

# One-step 순방향 추정 오차 필터를 이용한 임의의 결정지연을 갖는 블라인드 등화

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## Blind Equalization with Arbitrary Decision Delay using One-Step Forward Prediction Error Filters

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### 요 약

통신 채널에서 블라인드 등화는 전송효율을 저하시키는 훈련신호나 채널의 사전 정보가 필요치 않은 장점 때문에 많은 연구가 진행되어 왔다. 선형예측을 이용한 블라인드 등화는 등화기의 차수 추정 오차에 강인하며 적응 알고리즘을 이용하여 효율적으로 구현할 수 있는 장점이 있다. 하지만 기존의 one-step 선형예측을 이용한 블라인드 등화기는 임의의 결정 지연에 대해서는 구현할 수 없는 단점이 있다. 본 논문에서는 SIMO 채널에서 one-step 순방향 선형예측 필터를 이용하여 임의의 결정 지연을 갖는 블라인드 등화기를 제안한다. 제안한 알고리즘은 순방향 추정 오차를 훈련신호로 사용하여 최적의 결정 지연을 갖는 블라인드 등화기를 구하였으며 모의실험을 통하여 본 논문에서 제안한 알고리즘의 성능을 확인하였다.

### ABSTRACT

Blind equalization of communication channel is important because it does not need training signal, nor does it require a priori channel information. So, we can increase the bandwidth efficiency. The linear prediction error method is perhaps the most attractive in practice due to the insensitive to blind channel equalizer length mismatch as well as for its simple adaptive implementation. Unfortunately, the previous one-step prediction error method is known to be limited in arbitrary decision delay. In this paper, we propose method for fractionally spaced blind equalizer with arbitrary decision delay using one-step forward prediction error filter from second-order statistics of the received signals for SIMO channel. Our algorithm utilizes the forward prediction error as training signal and computes the best decision delay from all possible decision delay. Simulation results are presented to demonstrate the performance of our proposed algorithm.

### 1. Introduction

Multipath propagation appears to be a typical limitation in mobile digital communication where it leads to severe intersymbol interference (ISI). The classical techniques to overcome this problem use either

periodically sent training sequence or blind techniques exploiting higher order statistics (HOS). Adaptive equalization using training sequence wastes the bandwidth efficiency but in blind equalization, no training is needed and the equalizer is o

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obtained only with the utilization of the received signal. Since the seminal work by Tong *et al.*, the problem of estimating the channel response of multiple FIR channel driven by an unknown input symbol has interested many researchers in the signal processing areas and communication fields [2]-[4].

For the most part, algebraic and second-order statistics (SOS) techniques have been proposed that exploit the structural techniques (Hankel, Toeplitz matrix, *et al.*) of the single-input multiple-output (SIMO) channel or data matrices. The information on channel parameters or transmitted data is typically recovered through subspace decomposition of the received data matrix (deterministic method) or that of the received data correlation matrix (stochastic method). Although very appealing from the conceptual and signal processing techniques point of view, the use of the aforementioned techniques in real world applications faces serious challenges. Subspace-based techniques lay in the fact that they rely on the existence of numerically well-defined dimensions of the noise-free signal or noise subspaces. Since these dimensions are obviously closely related to the channel length, subspace-based techniques are extremely sensitive to channel order mismatch [5].

The prediction error method (PEM) offers an alternative to the class of techniques above. PEM, which was first introduced by Slock *et al.* and later refined by Meraim *et al.*, exploited the i.i.d. property of the transmitted symbols and apply a linear prediction error filter on the received data. The PEM offers great practical

advantages over most other proposed techniques. First, channel estimation using the PEM remains consistent in the presence of the channel length mismatch. This property guarantees the robustness of the technique with respect to the difficult channel length estimation problem. Another significant advantage of the PEM is that it lends itself easily to a low-cost adaptive implementation such as adaptive lattice filters. But the decision delay cannot be controlled with existing one-step prediction error method [5]-[7].

In this paper, we propose method for blind equalizers with arbitrary decision delay using one-step forward prediction error filter (FPEF). This paper makes three results. First, we utilize the forward prediction error (FPE) as training signal, and, therefore, we can control the arbitrary decision delay. Second, we derive an adaptive algorithm for zero forcing (ZF) and minimum mean square error (MMSE) blind equalization. Finally, we deduce the fractionally-spaced blind equalization with best decision delay.

Most notations are standard: vectors and matrices are boldface small and capital letters, respectively; the matrix transpose, the complex conjugate, the Hermitian, and Moore-Penrose pseudoinverse are denoted by  $(\cdot)^T$ ,  $(\cdot)^*$ ,  $(\cdot)^H$ , and  $(\cdot)^+$ , respectively;  $I_P$  is the  $P \times P$  identity matrix;  $E[\cdot]$  is the statistical expectation. This paper is organized as follows. In section II, we will formulate the problem and describe the FPEF-based blind ZF- and MMSE-blind equalization. In section III, we discuss the principle of our proposed method and develop the adaptive blind e

qualizer with arbitrary decision delay using one-step FPEF. Simulation results and performance comparison of our algorithms with some well-known existing algorithms are presented in section IV. A conclusion is given in section V.

## II. Problem Formulation

### 1. System Model

Let  $x(t)$  be the continuous-time signal at the output of a noisy communication channel

$$x(t) = \sum_{k=-\infty}^{\infty} s(k)h(t-kT) + v(t) \quad (1)$$

where  $s(k)$  denotes the transmitted symbol at time  $kT$ ,  $h(t)$  denotes the continuous-time channel impulse response, and  $v(t)$  is additive noise. The fractionally spaced discrete-time model can be obtained either by time oversampling or by the sensor array at the receiver [5]. The oversampled single-input single-output (SISO) model results SIMO model as in Fig. 1. The corresponding SIMO model is described as follows

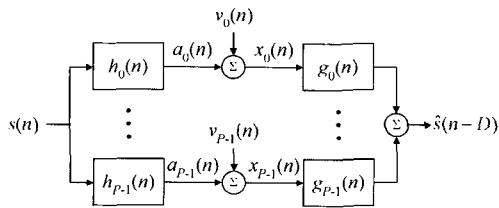


Fig. 1. The multichannel representation of a  $T/P$ -spaced equalizer.

$$\begin{aligned} x_i(n) &= \sum_{k=0}^L h_i(k)s(n-k) + v_i(n) \\ &= a_i(n) + v_i(n), \quad i=0, \dots, P-1 \end{aligned} \quad (2)$$

where  $P$  is the number of subchannel, and  $L$  is the maximum order of the  $P$

subchannel.

Let

$$\begin{aligned} \mathbf{x}(n) &= [x_0(n), \dots, x_{P-1}(n)]^T \\ \mathbf{h}(n) &= [h_0(n), \dots, h_{P-1}(n)]^T \\ \mathbf{v}(n) &= [v_0(n), \dots, v_{P-1}(n)]^T \end{aligned} \quad (3)$$

We represent  $x_i(n)$  in a vector form as

$$\mathbf{x}(n) = \sum_{k=0}^{L-1} s(k)\mathbf{h}(n-k) + \mathbf{v}(n) \quad (4)$$

Stacking  $N$  received vector samples into an  $(NP \times 1)$ -vector, we can write a matrix equation as

$$\mathbf{x}_N(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{v}_N(n) \quad (5)$$

where  $\mathbf{H}$  is a  $NP \times (N+L-1)$  block Toeplitz matrix,  $\mathbf{s}(n)$  is  $(N+L-1) \times 1$ ,  $\mathbf{x}_N(n)$ , and  $\mathbf{v}_N(n)$  are  $NP \times 1$  vectors.

$$\begin{aligned} \mathbf{s}(n) &= [s(n), \dots, s(n-L-N+2)]^T \\ \mathbf{x}_N(n) &= [\mathbf{x}^T(n), \dots, \mathbf{x}^T(n-N+1)]^T \\ \mathbf{v}_N(n) &= [\mathbf{v}^T(n), \dots, \mathbf{v}^T(n-N+1)]^T \end{aligned} \quad (6)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \dots & \mathbf{h}(L-1) & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{h}(0) & \dots & \mathbf{h}(L-1) \end{bmatrix} \quad (7)$$

We assume the followings throughout in this paper.

- A1) The input sequence  $\mathbf{s}(n)$  is zero-mean and white with variance  $\sigma_s^2$ .
- A2) The additive noise  $\mathbf{v}(n)$  is stationary with zero mean and white with variance  $\sigma_v^2$ .
- A3) The sequences  $\mathbf{s}(n)$  and  $\mathbf{v}(n)$  are uncorrelated.
- A4) The matrix  $\mathbf{H}$  has full rank, i.e., the subchannels  $h_i(n)$  have no common zeros to satisfy the Bezout equation.
- A5) The dimensions of  $\mathbf{H}$  obey  $NP > L+N-1$ .

Consider an FIR linear ZF- or MMSE-equalizer shown in Fig. 1, where  $g_i(n)$  for  $i=0,1,\dots,P-1$  is the order  $N$  equalizer of the  $i$ th subchannel. The equalizer impulse response in vector form is

$$\mathbf{g}(n)=[g_0(n),\dots,g_{P-1}(n)]^T \quad (8)$$

A  $D$ -delay equalizer vector of length  $NP$  is given as

$$\mathbf{g}_D=[\mathbf{g}^T(0),\dots,\mathbf{g}^T(N-1)]^T \quad (9)$$

and the symbol is estimated from

$$\hat{s}(n-D)=\mathbf{g}_D^T \mathbf{x}_N(n) \quad (10)$$

The output of the equalizer approaches  $s_{n-D}$  for some delay  $D$ . Then this equalizer is known as the  $D$ -delay ZF-equalizer. According to (5)-(7),  $\mathbf{x}_N(n)$  has nonzero correlation with only  $s(n),\dots,s(n-N-L+1)$ . Therefore, decision delay  $D$  is usually in the interval  $[0, N+L-1]$ . For finite SIMO channels, ZF- or MMSE-equalizer of the finite length can be found if assumption A4) holds and the equalizer length  $N \geq L$  [7]. A ZF equalizer does not perform optimally in the presence of noise. An MMSE equalizer minimize the cost function

$$J_{\text{MMSE}}=E[|s(n)-\hat{s}(n-D)|^2] \quad (11)$$

## 2. FPEF-Based Blind Equalization

Consider the following multichannel one-step forward prediction problem

$$\begin{aligned} \mathbf{f}_N(n) &= \mathbf{x}(n) - [\mathbf{p}_1 \mathbf{x}(n-1) + \dots + \mathbf{p}_N \mathbf{x}(n-N)] \\ &= [\mathbf{I}_P - \mathbf{P}_N] \mathbf{x}_{N+1}(n) \end{aligned} \quad (12)$$

where  $-\mathbf{p}_k$  for  $k=1,\dots,N$  are  $P \times P$  matrices

of a FPEF of order  $N$ . The FPEF coefficients are selected such that mean square value of  $\mathbf{f}_N(n)$ , i.e.,  $E[\|\mathbf{f}_N(n)\|^2]$ , is minimized. Therefore, for any set of FPEF coefficients  $\mathbf{p}_k$  ( $1 \leq k \leq N$ )

$$\frac{\partial E[\mathbf{f}_D(n) \mathbf{f}_D^H(n)]}{\partial \mathbf{p}_k^H} = 0, \text{ for } 1 \leq k \leq N \quad (13)$$

Due to the linearity of the expectation and differentiation operators, we can interchange these two operators [1]. We obtain as following

$$\begin{bmatrix} \mathbf{r}(0) & \dots & \mathbf{r}(N-1) \\ \mathbf{r}^H(1) & \dots & \mathbf{r}^H(N-2) \\ \vdots & \ddots & \vdots \\ \mathbf{r}^H(N-1) & \dots & \mathbf{r}(0) \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_N \end{bmatrix} = \begin{bmatrix} \mathbf{r}(1) \\ \mathbf{r}(2) \\ \vdots \\ \mathbf{r}(N) \end{bmatrix} \quad (14)$$

where  $\mathbf{r}(i-j)=E[\mathbf{x}(n-i)\mathbf{x}^H(n-j)]$

When the FPEF is optimum in the sense of MSE, the input signal vector  $\mathbf{x}_{N+1}(n)$  and the prediction error  $\mathbf{f}_N(n)$  are orthogonal. Therefore,

$$E[\mathbf{x}_{N+1}(n) \mathbf{f}_N^H(n)] = 0 \quad (15)$$

As shown in [5] and [6], we obtain

$$\mathbf{f}_N(n) = \mathbf{h}(0) s(n) \quad (16)$$

As described in (16), the FPE contains the transmitted symbol.

Let us first consider MMSE-equalization. A zero-delay MMSE equalizer can be obtained as shown in [6] and [7]

$$\mathbf{g}_0^{\text{MMSE}} = \sigma_s^2 \mathbf{h}^H(0) \mathbf{F}^{-1} [\mathbf{I}_P - \mathbf{P}_N] \quad (17)$$

where  $\mathbf{F}$  is the covariance matrix of  $\mathbf{f}_N(n)$  as following

$$\mathbf{F} = E[\mathbf{f}_N(n)\mathbf{f}_N^H(n)] = \sigma_s^2\mathbf{h}(0)\mathbf{h}^H(0) \quad (18)$$

A ZF-equalizer can also be obtained from FPEF as shown in [6] and [7]

$$\mathbf{g}_0^{ZF} = \frac{\mathbf{h}^H(0)}{\|\mathbf{h}(0)\|^2}[\mathbf{I}_P \quad -\mathbf{P}_N] \quad (19)$$

where  $\|\mathbf{h}(0)\|$  is the Euclidean norm of  $\mathbf{h}(0)$ .

When additive noise  $\mathbf{v}(n)=0$ , the MMSE- and ZF-equalizer become equivalent. But in noisy environment, MMSE equalizer has better performance than ZF equalizer generally [2], [7].

### III. Proposed Method

#### 1. Principle of the Proposed Method

As described in [13], multistep prediction has been suggested as a solution to the arbitrary decision delay equalization problem. The multistep prediction error can be modeled as an output of a truncated channel with no additive noise. The equalizers are result in various types. The equalization of [12] is proposed a combination of two multistep FPEF. The equalization of [7] consists of a cascade of a multistep FPEF and one-step BPEF. Multistep prediction-based method require two prediction error filters and need to estimate channel coefficient corresponding to decision delay [9].

It is obvious that multistep prediction-based methods are required more computational complexity than one-step FPEF-based methods and, moreover, needed to channel identification procedure before equalization. But one-step FPEF-based method are needed to first channel coefficient,

$\mathbf{h}(0)$ , only. A feasible solution for estimation of the  $\mathbf{h}(0)$  is given in [5], where the additive noise ignored. More accurate method is eigen-pair tracking using the covariance matrix of PE in (18) [6], [11].

We propose new method for arbitrary decision delay blind equalizer based on one-step FPEF. As described in (16), FPE contains transmitted symbol and it can be used as a training sequence, which has been used to traditional adaptive equalization method. Fig. 2 summarizes the ide

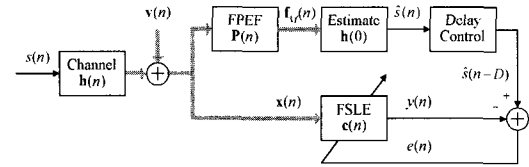


Fig. 2. The block diagram of the proposed method.

as presented above. Proposed method consists of four functional blocks. The first part functions FPEF to produce FPE, the second part estimates channel coefficient,  $\mathbf{h}(0)$ , the third part choose best decision delay, and the fourth part is fractionally-spaced linear equalizers.

After estimating FPE and  $\mathbf{h}(0)$  using one-step FPEF, we can extract transmitted symbol from FPE. And we use one-step FPEF-based equalized signal as training sequence.

#### 2. Adaptive Algorithm

In this section, we consider adaptive algorithms for the proposed method. The first is (12); we are required to compute the FPEF  $\mathbf{P}_N$  and to estimate the FPE  $\mathbf{f}_N(n)$ . To achieve fast convergence, we can use the RLS algorithm to update the FPEF as following

- Compute output:

$$\hat{\mathbf{x}}(n) = \mathbf{P}_N(n)\mathbf{x}_N(n-1) \quad (20)$$

• Compute FPE:

$$\mathbf{f}_N(n) = \mathbf{x}(n) - \hat{\mathbf{x}}(n) \quad (21)$$

• Compute Kalman gain:

$$\mathbf{K}(n) = \frac{\lambda^{-1}\mathbf{Q}(n-1)\mathbf{x}_N(n-1)}{1 + \lambda^{-1}\mathbf{x}_N^H(n-1)\mathbf{Q}(n-1)\mathbf{x}_N(n-1)} \quad (22)$$

• Update inverse of the correlation matrix:

$$\mathbf{Q}(n) = \lambda^{-1}\mathbf{Q}(n-1) - \lambda^{-1}\mathbf{K}(n)\mathbf{x}_N^H(n-1)\mathbf{Q}(n-1) \quad (23)$$

• Update FPEF coefficients:

$$\mathbf{P}_N(n) = \mathbf{P}_N(n-1) - \mathbf{f}_N(n)\mathbf{K}^H(n) \quad (24)$$

The term  $\lambda(0 \leq \lambda \leq 1)$  is intended to reduce the effect of past values on the statistics when the filter operates in nonstationary environment. It affects the convergence speed and the tracking accuracy of the algorithm [1]. The FPEF coefficients can also be computed by an LMS algorithm. In a simple manner, the FPE can be computed by (20) and (21), and the FPEF coefficients can be updated by the term

$$\mathbf{P}_N(n) = \mathbf{P}_N(n-1) - \mu \mathbf{f}_N(n)\mathbf{x}_N^H(n-1) \quad (25)$$

where  $\mu$ , the adaptation step-size, is a positive constant.

The second is estimation of  $\mathbf{h}(0)$ . From the covariance matrix of FPE in (18), its estimation of adaptive manner is given by

$$\mathbf{F}_N(n) = \mu \mathbf{F}_N(n-1) + \mathbf{f}_N(n)\mathbf{f}_N^H(n) \quad (26)$$

Compared with (18),  $\mathbf{h}(0)$  is the column of  $\mathbf{F}_N(n)$  with the largest norm [5], [6]. We can use either of the following equations to obtain the ZF- and MMSE-equalizer outputs [6], [7].

$$\begin{aligned} \hat{s}_{\text{MMSE}}(n) &= \sigma_v^2 \mathbf{h}^H(0) \mathbf{F}^{-1}(n) \mathbf{f}_N(n) \\ \hat{s}_{\text{ZF}}(n) &= \frac{\mathbf{h}(0)}{\|\mathbf{h}(0)\|^2} \mathbf{f}_N(n) \end{aligned} \quad (27)$$

The third is choice of best decision delay. It should be noted that the MMSE equalizer is designed for transmitted symbol recovery at specific decision delay. Thus, different decision delay can result in different performance. A recursive form to get best decision delay is discussed in [8], [10], and [14]. To get best decision delay choice, [8] and [10] propose the minimizing MSE is given by

$$J_{\text{MSE}}(D) = 1 - \mathbf{H}^H(D) \mathbf{R}^{-1} \mathbf{H}(D) \quad (28)$$

where  $\mathbf{H}(D)$  is the  $(D+1)$ th block column of the channel convolution matrix  $\mathbf{H}$  and  $\mathbf{R}$  is the autocorrelation matrix of oversampled received signal. But it is not very useful because  $\mathbf{H}$  is unknown. If the transmitted symbols have constant modulus (CM), which is practical case in digitally modulated signal such as QAM and PSK, the best decision delayed blind equalizer can be determined by the following CM index [14]:

$$J_{\text{CM}}(D) = \sum (|\mathbf{g}_D^H \mathbf{x}_N|^2 - 1)^2 \quad (29)$$

The blind equalizer having the smallest  $J_{\text{MSE}}$  or  $J_{\text{CM}}$  value will be considered as the best decision delayed blind equalizer. In many practical channels, it has been observed [2] that selecting  $D \approx (N+L)/2$  re

sults in good performance. For given MMSE- or CM-sense optimized decision delay  $D$ , we can get training-like sequence,  $t(n)$ , as following

$$\begin{aligned} t_{ZF}(n) &= \hat{s}_{ZF}(n-D) \\ t_{MMSE}(n) &= \hat{s}_{MMSE}(n-D) \end{aligned} \quad (30)$$

The forth is fractionally-spaced linear equalizer (FSLE) [15]. In Fig. 2, the output of FSLE is given by

$$y(n) = \sum_{k=0}^{P-1} \mathbf{c}_k^H(n) \mathbf{x}_k(n) \quad (31)$$

where  $\mathbf{c}_k(n)$  is the equalizer coefficients of the  $k$ th subchannel and  $\mathbf{x}_k(n)$  is the input vector of the  $k$ th subchannel

$$\begin{aligned} \mathbf{c}_k(n) &= [c_{k,0}(n), \dots, c_{k,N-1}(n)]^T \\ \mathbf{x}_k(n) &= [x_k(n), \dots, x_k(n-N+1)]^T \end{aligned} \quad (32)$$

We can use the LMS algorithm to update the equalizer coefficients as following

$$\mathbf{c}_k(n+1) = \mathbf{c}_k(n) + \mu e^*(n) \mathbf{x}_k(n) \quad (33)$$

where  $e(n)$  is either  $t_{ZF}(n) - y(n)$  for ZF criterion or  $t_{MMSE}(n) - y(n)$  for MMSE criterion. Table 1 summarizes the adaptive algorithms for the proposed method described earlier.

#### IV. Simulation Results

In this section, we use computer simulations to evaluate the performance of the proposed algorithms through following simulations.

- The Convergence of the FPE in white noise for several value of the SNR.
- The convergence of the first channel coefficient,  $\mathbf{h}(0)$ .

Table 1. Summary of the proposed algorithm.

- The best decision delay choice rule.
- The convergence of the proposed algorithm.
- The robustness of the proposed method to FPEF order overestimation.
- The performance of the proposed algorithm in comparison with existing algorithms.

The source symbols are drawn from a 16-QAM constellation with a uniform distribution. The noise is drawn from a

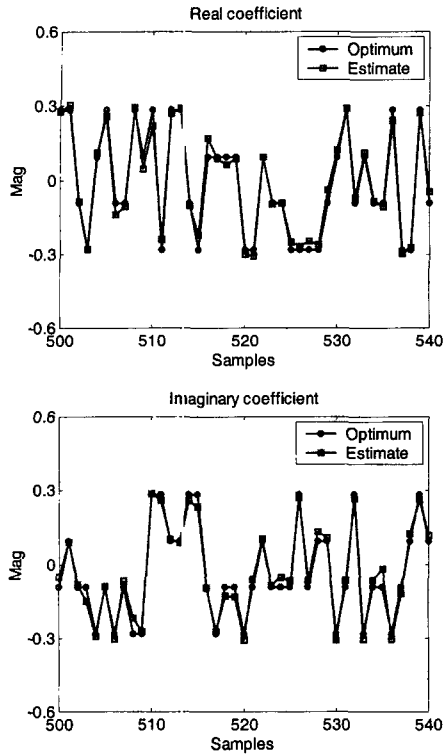


Fig. 3. The coefficients of  $h_0(0)$  and  $f_{N,0}(n)$ .

white Gaussian distribution at a varying SNR. As shown in Fig. 1, we can define the SNR as follows

$$\text{SNR} = \frac{E[\sum_{j=0}^{P-1} |a_j(n)|^2]}{E[\sum_{j=0}^{P-1} |v_j(n)|^2]} \quad (34)$$

As a performance index, we estimate the MSE, which is defined in (17). All results concerning MSE are ensemble averages of 50 independent Monte Carlo runs. Algorithm initialization parameters are  $\delta=10^{-3}$ ,  $\lambda=0.995$ , and  $\mu=0.005$ . The number of subchannels is set to  $P=2$ . For all simulations, we use the RLS algorithm for updating FPEF coefficient matrix in Table 1 and use MSE criteria in (30) and (33). The simulated channel is a length-16 version of an empirically

measured  $T/2$ -spaced digital microwave radio channel ( $P=2$ ) with 230 taps, which

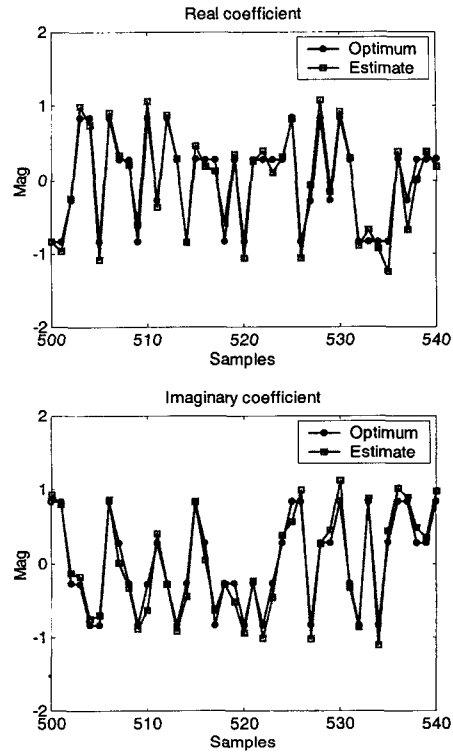


Fig. 4. The coefficients of  $h_1(0)$  and  $f_{N,1}(n)$ .

we truncated to obtain a channel with  $L=8$ . The Microwave channel *chan1.mat* is founded at <http://spib.rice.edu/microwave.html>. The shortened version is derived by linear decimation of the FFT of the full-length  $T/2$ -spaced impulse response and taking the IFFT of the decimated version (see [16] for more details on this channel).

#### ■ Convergence of the FPE

Fig. 3 and Fig. 4 show the optimum and estimated coefficients in (16) for an SNR of 20dB. Fig. 5 presents the mean square value of the difference between optimum and estimated coefficient for an SNR of 20dB, 30dB, and 40dB. For this



simulation, we set FPEF length to 8 and use RLS algorithm in (20)-(24). From

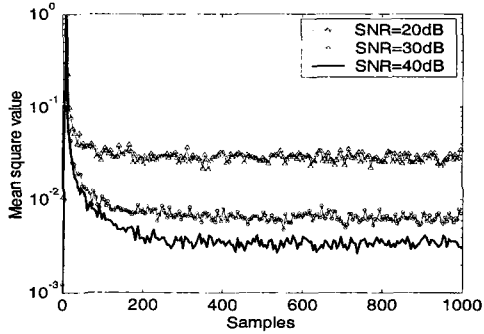


Fig. 5. The mean square value of the  $s(n)h(0)-f_n(n)$ .

these figures, we can see that the FPEF acts well.

■ Convergence of  $h(0)$

We examine the trajectories of the first channel coefficient estimation, which has an effect on the PEF-based blind equalizer. Accurate estimation can be

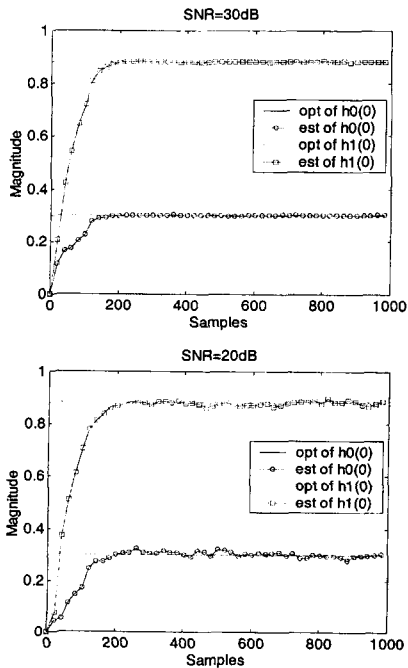


Fig. 6. Magnitude of the exact and estimated value of the  $h_0(0)$  and  $h_1(0)$ .

shown in Fig. 6, where the absolute value of the estimate for simulated channel is presented under SNR=30dB and 20dB.

■ Best decision delay choice rule

The performance of the blind equalizer is affected by a different decision delay. In Fig. 7, we show the  $J_{MSE}(D)$  and  $J_{CM}(D)$  versus decision delay  $D$  under SNR=30dB. It is available that selecting  $D \approx (N+L)/2=7$  results in good performance.

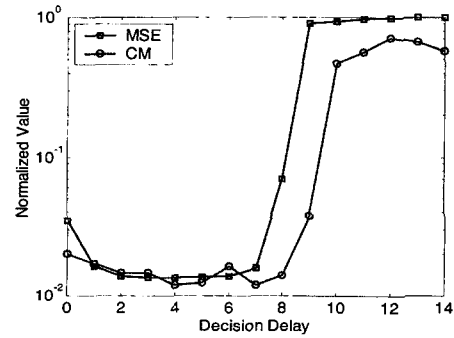


Fig. 7. The best decision delay choice rule.

■ Convergence of the proposed algorithm

Fig. 8 presents MSE curves for the proposed algorithm in Table 1 for different SNRs. We set FPEF length to 8, equalizer length  $N=8$ , and choose the optimum delay,  $D=7$  for this simulation. We use 2000 samples for our algorithm under SNR=15dB, 20dB, and 30dB. Convergence occurs after approximately 400 samples. The equalized received signal constellation plots are shown in Fig. 9 for an SNR of 20dB and 30dB.

■ Robustness to order overdetermination

Aforementioned advantage of the PEM-based blind equalizer is robust to equalizer order over-determination. Fig. 10 presents the proposed algorithm performance after 2000 samples for several FPEF orders. From Fig. 10, we can conclude that the exact order is not needed in the proposed algorithm.

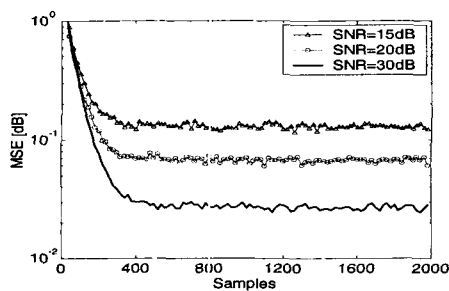


Fig. 8. The MSE of the proposed algorithm for different SNR's.

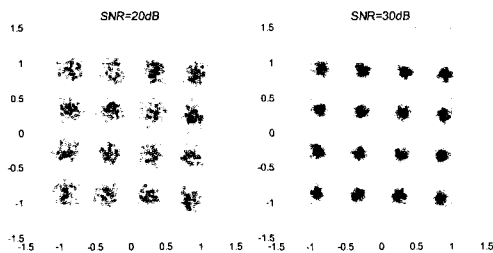


Fig. 9. Scatter plots after equalization under SNR=20dB and 30dB.

■ Performance comparison with existing algorithms

We compare the performance of the proposed algorithm with some existing algorithms: the constant modulus algorithm in [1] (denotes CMA), the one-step FPEF-based algorithm in [6] and [7] (denotes FPEF), and the one-step BPEF-based algorithm in [6] and [7] (denotes BPEF). We set both FPEF length and FSLE order to 8 for proposed algorithm. Let the equalizer order be  $N=18$  for CMA,  $N=8$  for

FPEF, and  $N=8$  for BPEF. Fig. 11 and Fig. 12 show the MSE curves for the proposed algorithm and existing algorithms under SNR=20dB and 30dB, respectively.

V. Conclusion

We have developed adaptive blind equalization based on one-step FPEF with arbitrary decision delay control. Our proposed method ensures flexible decision delay control and provides flexibility for a practical implementation since various well-known

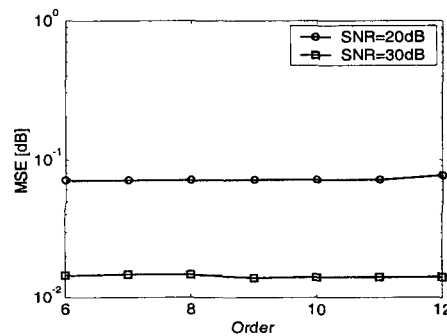


Fig. 10. The MSE performance of the proposed algorithm after 2000 samples for several FPEF orders.

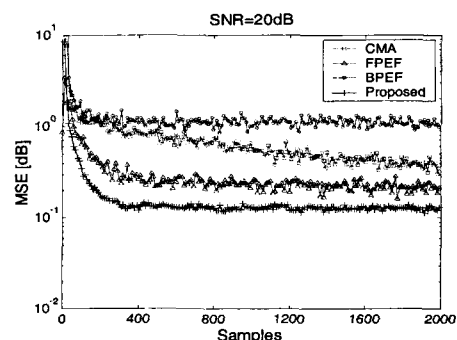


Fig. 11. The MSE comparison of our algorithm and existing algorithms under SNR=20dB.

adaptive algorithms, including RLS and LMS algorithm, can be used to implement the proposed method. Furthermore, our methods are robust to channel order estimation

n error in nature of linear prediction characteristics. We consider FPE as training sequence and utilize it for arbitrary decision delay blind equalization. Compared with HOS-based algorithm such as CMA or cumulant algorithm, proposed method is based on SOS; thus faster convergence can be achieved with little computational complexity. Simulation results show that our algorithms outperform many existing algorithms. The weakness of the proposed method lies as well; the magnitude of the first channel coefficient,  $h(0)$ , should be sufficiently large. Further research on the effect of this fact is needed. This aspect faces also to previous PEF-based blind equalization problem.

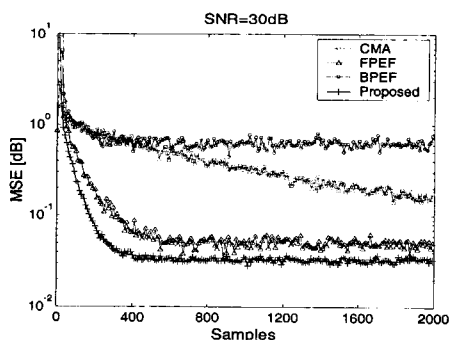


Fig. 12. The MSE comparison of our algorithm and existing algorithms under SNR=30dB

### References

[1] S. Haykin, Adaptive Filter Theory. Englewood Cliffs, NJ: Prentice-Hall, 1996.  
 [2] J. G. Proakis, Digital Communication. McGraw Hill, 2001.  
 [3] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," IEEE Trans. Inform. Theory., vol. 40, no. 2, pp. 3

40-349, Mar. 1994.  
 [4] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind channel identification based on second-order statistics: A frequency-domain approach," IEEE Trans. Inform. Theory, vol. 41, no. 1, pp. 329-334, Jan. 1995.  
 [5] K. Abed-Meraim, E. Moulines, and P. Loubaton, "Prediction error method for second-order blind identification," IEEE Trans. Signal Processing, vol. 45, no. 3, pp. 694-704, Mar. 1997.  
 [6] J. Mannerkoski and D. P. Taylor, "Blind equalization using least-squares lattice prediction," IEEE Trans. Signal Processing, vol. 47, no. 3, pp. 630-640, Mar. 1999.  
 [7] C. B. Papadias and D. T. M. Slock, "Fractionally spaced equalization of linear polyphase channels and related blind techniques based on multichannel linear prediction," IEEE Trans. Signal Processing, vol. 47, no. 3, pp. 641-653, Mar. 1999.  
 [8] J. Shen and Z. Ding, "Direct blind MMSE channel equalization based on second order statistics," IEEE Trans. Signal Processing, vol. 48, no. 4, pp. 1015-1022, Apr. 2000.  
 [9] J. Mannerkoski, V. Koivunen, and D. P. Taylor, "Performance bounds for multistep prediction-based blind equalization," IEEE Trans. Signal Processing, vol. 49, no. 1, pp. 84-93, Jan. 2001.  
 [10] D. K. Borah, R. A. Kennedy, Z. Ding, and I. Fijalkow, "Sampling and prefiltering effects on blind equalizer design," IEEE Trans. Signal Processing, vol. 49, no. 1, pp. 209-218, Jan. 2001.

