of this B.I.W. model.

4.2. Design Variables of Each Pillar

Table 2 illustrates the design variables and their limits for the FBHP upper, FBHP lower, and B-pillar middle section. Before the RBDO, the Monte Carlo simulation is performed to estimate the probability distribution on the section properties of each pillar. In the Monte Carlo simulation, five hundred random variables for the panel

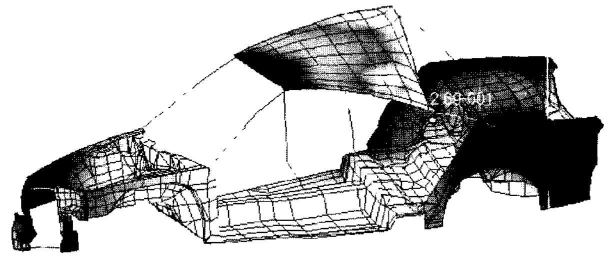

Figure 5. 1st torsion vibration mode of B.I.W.

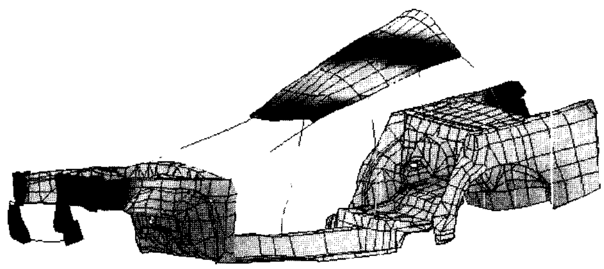

Figure 6. 1st bending vibration mode of B.I.W.

Table 1. Mass and natural frequencies of B.I.W.

Mass		290.99 kg
Natural frequency	1st torsion mode	31.21 Hz
	1st bending mode	43.13 Hz

Table 2. Design variables and their limits.

Unit: mm

Section name	Design variable	Initial value	Lower bound	Upper bound
FBHP upper	T_1	0.75	0.5	1.5
	T_3	1.60	0.1	2.0
	SV_1	1.00	0.8	1.5
FBHP lower	T_1	0.75	0.5	1.5
	T_3	1.60	0.1	2.0
	SV_1	1.00	0.8	1.5
B-pillar middle	T_1	1.00	0.5	1.5
	T_3	1.40	0.1	2.0
	SV_1	1.00	0.8	1.5

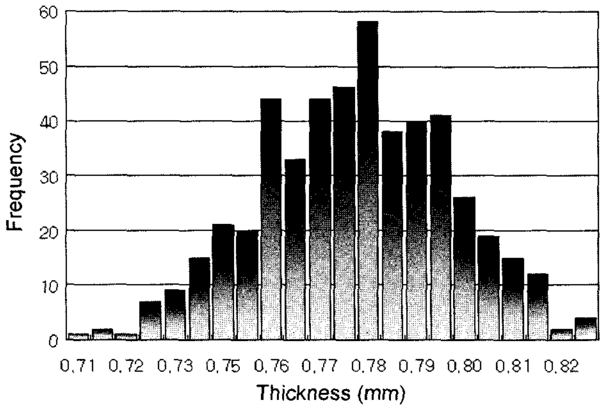

Figure 7. Probability distribution for T_3 of FBHP upper section.

thickness and the scale vector of each pillar are generated between the lower bound and the upper bound of each design variable. Figure 7 shows the probability distribution for the design variable, T_3, of the FBHP upper section.

4.3. Optimization of Pillar Sections

We perform the deterministic optimization and the RBDO for each section of the vehicle structure. The formulation of the RBDO with a reliability goal of 99.5% from B.I.W. vibration can be expressed as Equation (10).

Minimize : Mass
Subject to :
· 1st Torsion Frequency > 32 Hz
· 1st Bending Frequency > 44 Hz (10)
· Reliability > 99.5%

Each optimization result for the mass, the 1[st] torsion natural frequency, and the 1[st] bending natural frequency of the B.I.W. are listed in Table 3. As shown in this table, the objective function value of the RBDO is somewhat greater than that of the deterministic optimization.

Table 3. Comparison of mass and natural frequencies for B.I.W.

	Initial	Deterministic optimization	RBDO
Mass (kg)	290.99	278.05	281.39
1[st] torsion mode frequency (Hz)	31.21	32.24	32.28
1[st] bending mode frequency (Hz)	43.13	45.65	45.72

Table 4. Comparison of design variables.

Unit: mm

Design variables		Initial	Deterministic optimization	RBDO
FBHP upper	T_1	0.75	0.50	0.52
	T_3	1.60	0.72	0.77
	SV_1	1.00	1.47	1.50
FBHP lower	T_1	0.75	0.50	0.53
	T_3	1.60	0.10	0.35
	SV_1	1.00	0.80	1.24
B-pillar middle	T_1	1.00	1.00	0.99
	T_3	1.40	0.10	0.15
	SV_1	1.00	0.80	0.83

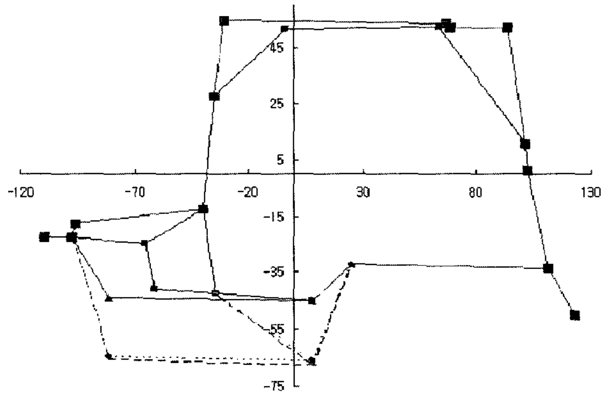

Figure 8. Configuration of FBHP upper.

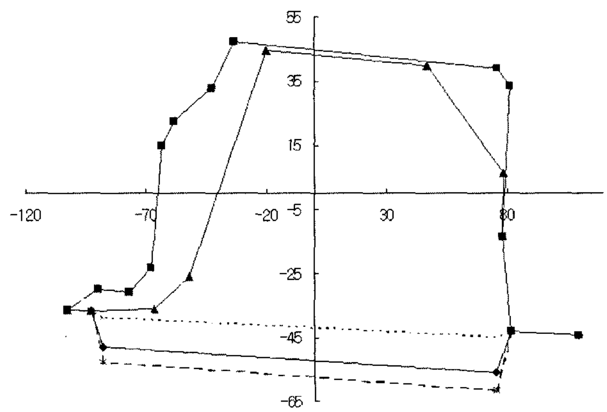

Figure 9. Configuration of FBHP lower.

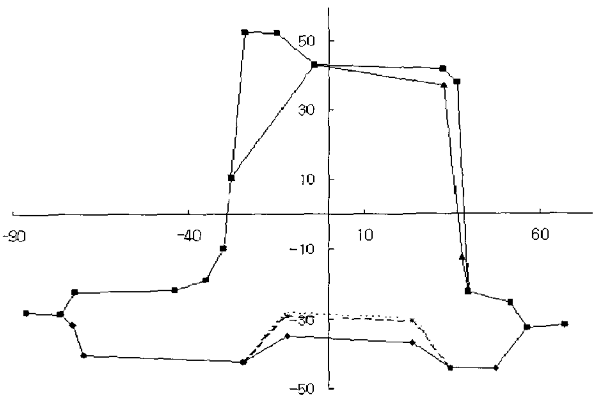

Figure 10. Configuration of B-pillar middle.

However, because of the RBDO considers the probable variation of the design variables, the pillar section that is determined through the RBDO method can guarantee the design constraints (i.e., natural frequencies of the vehicle structure) that engineers require. Table 4 illustrates the optimization result for the design variables. Figures 8-10 show the comparisons of the shape change for the FBHP upper, the FBHP lower, and the B-pillar middle section according to the initial design, the deterministic optimization, and the RBDO. In these figures, the solid lines represent the initial shape, the dotted lines represent the deterministic optimization results, and the dashed lines represent the RBDO results.

To verify the optimization results obtained from the RBDO, we perform the reliability analysis, and the reliability estimation result shows that the failure probability is to be 0.472%. Therefore, this RBDO result satisfies 99.5% confidence level set up.

5. CONCLUSIONS

This paper presents the pillar section optimization technique considering the reliability of the vehicle body structure consisted of complicated thin-walled panels. The RSM is utilized to obtain the response surface models that describe the approximate performance functions on the section properties of the vehicle pillar and on the mass and the natural frequencies of the B.I.W. The reliability-based design optimization on the pillar sections is performed by using the optimization program, DOT, and compared with the deterministic optimization. The FORM and the MVFOSM are applied for reliability analysis of the vehicle structure. By applying the proposed RBDO technique, it is possible to optimize the pillar sections considering the reliability that engineers require.

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