Journal of the Society of Korea Industrial and Systems Engineering
Vol, 26, No. 2, pp.62-67, March 2003.

복잡한 통신 시스템의 성능분석을 위한 유한소스 대기 모형

김 제 숭

상지대학교 산업공학과

Finite Source Queueing Models for Analysis of Complex Communication Systems †

Che-Soong Kim

Dept. of Industrial Engineering, Sangji University

This paper deals with a First-Come, First-Served queueing model to analyze the behavior of heterogeneous finite source system with a single server. Each sources and the processor are assumed to operate in independently Markovian environments, respectively. Each request is characterized by its own exponentially distributed source and service time with parameter depending on the state of the corresponding environment, that is, the arrival and service rates are subject to random fluctuations. Our aim is to get the usual stationary performance measures of the system, such as, utilizations, mean number of requests staying at the server, mean queue lengths, average waiting and sojourn times.

In the case of fast arrivals or fast service asymptotic methods can be applied. In the intermediate situations stochastic simulation is used. As applications of this model some problems in the field of telecommunications are treated.

Keywords : Finite Source, Queueing Communication System, Markov-Modulation, Random Environment, Fast Arrival, Fast Service.

1. Introduction

Performance modeling of recent computer and communication system has become more complicated as the size and complexity of the system has increased, (see Haverkort [5]). One of the measures of greatest interest is the distribution of the time to the first system failure. It is well-known that the majority of the problems can be treated by the help of Semi-Markov Processes. Since the failure free operation time of the system corresponds to sojourn time problems, we can use the results obtained for SMP.If the exit from a given subset of the state space is a "rare"event, that is it occurs with a small probability it is natural to investigate the asymptotic behavior of the sojourn time in that subspace, see Gertsbakh [4] and Kovalenko [6]. Realistic consideration of certain stochastic systems, however, often requires the introduction of a random environment, sometimes referred as to Markov Modulation, where system parameters are subjected to randomly fluctuations or bursts. This situation may be attributed to certain changes in the physical environment such as weather, or sudden personal changes and workload alterations. Gaver et al. [3] proposed an efficient computational approach for the analysis of a generalized structure involving finite state space birth-and-death processes in a Markovian environment.

This study deals with a First-Come, First-Served (FCFS) queueing model to analyze the behavior of heterogeneous

† This work was supported by Sangji University Research Grant 2002

finite-source system with a single server. The sources and the server are supposed to operate in independent random environments, respectively, allowing the arrival and service processes to be Markov-modulated ones. Each request is characterized by its own exponentially distributed source and service time with parameter depending on the state of the corresponding environment, that is, the arrival and service rates are subject to random fluctuations. Our aim is to get the usual stationary performance measures of the system, such as, utilizations, mean number of requests staying at the server, mean queue lengths, average waiting and sojourn times. The main problem is that the state space of the underlying continuous time Markov chain will be very large, so we have the state space explosion problem. To avoid it in the case of "fast"arrival or "fast"service situations asymptotic methods can be applied. In the intermediate situations stochastic simulation is used.

This study generalizes the results of Sztrik and Rigo [10] where assuming "fast" arrivals the sources are homogeneous and the whole system is governed by two random environments. In the case of "fast" service it extends the results of Sztrik [8] where the request are heterogeneous and arrival and service rates are depend on the state of two governing Markov-chains, respectively. The technique used here is similar to the one applied in Sztrik [8], [9], [10] and closely related to the theoretic investigations of Anisimov [1-2].

2. The Queueing Model

Consider a finite-source queuing system with N heterogeneous sources and a single server. The sources and the server operate in independent random environments. The environmental changes are reflected in the values of the access and service rates that prevail at any point of time. The main objective is to adapt these parameters to respond to random changes effectively and thus maintain derived level of the system performance.

Source P is assumed to operate in a random environment governed by an ergodic Markov chain $(\xi_p(t), t \geq 0)$ with state space $(1,2,\Lambda,r_p)$ and with transition density matrix

$$\left(a_{i_p j_p}^{(p)}, i_p, j_p = 1,\Lambda, r_p, \; a_{i_p i_p}^{(p)} = \sum_{k \neq i_p} a_{i_p k}^{(p)} \right)$$

Whenever the environmental process $\xi_p(t)$ is in state i_p the probability that the source generates a request in the time interval $(t,t+h)$ is $\lambda_p(i_p,\varepsilon)h + o(h)$, $p=1,\Lambda,N$. Each request is transmitted to a server where the service immediately starts if it is idle, otherwise a queueing line is formed. The service discipline is FCFS. The server is also supposed to operate in a random environment governed by an ergodic Markov chain $(\xi_p(t), t \geq 0)$ with state space $(1,2,\Lambda,r_{N+1})$ and with transition density matrix.

$$\left(a_{i_{N+1} j_{N+1}}^{(N+1)}, i_{N+1}, j_{N+1} = 1,\Lambda, r_{N+1}, \; a_{i_{N+1} j_{N+1}}^{(N+1)} = \sum_{k \neq i_{N+1}} a_{i_{N+1} k}^{(N+1)} \right)$$

Whenever the environmental process $(\xi_p(t), t \geq 0)$ is in state i_p the probability that the service of request P is in the time interval $(t,t+h)$ is $\mu_p(i_p,\varepsilon)h + o(h)$. If a given source has sent a request it stays idle and it cannot generate another one. After being serviced each request involved here and the random environmental are supposed to be independent of each other.

In practical applications it is very important to know the distribution of time until the server becomes idles, which is actually the busy period of the processor, or the distribution of time until the number of requests staying at the service facility reaches a certain level. So that in the following we will use two special situations, namely, "fast"arrival and "fast"service assumptions to get the above mentioned distributions, respectively.

2.2 Fast Service Case

Let us consider the system under the assumption of "fast" arrivals, i.e., $\lambda_p(i_p,\varepsilon) \to \infty$ as $\varepsilon \to 0$, and $\mu_p(i_{N+1},\varepsilon) = \mu_p(i_{N+1})$. For simplicity, let $\lambda_p(i_p,\varepsilon) = \lambda_p(i_p)/\varepsilon$, $p=1,\Lambda,N$. Denote by $Y_\varepsilon(t)$ the number of requests staying in the sources at time t, and let $\Omega_\varepsilon(N-1) = inf\{t : t \geq 0, Y_\varepsilon(t) = N / Y_\varepsilon(0) \leq N-1\}$, that is, the case when the server becomes idles, i.e. $\Omega_\varepsilon(N-1)$ is the busy period length of the server, or processor. Denote by $\pi_0(i_1,i_2,\Lambda,i_N,i_{N+1} : 0; k_1,k_2,\Lambda,k_N)$ the steady-state probability that random environment $\xi_p(t)$ is in state i_p, ..., there is no request in the source and the order of their arrivals to the server is (k_1,k_2,Λ,k_N). Similarly, denote by $\pi_0(i_1,\Lambda, i_{N+1} : 1; k_1,\Lambda,k_N)$ the steady state probability that p^{th} random environment is in state i_p, $p=1,\Lambda,N+1$, source k_1 is generating a new request and the order sources sent their requests in the order (k_1,k_2,Λ,k_N). Clearly, $(k_1,k_2,\Lambda,k_N)\in V_N^{N-s+1}$, $s=1,2$ where V_N^{N-s+1} denotes the set of all variations of order $N-s+1$ of elements $1,\Lambda,N$. Now we have:

Theorem 1. *For the system in question under the above assumptions, independently of the initial state, the distribution of the normalized random variable* $\varepsilon^{N-1}\Omega_\varepsilon(N-1)$ *converges weakly to an exponentially distributed random variable with parameter.*

$$\Lambda=\sum_{i_1=1}^{r_1}\Lambda\sum_{i_{N+1}=1}^{r_{N+1}}\sum_{(k_1,k_2,\Lambda,k_N)\in V_N^N}\pi_0(i_1,\Lambda,i_{N+1}:1;k_2,\Lambda,k_N)$$
$$\times\frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})}\times\frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\lambda_{k_2}(i_{k_2})}\times$$
$$\Lambda\times\frac{\lambda_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\Lambda+\lambda_{k_{N-1}}(i_{k_{N-1}})}\frac{1}{D}$$

Where

$$D=\sum_{p=1}^{N+1}\sum_{j_p=1}^{r_p}\sum_{\substack{i_p=1\\ i_p\neq j_p}}^{r_p}\sum_{(k_1,\Lambda,k_N)\in V_N^N}\pi_0(i_1,\Lambda,i_{N+1}:0;k_1,\Lambda,k_N)$$
$$\times\frac{a_{i_pj_p}^{(p)}}{\left(\sum_{q=1}^{N+1}a_{i_qi_q}^{(q)}+\mu_{k_1}(i_{N+1})\right)^2}$$

Proof. It is similar to the one used in Sztrik[9,10] and due to the limited page numbers only the main parts are discussed. Let us introduce the following stochastic process $Z_\varepsilon(t)=(\xi_1(t),\Lambda,\xi_{N+1}(t):Y_\varepsilon(t);\beta_1(t),\Lambda,\beta_{Y_\varepsilon(t)})$, where $(\beta_1(t),\Lambda,\beta_{Y_\varepsilon(t)})$ denote the indices of the messages in order of their arrival at the processor. It is easy to see that $(Z_\varepsilon(t),t\geq 0)$ is a multi-dimensional Markov Chain with rather complex state space

$$E=((i_1,i_2,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),i_p=1,\Lambda,r,$$
$$p=1,\Lambda,N+1,(k_1,\Lambda,k_{N-s})\in V_N^{N-s},s=0,\Lambda,N)$$

Where $k_0=\{0\}$ by definition. Furthermore, let

$$\langle\alpha_m\rangle=((i_1,i_2,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),i_p=1,\Lambda,r,$$
$$p=1,\Lambda,N+1,(k_1,\Lambda,k_{N-s})\in V_N^{N-s},s=0,\Lambda,m)$$

be a subset of the states.

Let us denote by $g_\varepsilon(\langle\alpha_m\rangle)$ the steady state probability of exit from $\langle\alpha_m\rangle$, that is

$$g_\varepsilon(\langle\alpha_m\rangle)=\sum_{i^{(m)}\in X_m}\pi_\varepsilon(i^{(m)})\sum_{j^{(m+1)}\in X_{m+1}}p_\varepsilon(i^{(m)},j^{(m+1)})$$

Hence our aim is to determine the distribution of the first exit time of $Z_\varepsilon(t)$ from $\langle\alpha_m\rangle$, provided that $Z_\varepsilon(t)\in\langle\alpha_m\rangle$. It can easily be verified that the transition probabilities for the embedded Markov chain are

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),(i_1,\Lambda,j_p,\Lambda,i_{N+1}:s;k_1,\Lambda k_{N-s}))$$
$$=\frac{a_{i_pj_p}^{(p)}}{\gamma_\varepsilon(i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s})},for\, s=0,\Lambda N,p=1,\Lambda,N+1$$

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:s;k_2,\Lambda,k_{N-s}),(i_1,\Lambda,i_{N+1}:s+1;k_1,\Lambda k_{N-s+1}))$$
$$=\frac{\mu_{k_1}(i_{N+1})}{\gamma_\varepsilon(i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s})},for\, s=0,\Lambda N-1$$

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),(i_1,\Lambda,i_{N+1}:s-1;k_1,\Lambda k_{N-s-1}))$$
$$=\frac{\lambda_{k_{N-s}}(i_{k_{N-s}})/\varepsilon}{\gamma_\varepsilon(i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s})},for\, s=1,\Lambda N$$

As $\varepsilon\rightarrow 0$ implies

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:0;k_1,\Lambda,k_N),(i_1,\Lambda,j_p,\Lambda,i_{N+1}:0;k_1,\Lambda k_N))$$
$$=\frac{a_{i_pj_p}^{(p)}}{a_{i_1i_1}^{(1)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu_{k_1}(i_{N+1})},\ p=1,\Lambda,N+1$$

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),(i_1,\Lambda,j_p,\Lambda,i_{N+1}:s;k_1,\Lambda k_{N-s}))$$
$$=0(1),for\, s=0,\Lambda N,p=1,\Lambda,N+1$$

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:0;k_1,\Lambda,k_N),(i_1,\Lambda,i_{N+1}:1;k_2,\Lambda k_N))$$
$$=\frac{\mu_{k_1}(i_{N+1})}{a_{i_1i_1}^{(1)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu_{k_1}(i_{N+1})},\ p=1,\Lambda,N+1$$

$$p_\varepsilon((i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}),(i_1,\Lambda,i_{N+1}:s+1;k_2,\Lambda k_{N-s}))$$
$$=\frac{\mu_{k_1}(i_{N+1})\varepsilon}{\sum_{p\neq k_1,\Lambda,k_{N-s}}\lambda_p(i_p)}(1+0(1)),s=1,\Lambda,N-1$$

The probabilities

$\pi_0\left(i_1,i_2,\Lambda,i_N,i_{N+1}:0;k_1,k_2,\Lambda,k_N\right)$,

$\pi_0\left(i_1,i_2,\Lambda,i_N,i_{N+1}:1;k_1,k_2,\Lambda,k_{N-1}\right)$

$i_p=1,\Lambda,r_p,p=1,\Lambda,N+1,(k_1,\Lambda,k_{N-s})\in V_N^{N-s},s=0,1$

satisfy the following system of equations

$$\begin{aligned}
&\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_N\right)\\
&=\sum_{i_1\neq j_1}\pi_0(i_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_N)\\
&\times a_{i_1j_1}^{(1)}\,(a_{i_1i_1}^{(1)}+a_{j_2j_2}^{(2)}+\Lambda+a_{j_{N+1}j_{N+1}}^{(N+1)}+\mu_{k_1}(j_{N+1}))^{-1}\\
&+\sum_{i_2\neq j_{21}}\pi_0(j_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_N)\\
&\times a_{i_2j_2}^{(2)}\,(a_{j_1j_1}^{(1)}+a_{i_2i_2}^{(2)}+\Lambda+a_{j_{N+1}j_{N+1}}^{(N+1)}+\mu_{k_1}(j_{N+1}))^{-1}\\
&\qquad+\ \Lambda\ +\\
&+\sum_{i_{N+1}\neq j_{N+1}}\pi_0(j_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_N)\\
&\times a_{i_{N+1}j_{N+1}}^{(N+1)}\,(a_{j_1j_1}^{(1)}+a_{j_2j_2}^{(2)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu_{k_1}(i_{N+1}))^{-1}\\
&+\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:1;k_1,\Lambda,k_{N-1}\right) \qquad (1)
\end{aligned}$$

$$\begin{aligned}
&\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:1;k_1,\Lambda,k_{N-1}\right)\\
&=\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_{N-1}\right)\\
&\times\mu_{k_N}(j_{N+1})(a_{j_1j_1}^{(1)}+a_{j_2j_2}^{(2)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu_{k_1}(i_{N+1}))^{-1} \qquad (2)
\end{aligned}$$

To apply the results of Anisimov [1, 2] we need the solution of (1), (2) with normalizing condition

$$\begin{aligned}
&\sum_{p=1}^{N+1}\sum_{i_p=1}^{r_p}\sum_{(k_1,\Lambda,k_N)}\{\pi_0(i_1,\Lambda,i_{N+1}:0;k_1,\Lambda,k_N)\\
&+\pi_0\left(i_1,\Lambda,i_{N+1}:1;k_1,\Lambda,k_{N-1}\right)=1 \qquad (3)
\end{aligned}$$

Suppose that we have this solution. Then we get

$$\begin{aligned}
&g_\varepsilon\langle\alpha^m\rangle=\varepsilon^m\sum^{r_1}\Lambda\sum^{r_{N+1}}\sum\pi_0(i_1,\Lambda,i_{N+1}:1:k_2,\Lambda,k_N)\\
&\times\frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})}\times\frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\lambda_{k_2}(i_{k_2})}\times\Lambda\\
&\Lambda\times\frac{\lambda_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\Lambda+\lambda_{k_{N-1}}(i_{k_{N-1}})}(1+0(1))
\end{aligned}$$

Taking into account the exponentially of $\tau_0\left(i_1,\Lambda,i_{N+1}:s;k_1,\Lambda,k_{N-s}\right)$ for fixed Θ we have

$$\begin{aligned}
&\mathrm{E}\exp\{i\varepsilon^m\Theta\tau_\varepsilon\left(i_1,\Lambda,i_{N+1}:0;k_1,\Lambda,k_{N-s}\right)\}\\
&=1+\varepsilon^m\frac{i\Theta}{a_{i_1i_1}^{(1)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu_{k_1}(i_{N+1})}(1+0(1))
\end{aligned}$$

Notice that $\beta_\varepsilon=\varepsilon^m$ and therefore we immediately get that $\varepsilon^m\Omega_\varepsilon(m)$ converges weakly to an exponentially distributed random variable with parameter

$$\begin{aligned}
&\Lambda=\sum_{i_1=1}^{r_1}\Lambda\sum_{i_{N+1}=1}^{r_{N+1}}\sum_{(k_1,k_2,\Lambda,k_N)\in V_N^N}\pi_0(i_1,\Lambda,i_{N+1};1;k_2,\Lambda,k_N)\\
&\times\frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})}\times\frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\lambda_{k_2}(i_{k_2})}\times\Lambda\\
&\Lambda\times\frac{\lambda_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\Lambda+\lambda_{k_{N-1}}(i_{k_{N-1}})}\frac{1}{D} \qquad (4)
\end{aligned}$$

Let $m=N-1$ from which our statement immediately follows. It should be noted that this asymptotic approach considerably reduces the calculations since to solve the system of equations (1), (2) is much easier than to get the stationary distribution of $(Z_\varepsilon(t),t\geq o)$ and system of stochastic relations concerning to its sojourn time in $\langle\alpha_m\rangle$. Notice, if $\mu_p(i_{N+1})=\mu(i_{N+1}),p=1,\Lambda,N$, a closed form solution can be obtained.

Denoted by $\left(\pi_{i_p}^{(p)},i_p=1,\Lambda,r_p\right)$ the steady state distribution of the governing Markov chains $(\xi_p(t),t\geq o)$, $p=1,\Lambda,N$, respectively. It can be verified, that the solution of (1), (2) is

$$\begin{aligned}
&\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:0;k_1,\Lambda,k_N\right)\\
&=B(\pi_{j_1}^{(1)},\Lambda,\pi_{j_{N+1}}^{(N+1)})(a_{j_1j_1}^{(1)}+a_{j_2j_2}^{(2)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+\mu(j_{N+1}))\\
&\pi_0\left(j_1,j_2,\Lambda,j_{N+1}:1;k_1,\Lambda,k_{N-1}\right)\\
&=B(\pi_{j_1}^{(1)},\Lambda,\pi_{j_{N+1}}^{(N+1)})\,\mu(j_{N+1})
\end{aligned}$$

Where B is the normalizing constant, i.e.

$$\begin{aligned}
&\frac{1}{B}=N!\sum_{i_1=1}^{r_1}\Lambda\sum_{i_{N+1}=1}^{r_{N+1}}(\pi_{i_1}^{(1)},\Lambda,\pi_{i_{N+1}}^{(N+1)})\\
&\times(a_{j_1j_1}^{(1)}+a_{j_2j_2}^{(2)}+\Lambda+a_{i_{N+1}i_{N+1}}^{(N+1)}+2\mu(j_{N+1}))
\end{aligned}$$

Thus from (4) it follows that $\varepsilon^{N-1}\Omega(N-1)$ converges weakly to an exponentially distributed random variable with parameter.

$$\Lambda = \frac{1}{N!}\sum_{i_1=1}^{r_1}\Lambda \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,k_2,\Lambda,k_N)\in V_N^N}\left(\pi_{i_1}^{(1)}\Lambda\ \pi_{i_{N+1}}^{(N+1)}\right)\mu(i_{N+1})$$
$$\times\frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})}\times\frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\lambda_{k_2}(i_{k_2})}\times\Lambda$$
$$\Lambda\ \times\frac{\lambda_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\Lambda\ +\lambda_{k_{N-1}}(i_{k_{N-1}})} \qquad (5)$$

Thus from (5) we get that the busy period length of the server is asymptotically an exponentially distributed random variable with parameter.

$$\varepsilon^{N-1}\Lambda = \frac{\varepsilon^{N-1}}{N!}\sum_{i_1=1}^{r_1}\Lambda \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,k_2,\Lambda,k_N)\in V_N^N}\left(\pi_{i_1}^{(1)}\Lambda\ \pi_{i_{N+1}}^{(N+1)}\right)\mu(i_{N+1})$$
$$\times\frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})}\times\frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\lambda_{k_2}(i_{k_2})}\times\Lambda$$
$$\Lambda\ \times\frac{\lambda_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1})+\Lambda\ +\lambda_{k_{N-1}}(i_{k_{N-1}})} \qquad (6)$$

In the case when there are no random environments, that is $\mu(i_{N+1})=\mu,\ \lambda_p(i_p)=\lambda_p,\quad i_p=1,\Lambda,r_p,\ p=1,\Lambda,N$, from (6) we get

$$\varepsilon^{N-1}\Lambda = \frac{\varepsilon^{N-1}}{N!}\sum_{(k_1,\Lambda,k_N)\in V_N^N}\mu\frac{\mu}{\lambda_{k_1}}\frac{\mu}{\lambda_{k_1}+\lambda_{k_2}}\times$$
$$\Lambda\ \times\frac{\mu}{\lambda_{k_1}+\Lambda\ +\lambda_{k_{N-1}}} \qquad (7)$$

Finally, for the totally homogeneous case from (7) we obtain

$$\varepsilon^{N-1}\Lambda = \frac{1}{(N-1)!}\frac{\mu^N}{(\lambda/\varepsilon)^{N-1}}$$

The utilization $U_r$of the receiver, which is the long run fraction of time during which it is busy, can be given by

$$U_r = \frac{1}{\varepsilon^{N-1}\Lambda}\left(\frac{1}{\varepsilon^{N-1}\Lambda}+I\right)^{-1}$$

Where I denotes its idle period length.

2.2 Fast Service Case

In this section let us assume that the service is "fast", that is $\mu_p(i_{N+1},\varepsilon)\to\infty$ as $\varepsilon\to 0$, and $\lambda_p(i_p,\varepsilon)=\lambda_p(i_p)$. For simplicity, let $\mu_p(i_{N+1},\varepsilon)=\lambda_p(i_{N+1})/\varepsilon,\ p=1,\Lambda,N$. Our aim is to get the distribution of time until the number of requests staying at the service facility reaches a certain level. In this case denote by $Y_\varepsilon(t)$ the number of requests staying at the service facility at time t, and let

$$\Omega_\varepsilon(m)=in\,f\{t:\ t\ge 0,\ Y_\varepsilon(t)=m+1/Y_\varepsilon(0)\le m\}$$

The following theorem has a practical importance in the field of reliability theory where $Y_\varepsilon(t)$ can be interpreted as the failure free operational time of the system.

Theorem 2 *For the system in question under the above assumptions, independently of the initial state, the distribution of the normalized random variable* $\varepsilon^m\Omega_\varepsilon(m)$ *converges weakly to an exponentially distributed random variable with parameter.*

$$\Lambda = \sum_{i_1=1}^{r_1}\Lambda\sum_{i_{N+1}=1}^{r_{N+1}}\sum_{(k_1,k_2,\Lambda,k_N)\in V_N^N}\left(\prod_{p=1}^{N+1}\pi_{i_p}^{(p)}\right)\frac{\prod_{s=1}^{m+1}\lambda_{k_s}(i_{k_s})}{\prod_{s=1}^{m}\mu_{k_1}(i_{N+1})}$$

Proof. It is similar to the one used in C.S.Kim & Sztrik[8] and the above analysis, thus it is omitted.

2.3 Stochastic Simulation

As it can be seen the above discussed two special cases result rare events, since in both situations we get events of very small probabilities. It is also well known that simulation of rare events are quite complicated and time consuming. Even so, the validation of the asymptotic results by simulation is in progress. In the intermediate situations, that is when the arrival and service rates are in the same scales for moderate source size under different service disciplines, a toll has recently been developed, see Sztrik [11] by the help of which not only exponentially distributed source and service times, but Coxian ones can be investigated. By using the Law-Carson algorithm, see

Law and Kelton [7] confidence intervals for the main performance measures are obtained, such as, utilizations, mean number of requests staying at the server, mean queue lengths, average waiting and sojourn times.

3. Conclusions

In this paper, we have presented as a queueing model to analyze the behavior of a First Come First Service heterogeneous finite source computer and communication system with a single processor. The system operates in Markovian environments and the messages arrival fast and service fast. An asymptotic approach has been provided to obtain the distribution of the time to the first system failure.

Reference

[1] Anisimov V. V., Asymptotic Analysis of Switching Queueing Systems in Condition of Low and Heavy Loading, Matrix Analytic Methods in Stochastic Models, Lecture Notes in Pure and Appl. Mathem. Series, Marcel Dekker, Inc. 1996

[2] Anisimov V. V., Asymptotic Analysis of Reliability for Switching Systems in Light and Heavy Traffic Conditions Recent Advances in Reliability Theory: Methodology, Practice and Inference, Birhauser Bosten Inc. 2000.

[3] Gaver D. P., Jacobs P.A., Latouche G., "Finite birth and death models is randomly changing environments", Advances in Applied Probability 16, pp. 715-731, 1984.

[4] Gertsbakh I.B., "Asymptotic methods in reliability theory; A review", Advances in Applied Probability 16, pp. 147-175.

[5] Haverkort B., Performance of Computer Communication Systems, John Wiley & Sons, 1998.

[6] Kovalenko I.N., "Rare events in queueing systems, A urvey", Queueing Systems 16, pp. 1-49, 1994.

[7] Law A.L., Kelton W.D., Simulation Modeling and Analysis, McGraw-Hill, 1991.

[8] Sztrik J., Kim C. S., "Markov Modulated Finite-Source Queueing Models in Evaluation of Computer and Communication Systems", Computers & Mathematics with Applications, in press.

[9] Sztrik J., Kouvatsos D.D., "Asymptotic Analysis of Heterogeneous Multiprocessor System in a Randomly Changing Environments", IEEE Transactions on Software Engineering 17, pp. 1069-1075, 1991.

[10] Sztrik J., Moeller O., "A tool for simulation of Markov-Modulated Finite-Source Queueing Systems", Proceeding of Messung Modelling und Berwertung, Trier, Germany, pp. 109-114, 1999.