

# Mathematical Models for Optimal Bead Geometry for GMA Welding Process

C. E. Park, C. S. Li and I. S. Kim

## Abstract

A major concern in Gas Metal Arc (GMA) welding process is the determination of welding process variables such as wire diameter, gas flow rate, welding speed, arc current and welding voltage and their effects on the desired weld bead dimensions and shape. To successfully accomplish this objective, 81 welded samples from mild steel AS 1204 flats adopting the bead-on-plate technique were employed in the experiment. The experimental results were used to develop a mathematical model to predict the magnitude of bead geometry as follows; weld bead width, weld bead height, weld bead penetration depth, weld penetration shape factor, weld reinforcement shape factor, weld bead total area, weld bead penetration area, weld bead reinforcement area, weld bead dilution, length of weld bead penetration boundary and length of weld bead reinforcement boundary, and to establish the relationships between weld process parameters and bead geometry. Multiple regression analysis was employed for investigating and modeling the GMA process and significance test techniques were applied for the interpretation of the experimental data.

**Key Words :** Weld process simulation, Microstructure evolution, Finite element analysis, Hot-tap welding, Repair welding.

## 1. Introduction

GMA welding process in which the welding electrode is melted and molten metals is transferred to the workpiece, is the method chosen for assembling metal structures such as ships, cars, trains, pipelines and bridges. One of the important tasks in the GMA welding process is to understand how welding process parameters affect the welding bead geometry and subsequently develop the analytical models for predicting the desired weld process outputs. By carefully choosing and closely controlling welding process parameters, high quality welds may be made in all circumstances for arc welding process<sup>1)</sup>.

Many attempts have been made to understand and estimate the effect of welding process parameters on the optimal bead geometry. These include theoretical studies<sup>3,7,13-14,17,19)</sup> and empirical models based on actual

welding application<sup>2,4-6,8-10,12,16,18)</sup>. Generally, theoretical models require accurate knowledge of the heat and mass addition to the molten weld pool, interaction of the arc with the surface, the relationship between welding process parameters and arc efficiency, and electrode detachments and droplet formation. Furthermore, the physical knowledge of the welding process, boundary conditions, and material properties should be considered as hypotheses, but have never been incorporated in a comprehensive model. Even if existing theoretical models are reasonably accurate for predicting the Fusion Zone (FZ) and Heat Affected Zone (HAZ), the prediction of desired weld bead dimensions and shape is less than successful due to lack of information regarding actual phenomena dependent on the physical and chemical properties of materials over a wide temperature range<sup>3)</sup>.

In automating the GMA welding process, analytical equations that describe the interaction of the weld process parameters and their influence on optimal bead geometry are required for the development of process control algorithms. An early attempt of procedure optimization, called a tolerance box, was developed to preserve all the compiled information, to allow a rigorous determination of the effects on the quality of any modification of the welding process parameters and to offer a well-informed choice of the welding process

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parameters in terms of the constraints imposed by the production process<sup>4,8)</sup>. However, this approach required a large number of tests and was found to be impractical for process control purpose when dealing with more than three welding process input parameters. The above mentioned work has been summarized by Shinoda and Doherty<sup>16)</sup>.

However, the situation has been recently altered with the advent of increasing computer efficiency and better understanding of the usefulness of statistically designed experimentation based on factorial techniques, the latter of which can reduce cost and provide the required information about main and interaction effects on the response factors. Such techniques for establishing relationship between welding process parameters and bead geometry have been reported for the submerged arc butt welds in order to accomplish control over arc behavior for fully mechanized and automatic welding<sup>9-10)</sup>. The weld process parameters included in these studies were welding current, arc voltage, welding speed, bevel angle and electrode diameter. Similar mathematical models relating welding process variables and bead geometry for the selection and control of the procedural variables for flux cored arc welding processes have also been reported<sup>5)</sup>. Raveendra and Parmar<sup>12)</sup> represented a mathematical model which used fractional factorial techniques based on regression and analysis of variance techniques. This model was developed for studying the effect of touch angle on the bead geometry for CO<sub>2</sub> shielded flux cored arc welding. Their experimental results showed that mathematical modeling can be an effective tool for predicting bead geometry and estimating control variables for achieving a desired bead geometry.

Chande<sup>12)</sup> first applied this technique to GMA process and included the effects of voltage, current, welding speed, composition of the shielding gas, electrode diameter, and the electrode extension. These results showed that electrode extension and electrode polarity were important variables and mathematical models derived from experimental results could be used to predict the bead geometry fairly accurately. Recently, Yang, et al.<sup>18)</sup> extended the study to the weld deposit area and presented the effects of electrode polarity, extension, electrode diameter, welding current, arc voltage, travel speed, power source characteristics and flux basicity on the weld deposit area. Experimental results indicated that a small-diameter electrode, long electrode extension, low voltage and high welding speed produced large deposit area, whereas the power source and flux type did not seem to have any significant effect on the weld deposit

area.

Bead geometry depends on the amount and distribution of the input energy on the workpiece surface and the dissipation of input energy in the workpiece<sup>17)</sup>. In GMA welding process, heat and mass inputs are coupled and transferred by the weld arc to the molten weld pool and by the molten metal that is being transferred to the weld pool. The amount and distribution of the input energy are basically controlled by the obvious and careful choices of welding process parameters in order to accomplish the optimal bead geometry and the desired mechanical properties of the weldment. To make effective use of automated and robotic GMA welding, it is imperative that the mathematical models are used to predict bead geometry, applicable to all welding positions and covering a wide range of material thickness.

The objective of work presented in this paper is to develop mathematical models for studying the influence of welding process parameters on bead geometry assisting in the GMA welding process optimization and the generation of process control algorithms, and correlating welding process variables to bead geometry of bead-on-plates deposited by GMA welding process.

## 2. Design of Experiment

To achieve a satisfactory bead geometry and the weld quality, and control the welding process parameters, it is necessary to study the effects of those parameters affecting the mode of metal transfer and the amount of heat input to the workpiece in GMA welding. To develop the mathematical models, the welding process parameters included were wire diameter, gas flow rate, welding speed, arc current and welding voltage. All other parameters except these parameters under consideration were kept constant. Table 1 shows these welding process

Table 1 Welding process parameters and limits

Parameter	Symbol	Unit	Values used
Wire diameter	D	mm	0.9, 1.2, 1.6
Gas flow rate	G	l	6, 10, 14
Welding speed	S	mm/min	250, 330, 410
Arc current	I	Amp	90, 190, 250, 180, 260, 360
Welding voltage	V	Volt	20, 25, 30

parameters and their limits. Arc current levels selected for 0.9 mm diameter wire were 90, 190, 250 Amp, whereas the levels for 1.2 and 1.6 mm diameter wire were 180, 260, 360 Amp.

The  $3^n$  factorial design provided the optimum number of treatment combinations where  $n$  factors can be studied in a complete factorial classification, and the main effect of each factor as well as the interactions between them should be defined<sup>11)</sup>. In this study, experimental results were used for fitting the response curve. Conventionally, in the experimental factorial design fields, welding process parameters are known as factors and bead geometry as effects. The experimental design includes a fractional factorial experiment, involving five parameters designed on three levels around the constraints described above. The levels were selected, partly in anticipation of known consequences of change and partly in respect of acceptable quality control criteria. In all planes, the five parameters are denoted by the capital letters D (wire diameter), G (gas flow rate), S (welding speed), I (arc current), V (welding voltage) and were investigated at the three levels of factors as low, intermediate, and high. They were designated by the digits 0 (low), 1 (intermediate) and 2 (high). Each treatment combination in the  $3^5$  fractional factorial design were designated by  $X_1, X_2, X_3, X_4, X_5$ , where  $X_1$  is the level of factor D,  $X_2$  is the level of factor G,  $X_3$  is the level of factor S,  $X_4$  is the level of factor I,  $X_5$  is the level of factor V. For example, for an experiment involving the weld process parameters D, G, S, I and V, the treatment combination 20111 indicates D at level 2, G at level 0, and S, I and V at level 1.

To measure certain weld bead geometric parameters that describe its appearance and induce other dimensions which are meaningful in determining the acceptability of the weld, a definition of bead geometry can be proposed as the basic of a control system for the automatic GMA welding process. These parameters generally determine the basic mechanical properties of the weld and can ensure the basic stress handling capabilities of the joints. Fig. 1 shows typical weld bead geometric parameters. The response surface is weld bead width, weld bead height, weld bead penetration depth, weld penetration shape factor, weld reinforcement shape factor, weld bead total area, weld bead penetration area, weld bead reinforcement area, weld bead dilution, length of weld bead penetration boundary, and length of weld bead reinforcement boundary. A three level full factorial design requiring 243 trials was found in order to describe the effect of input parameters on desired output. The half replicate of tests of 81 trials for fitting each equation,

called a fractional factorial design, was finally used to evaluate the main effect and interaction effects of five welding process parameters at three levels.

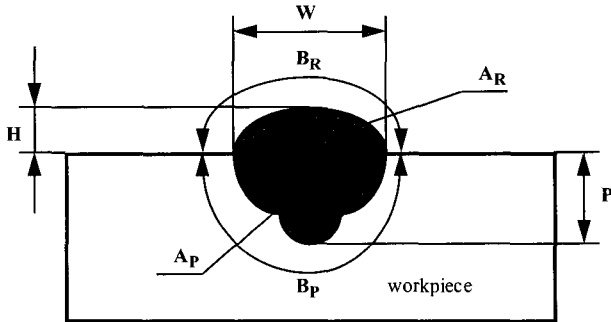
### 3. Experimental Procedure

The welding facility at the Center for Advanced Manufacturing and Industrial Automation (CAMIA) was chosen as the basis for the data collection and evaluation. The facility consists of a Lincoln gas metal arc welding unit that includes a welding power source, welder remote control unit and wire torch, and a Hitachi process robot manipulator that has a robot control unit and robot teach box. Torch positioning and motion control were obtained using the Hitachi six axis robot controller (M6060II).

The selection of the welding electrode wire was based principally upon matching the mechanical properties and physical characteristics of the base metal, weld size, and existing electrode inventory. Steel wires with diameters of 0.9, 1.2 and 1.6 mm with composition of C 0.07-0.15 %, Mn 1.00-1.50 %, Si 0.60-0.85 %, S 0.035 % max, P 0.025 % max and Cu 0.5 % max, were used as the welding consumables. To optimize the GMA welding process, two samples were taken for observation after discarding 50 mm on each side to eliminate the end effects. Welding was carried out on experimental plates of  $200 \times 75 \times 12$  mm mild steel AS 1204 flats with composition of C 0.25 %, Si 0.4 % and P 0.04 % adopting the bead-on-plate technique.

Experimental test plates were located in the fixture jig by the robot controller and the required input weld conditions were fed for the particular weld steps in the robot path. With welder and argon shield gas turned on, the robot was initialized and welding was executed. This continued until the predetermined fractional factorial experimental runs were completed. To measure the weld bead dimensions and shape, the transverse section of each weld was cut using a power hacksaw from the mid-length position of welds and the end faces were machined. Specimen end faces were polished and etched using a 2.5% nital solution to reveal grain boundaries and display the depth of penetration. An image analyzer was used to accurately measure the eleven weld bead dimensions such as weld bead width, weld bead height, weld bead penetration depth, weld penetration shape factor, weld reinforcement shape factor, weld bead total area, weld bead penetration area, weld bead reinforcement area, weld bead dilution, length of weld bead penetration boundary, and length of weld bead reinforcement boundary, as shown in Fig. 1. The results of the experiment were analyzed and relationships

between the five welding process parameters and the eleven weld bead geometric variables were established as discussed below.



- W : Weld bead width
- H : Weld bead height
- P : Weld bead penetration depth
- WSP : Weld penetration shape factor,  $W / P$
- WRFF : Weld reinforcement shape factor,  $W / H$
- $A_T$  : Weld bead total area,  $A_P + A_R$
- $A_P$  : Weld bead penetration area
- $A_R$  : Weld bead reinforcement area
- DI : Weld bead dilution,  $A_P / A_R \times 100$
- $B_P$  : Length of weld bead penetration boundary
- $B_R$  : Length of weld bead reinforcement boundary

Fig. 1 Schematic representation of bead geometry parameters

## 4. Results and Discussion

### 4.1 Mathematical Model

With the five independent parameters, the response parameter could be any of the bead geometry dimensions under consideration. The effects caused by changes in the five main factors and to their first order interactions were obtained by summing the appropriate measured effects for each factor combination. To quantitatively evaluate the effect of welding process parameters on the weld bead dimensions and appearance, conventional curvilinear types of equations which express the relationships between welding process parameters and bead geometry were developed. Generally, the response function <sup>1,11)</sup> can be represented as follows:

$$Y = f(D, G, S, I, V) \tag{1}$$

Assuming a linear relationship for the close range and considering all the main effects together with the two factor interactions, the above equation which uses two types of criteria; goodness of fit between experimental and predicted results and physical knowledge between welding process parameters and bead geometry, can be expressed as follows <sup>10)</sup>:

$$Y = a (D)^b (G)^c (S)^d (I)^e (V)^f \tag{2}$$

where the empirical coefficients  $a, b, c, d, e$  and  $f$  represent constants.

The analytical techniques chosen in this study were first the analysis of variance (ANOVA) performed on the fractional factorial design and quantifying the effect of welding process parameters on each weld bead geometric variable to verify the significance of each factor on the optimization parameter and to detect whether there were any interaction effects among the factors themselves. Second the multiple correlation coefficient and the Fisher's F-ratio (F) were used to gauge goodness of fit and indicate significance at the 10% level on Fisher's F-ratio to include the physical considerations about the logical shape of the equations. As a result, a function based on the analysis of variance was developed describing the experimental results using the method of least squares.

Best fit equations from the investigation of the interrelation between the five welding process parameters and eleven weld bead dimensions were calculated by using the standard statistical techniques such as multiple regression analysis. These analyses were carried out with the help of a standard statistical package program, SAS, using IBM compatible PC <sup>15)</sup>. The following curvilinear equations for bead geometry prediction were obtained:

$$W = D^{0.3647} S^{-0.4873} I^{0.4151} V^{0.9273} 10^{-0.0097} \tag{3}$$

$$H = D^{-0.4092} G^{-0.0844} S^{-0.4327} I^{0.7475} V^{-0.6649} 10^{-0.8607} \tag{4}$$

$$P = D^{0.3945} G^{0.1187} I^{0.9557} V^{0.1529} 10^{-2.2959} \tag{5}$$

$$WRSP = D^{0.0178} S^{-0.2746} I^{-0.5932} V^{0.4989} 10^{2.1229} \tag{6}$$

$$WRFF = D^{0.7738} G^{0.0752} I^{-0.3324} V^{1.5929} 10^{-0.9985} \tag{7}$$

$$A_T = D^{0.4162} S^{-0.6938} I^{1.027} V^{0.7669} 10^{-0.2244} \tag{8}$$

$$A_P = D^{1.6088} I^{0.7856} V^{2.2168} 10^{-4.1053} \tag{9}$$

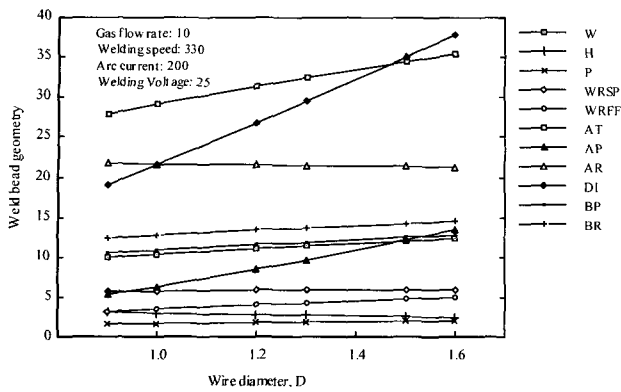


Fig. 2 Effect of wire diameter on bead geometry

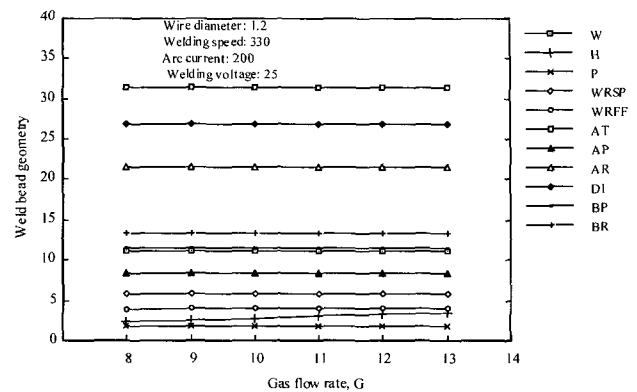


Fig. 3 Effect of gas flow rate on bead geometry

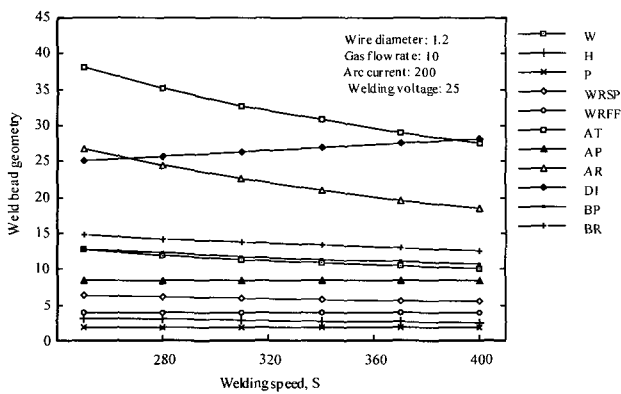


Fig. 4 Effect of welding speed on bead geometry

Table 2 Analysis of variance tests for mathematical models

No. of equation	SEE	R	100 R <sup>2</sup>
3	0.0254	0.9814	96.31
4	0.0525	0.9094	88.27
5	0.1263	0.9026	81.47
6	0.1863	0.6346	40.27
7	0.0564	0.9332	87.09
8	0.1278	0.8750	76.56
9	0.2226	0.8380	70.22
10	0.0231	0.8086	65.39
11	0.2109	0.6341	40.20
12	0.0654	0.8979	80.63
13	0.0597	0.8988	80.79

$$A_R = D^{-0.0381} S^{-0.7962} I^{1.0843} 10^{0.8442} \quad (10)$$

$$DI = D^{1.1929} S^{0.2454} I^{-0.2412} V^{1.4507} 10^{-0.7572} \quad (11)$$

$$B_P = D^{0.344} S^{-0.3637} I^{0.4881} V^{0.783} 10^{-0.2647} \quad (12)$$

$$B_R = D^{0.2731} S^{-0.3371} I^{0.4975} V^{0.5442} 10^{0.0487} \quad (13)$$

The adequacy of the mathematical models and the significance of coefficients were tested by applying the analysis of variance technique and student's test (T) respectively. Table 2 presents the standard error of estimation (SEE), the multiple correlation coefficient (R) and the squared multiple correlation coefficient (100 R<sup>2</sup>) for eleven weld bead geometric parameters respectively. The experimental results of bead geometry showed that the developed mathematical model could be effectively applied for prediction of the optimal welding conditions in GMA welding process.

### 4.2 Testing the Mathematical Models

The standard error of estimation that represents the standard deviation for the differences between the measured values and predicted values of each of experimental results, can be used to estimate the extent to which future results can be expected to differ from predictions made by using the above mathematical equations. The squared multiple correlation coefficients, given in Table 2, notes the percentage of the total variability measured in the weld bead geometric parameter that is respectively explained by the welding process parameters included in the above equations.

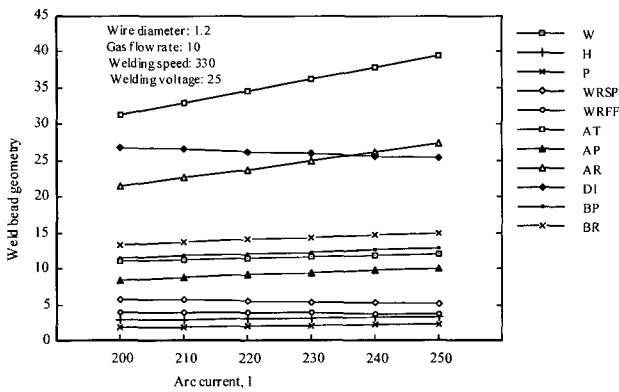


Fig. 5 Effect of arc current on bead geometry

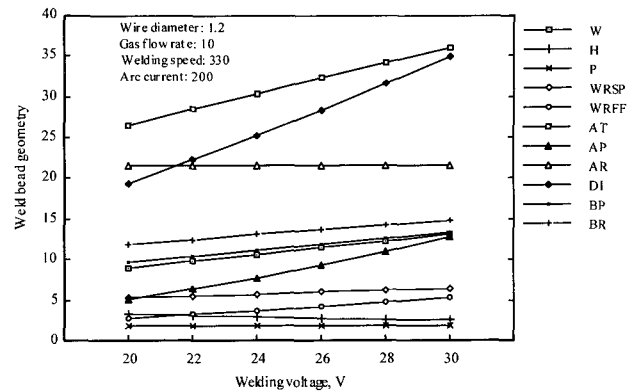


Fig. 6 Effect of welding voltage on bead geometry

These equations are useful for predicting the bead geometry.

It may be emphasized that the 100 R<sup>2</sup> values of the equation (3), given in Table 2, indicates that more than 90 % of the total variability is explained by the multiple regression analysis. Similarly, the 100 R<sup>2</sup> values of the equations (4), (5), (7), (12) and (13) show that the

variability explained by the equations is above 80 %. Equations (6), (8), (9), (10) and (11) explain more than 40 % of the variability.

### 4.3 Interpretation of the Mathematical Models

Based on the mathematical models that determine a

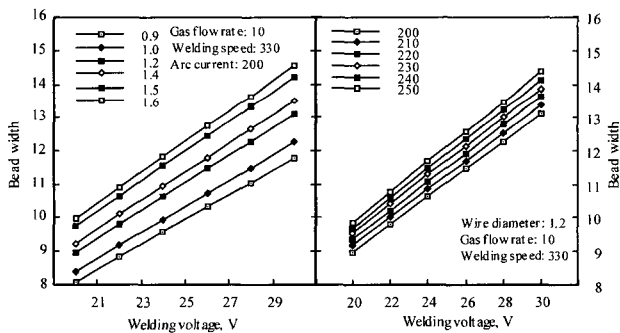


Fig. 7 Effect of weld process parameters on weld bead width

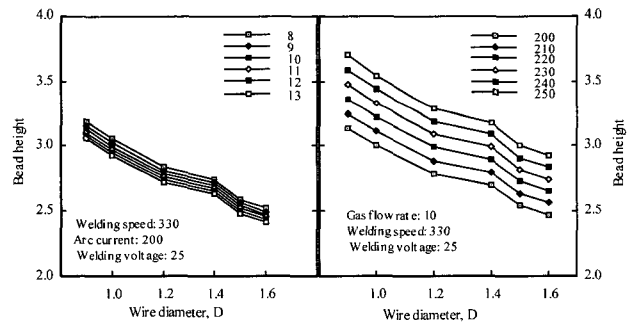


Fig. 8 Effect of weld process parameters on weld bead height

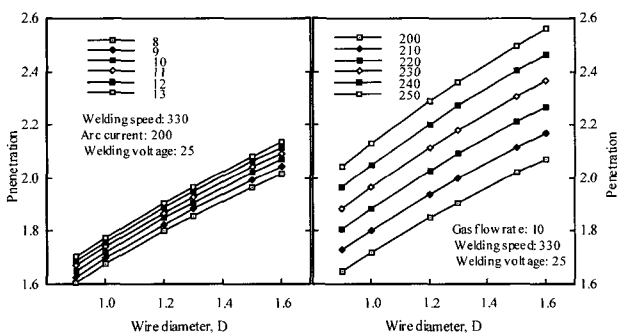


Fig. 9 Effect of weld process parameters on weld bead penetration depth

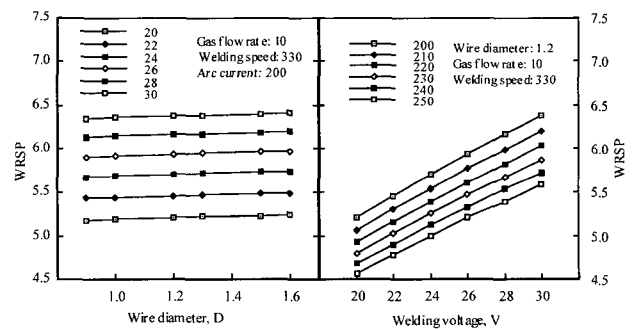


Fig. 10 Effect of weld process parameters on WRSP

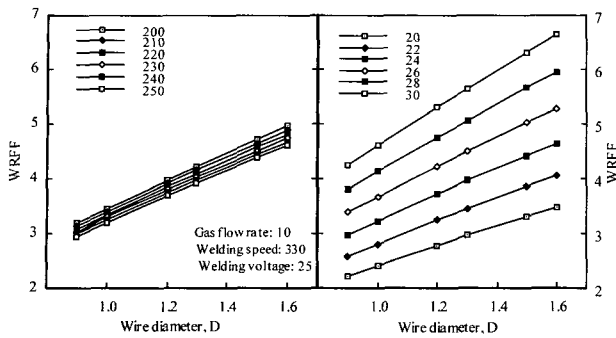


Fig. 11 Effect of weld process parameters on WRFF

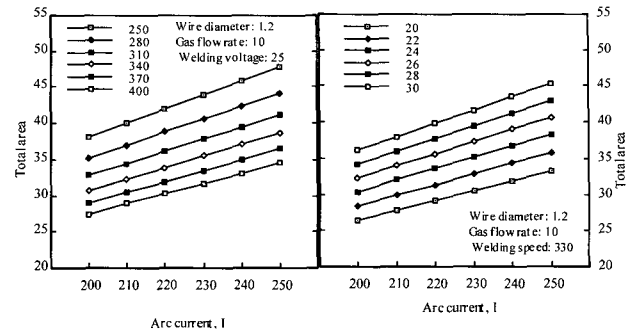


Fig. 12 Effect of weld process parameters on weld bead total area

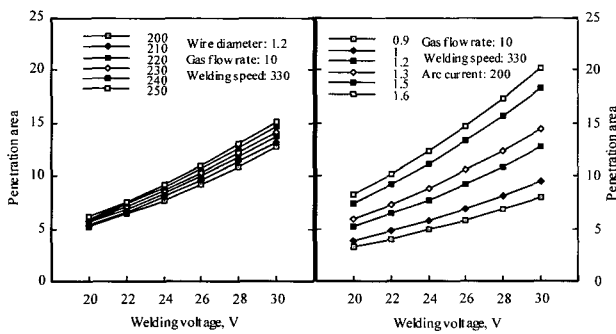


Fig. 13 Effect of weld process parameters on weld bead penetration area

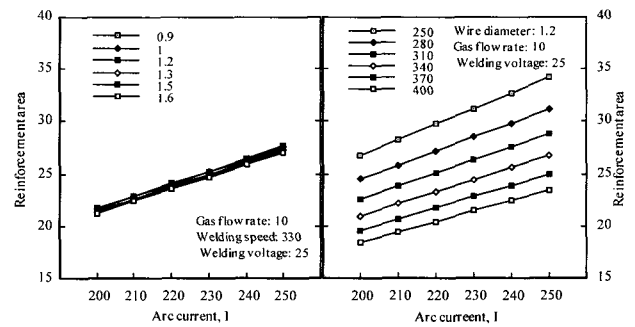


Fig. 14 Effect of weld process parameters on weld bead reinforcement area

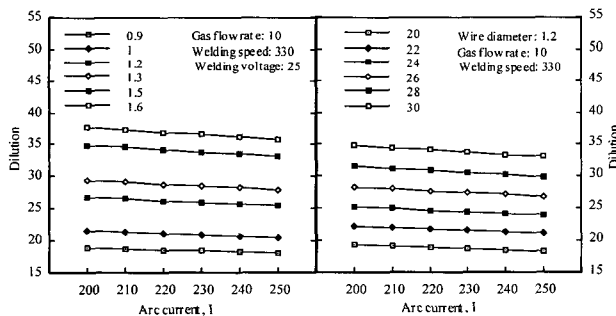


Fig. 15 Effect of weld process parameters on weld bead dilution

given bead geometry and provide useful guidelines for systems which control bead geometry, a limited range of welding conditions, the effect of each welding process parameters and their significant interactions on weld bead geometric parameters were computed and plotted. Fig. 2 to 6 indicate the effects of the five welding process parameters on the eleven weld bead geometric

parameter within the ranges studied. The estimated effects of welding process parameters on weld bead dimensions are presented only for parameters levels. The use of mathematical equations outside these boundaries is possible through extrapolation and should be resorted with care. To stress interaction effects between welding process parameters on bead geometry, Fig. 7 to 17 have been plotted taking into account the two major interactions for each of the mathematical models.

To make effective use of automated and robotic arc welding, it is imperative that mathematical models that can be programmed easily and fed to the robot controller having a high degree of confidence are developed. They should also cover a wide range of material thickness and be applied for all position welding. For the automatic, open loop control, welding system to use these data, the data must be available in the form of mathematical equations. It was in the light of these concluding remarks and suggestions for further developments outlined by previous researchers that the work in this paper was undertaken.

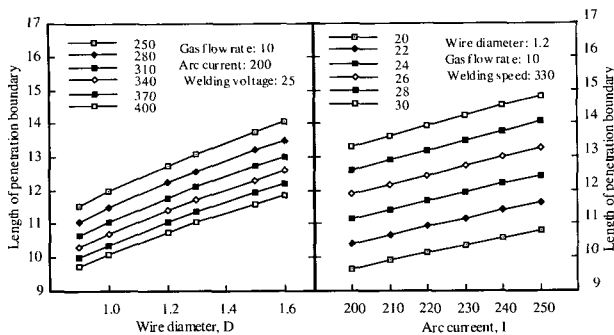


Fig. 16 Effect of weld process parameters on length of weld bead penetration boundary

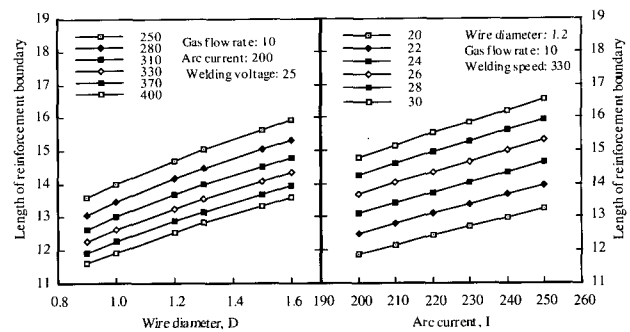


Fig. 17 Effect of weld process parameters on length of weld bead reinforcement boundary

## 5. Conclusion

The effect of welding process parameters on bead geometry when bead-on-plate welds are deposited using the GMA welding process has been studied and the following conclusions reached:

1. Results show that empirical equations can be found relating welding process parameters such as wire diameter, gas flow rate, welding speed, arc current and welding voltage on the development of eleven weld bead geometric variables of weld bead width, weld bead height, weld bead penetration depth, weld penetration shape factor, weld reinforcement shape factor, weld bead total area, weld bead penetration area, weld bead metal area, weld bead dilution, length of weld bead penetration boundary and length of weld bead reinforcement boundary.

2. Fractional factorial design provides a rapid and powerful means for establishing relationships between welding process input parameters and bead geometry.

3. Bead geometry can be expressed in terms of the welding process parameters by using regression equations obtained by statistical analysis.

4. Mathematical models developed from experimental results can be used to control the welding process parameters in order to achieve the desired bead geometry based on weld quality criteria.

5. These equations may prove useful and applicable for automatic control systems and expert system.

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