New Fault Location Algorithms by Direct Analysis of Three-Phase Circuit Using Matrix Inverse Lemma for Unbalanced Distribution Power Systems

Myeon-Song Choi* and Seung-Jae Lee*

Abstract - Unbalanced systems, such as distribution systems, have difficulties in fault locations due to single-phase laterals and loads. This paper proposes new fault locations developed by the direct three-phase circuit analysis algorithms using matrix inverse lemma for the line-to-ground fault case and the line-to-line fault case in unbalanced systems. The fault location for balanced systems has been studied using the current distribution factor, by a conventional symmetrical transformation, but that for unbalanced systems has not been investigated due to their high complexity. The proposed algorithms overcome the limit of the conventional algorithm using the conventional symmetrical transformation, which requires the balanced system and are applicable to any power system but are particularly useful for unbalanced distribution systems. Their effectiveness has been proven through many EMTP simulations.

Keywords: Power system protection, Unbalance distribution system, Fault location

1. Introduction

Service continuity is one of the most important concerns to utility companies, but faults are inevitable and often result in power interruption. Faults are more prominent in distribution systems because most of lines are exposed to air and are spread over a wide area. Fast fault location helps reduce the interruption time, but conventional visual inspection is not only time-consuming, but also requires significant manpower. In recent years, digital protective devices have been replacing old devices like electromechanical or static devices. The digital devices are generally equipped with a fault recording function, which enables an automatic fault location.

During last decade, many studies have attempted to solve the fault location problem, that is, to calculate the distance from a protective device to the fault.

Much of the research has dealt with transmission networks, which are generally operated in a balanced manner. The methods gleaned from previous research can be classified into three groups: those using traveling waves [1], those using harmonics [2], and those using apparent impedance based on fundamental components of voltage and current [3]. The last method can be further divided into two subgroups: one that uses one-end information and one that uses information from both ends of the faulted line. The latter method provides higher accuracy but requires additional devices for communication and data synchronization Therefore, the former is more widely used, with some accuracy improving technique adopted [4].

Much of research mentioned previously focuses on the fault location of the transmission lines because transmission networks are generally under the balanced operation that enables the circuit analysis based on the symmetrical component. Note that the sequence component method generates three independent sequence networks only in the case of the balanced systems and becomes a powerful tool in the circuit analysis. However, it cannot be applied to the distribution system case since most distribution systems are unbalanced due to the mixed use of single-phase and three-phase laterals and loads, and so on.

Although a direct circuit analysis of three-phase networks is another alternative, it has been omitted due to its high complexity and difficult analysis. Indeed, few studies have been reported on the fault location problem for distribution systems.

Fault location methods for unbalanced distribution systems utilize harmonics [5], fundamental components, and line parameters [6]. Additional calculation burdens, such as recalculation of the voltage and current at each node [7,8], are needed to compensate for the characteristics unique to the distribution system.

In this paper, fault location algorithms based on the direct circuit analysis are suggested. The fault location equation has been derived by applying matrix inverse lemma [9] and is relatively simple and easily applied to any system regardless of a phase balance condition. Therefore, it

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can handle not only the transmission systems, but also the distribution systems.

2. Algorithms for Line-to-Ground Fault Location

2.1 Proposed Algorithm

Since a-phase voltage at the fault is same as $I_f R_f$, a-phase voltage at A can be expressed as

$$V_{Sa} = (1-d)(Zl_{aa} I_{Sa} + Zl_{ab} I_{Sb} + Zl_{ac} I_{Sc}) + I_f R_f \quad (1)$$

where V_{Sa} is a-phase voltage, I_{Sa}, I_{Sb}, and I_{Sc} are phase currents at A,

$$Zl = \begin{bmatrix} Zl_{aa} & Zl_{ab} & Zl_{ac} \\ Zl_{ba} & Zl_{bb} & Zl_{bc} \\ Zl_{ca} & Zl_{cb} & Zl_{cc} \end{bmatrix}$$

is the line impedance matrix, I_f is the fault current, and R_f is the fault resistance.

Note that there are two parallel circuits at the fault position. One is associated with the fault and the other with the load. Thus, the fault current I_f can be obtained from I_s using the current distribution law of the parallel circuit.

$$I_f = Y_f [Y_f + (dZl + Zr)^{-1}]^{-1} I_S \quad (2)$$

where

$$I_f = [I_f \quad 0 \quad 0]' \ ,$$
$$I_S = [I_{Sa} \quad I_{Sb} \quad I_{Sc}]',$$

and Y_f is the fault admittance matrix given as

$$Y_f = \begin{bmatrix} 1/R_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The load admittance matrix is $(dZl + Zr)^{-1}$ where

$$Zr = \begin{bmatrix} Zr_{aa} & Zr_{ab} & Zr_{ac} \\ Zr_{ba} & Zr_{bb} & Zr_{bc} \\ Zr_{ca} & Zr_{cb} & Zr_{cc} \end{bmatrix}$$

is the load impedance matrix.

The inverse matrix $[Y_f + (dZl + Zr)^{-1}]^{-1}$ in Eq. (2) can be simplified by the matrix inverse lemma [9]:

$$(BCD + A^{-1})^{-1} = A - AB(C^{-1} + DAB)^{-1} DA \ . \quad (3)$$

Now, let's define A, B, C, and D as follows:

$$A \equiv (dZl + Zr) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (4)$$

$$B \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ C \equiv 1/R_f, \ D \equiv [1 \quad 0 \quad 0] \ . \quad (5)$$

Then, applying the lemma, we obtain

$$[Y_f + (dZl+Zr)^{-1}]^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} (R_f + a_{11})^{-1} [a_{11} \quad a_{12} \quad a_{13}] \quad (6)$$

Then the fault current equation, Eq. (6) can be rewritten as

$$\begin{bmatrix} I_f \\ 0 \\ 0 \end{bmatrix} = \frac{R_f}{R_f + al} \begin{bmatrix} 1/R_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ xx & xx & xx \\ xx & xx & xx \end{bmatrix} \begin{bmatrix} I_{Sa} \\ I_{Sb} \\ I_{Sc} \end{bmatrix} \quad (7)$$

where xx represents an element with which we have no concern. The final expression for the fault current becomes

$$I_f = \frac{1}{R_f + dZl_{aa} Zr_{aa}} \times [dZl_{aa} + Zr_{aa} \quad dZl_{ab} + Zr_{ab} \quad dZl_{ac} + Zr_{ac}] \begin{bmatrix} I_{Sa} \\ I_{Sb} \\ I_{Sc} \end{bmatrix} . \quad (8)$$

Substitution of Eq. (8) into the fault current equation of Eq. (1) yields

$$(V_{Sa} - (1-d)C_1)(R_f + dZl_{aa} + Zr_{aa}) - R_f (dC_1 + C_2) = 0 \quad (9)$$

where

$$C_1 = Zl_{aa}I_{Sa} + Zl_{ab}I_{Sb} + Zl_{ac}I_{Sc},$$
$$C_2 = Zr_{aa}I_{Sa} + Zr_{ab}I_{Sb} + Zr_{ac}I_{Sc} \ .$$

Eq. (9) can be rearranged as a second-order polynomial with respect to the distance variable d using expressions of the coefficients as

$$\begin{aligned} a_r + ja_i &= C_1 Zl_{aa} \\ b_r + jb_i &= V_{Sa}Zl_{aa} - C_1 Zl_{aa} C_1 Zr_{aa} \\ c_r + jc_i &= Zr_{aa}(V_{Sa} - C_1) \\ d_r + jd_i &= Vs_a - C_1 - C_2 . \end{aligned}$$

Then Eq. (9) becomes

$$d^2(a_r + ja_i) + d(b_r + jb_i) + c_r + jc_i + R_f(d_r + jd_i) = 0 \quad (10)$$

and substituting fault resistance using its imaginary part gives the final result as

$$d^2(a_r - \frac{d_r}{d_i}a_i) + d(b_r - \frac{d_r}{d_i}b_i) + c_r - \frac{d_r}{d_i}c_i = 0 . \quad (11)$$

Finally, the fault distance $(1-d)$ can be obtained by solving Eq. (11). Note that this fault location equation based on the direct circuit analysis can be applied to any system—balanced or unbalanced, three-phase or three-/single-phase systems.

2.2 Simulation Results

The suggested direct circuit analysis based algorithm has been applied to the unbalanced system in Fig. 2 for verification. The line impedance and the equivalent impedance at point B are given as

$$Zl_{abc} = \begin{bmatrix} 0.655+1.468j & 0.19+1.27j & 0.095+0.637j \\ 0.19+1.27j & 1.31+2.937j & 0.19+1.27j \\ 0.095+0.637j & 0.19+1.27j & 0.655+1.468j \end{bmatrix}$$

$$Z_{Babc} =$$
$$10^2 \begin{bmatrix} 1.07+0.392j & 0.005+0.067j & 0.014+0.059j \\ 0.005+0.067j & 0.512+0.322j & 0.005+0.067j \\ 0.014+0.059j & 0.005+0.067j & 1.07+0.392j \end{bmatrix} .$$

The single line-to-ground fault is assumed to have occurred between A and B. The results are compared with those of the conventional method using a distribution factor of the negative sequence current, assuming a balanced system. In the case study results in Figs. 3 and 4, nine different fault distances, varying from 0.1[pu] to 0.9[pu] by 0.1[pu] step, and three fault resistances of 0[Ω], 30[Ω], 50[Ω] have been considered. EMTP simulation has been performed in which a 1920 [Hz] sampling frequency is used. The percentage of errors of the fault location in Fig. 1 show the good performance of proposed algorithms.

The error of the fault location is calculated by the following equation.

$$\%Error = \frac{|d_{est} - d_{real}|}{L} \times 100 \quad (12)$$

where d_{est} is the estimated distance, d_{real} is the real distance, and L is the whole line length [pu].

A significant accuracy difference can be observed between the two algorithms. The maximum estimation error is 25% in the case of the conventional algorithm and 0.08% in the case of the proposed algorithm. The conventional algorithm's unacceptably high error rate is due to the system unbalance. Minimal calculation error contributes to the errors in the proposed algorithms, showing its effectiveness for the real application. Many other tests using the proposed algorithm have shown it to be very accurate and to provide reliable fault location results in both balanced and unbalanced systems.

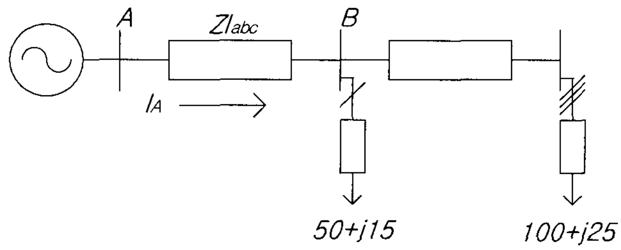

Fig. 2 A simple study system for an unbalanced power system

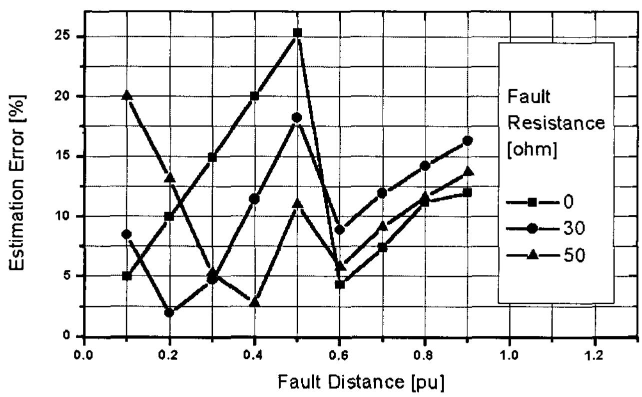

Fig. 3 Errors of conventional method in case of line to ground fault for the unbalanced system

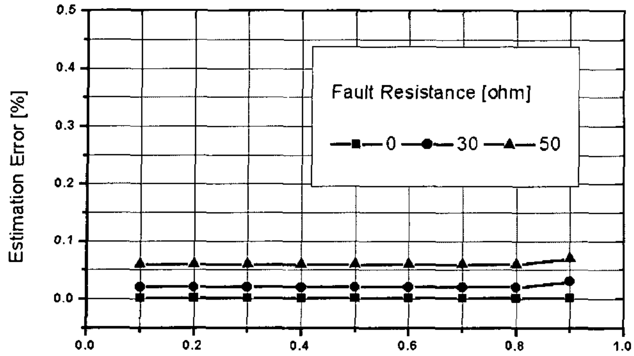

Fig. 4 Errors of the proposed method in the case of line-to-ground fault for the unbalanced system

3. Algorithm for Line-to-Line Fault Location

3.1 Proposed Algorithm

Fig. 5 shows a single line-to-line fault in a general three-phase system.

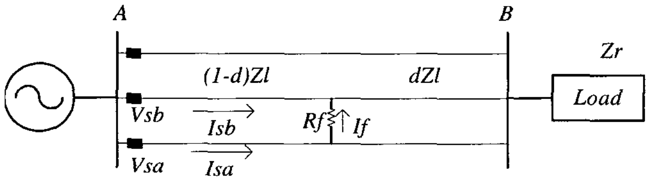

Fig. 5 A single line-to-line fault on a three-phase system

Since a-phase voltage at the fault is same as $I_f R_f$, a-phase voltage at A can be expressed as

$$V_{Sa} - V_{Sb} = (1-d)((Zl_{aa} - Zl_{ba})I_{Sa} + (Zl_{ab} - Zl_{bb})I_{Sb} + (Zl_{ac} - Zl_{cb})I_{Sc}) + I_f R_f \quad (13)$$

where V_{Sa} and V_{Sb} are phase voltages and I_{Sa}, I_{Sb}, and I_{Sc} are phase currents at A.

The current distribution law of the two parallel circuit, one for the fault and the other with the load, at the fault position can be represented the same as in the case of line-to-ground fault in Eq. (2). The fault admittance matrix Y_f is given as

$$Y_f = \begin{bmatrix} 1/R_f & -1/R_f & 0 \\ -1/R_f & 1/R_f & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The inverse matrix $[Y_f + (dZl + Zr)^{-1}]^{-1}$ in Eq. (2) can be simplified by the matrix inverse lemma in Eq. (3).

Now, let's define A the same as in Eq. (4) and B, C, and D as

$$B \equiv \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, C \equiv 1/R_f, \quad D \equiv [1 \ \ -1 \ \ 0] . \quad (14)$$

Then, applying the lemma, we can obtain Eq. (15).

The fault current equation, Eq. (13), can be rewritten as

$$\begin{bmatrix} I_f \\ -I_f \\ 0 \end{bmatrix} = \frac{1}{(R_f + C_1 - C_2)} \begin{bmatrix} C_1 & C_2 & C_3 \\ xx & xx & xx \\ xx & xx & xx \end{bmatrix} \begin{bmatrix} I_{Sa} \\ I_{Sb} \\ I_{Sc} \end{bmatrix} \quad (16)$$

where the coefficients are

$$C_1 = a_{11} - a_{21} = d(Zl_{aa} - Zl_{ba}) + Zr_{aa} - Zr_{ba} = dA_1 + B_1$$
$$C_2 = a_{12} - a_{22} = d(Zl_{ab} - Zl_{bb}) + Zr_{ab} - Zr_{bb} = dA_2 + B_2$$
$$C_3 = a_{13} - a_{23} = d(Zl_{ac} - Zl_{bc}) + Zr_{ac} - Zr_{bc} = dA_3 + B_3$$

The final expression for the fault current becomes

$$I_f = \frac{C_1 I_{Sa} + C_2 I_{Sb} + C_3 I_{Sc}}{(R_f + C_1 - C_2)} . \quad (17)$$

Substitution of Eq. (17) into the fault current equation of

$$[Y_f + (dZl + Zr)^{-1}]^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} - a_{12} \\ a_{21} - a_{22} \\ a_{31} - a_{32} \end{bmatrix} (R_f + a_{11} + a_{22} - a_{12} - a_{21})^{-1} [a_{11} - a_{21} \quad a_{12} - a_{22} \quad a_{13} - a_{23}]$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \frac{1}{(R_f + a_{11} + a_{22} - a_{12} - a_{21})} \begin{bmatrix} (a_{11} - a_{12})(a_{11} - a_{21}), (a_{11} - a_{12})(a_{12} - a_{22}), (a_{11} - a_{12})(a_{13} - a_{23}) \\ (a_{21} - a_{22})(a_{11} - a_{21}), (a_{21} - a_{22})(a_{12} - a_{22}), (a_{21} - a_{22})(a_{13} - a_{23}) \\ xx \quad xx \quad xx \end{bmatrix} \quad (18)$$

$$= \frac{1}{(R_f + a_{11} + a_{22} - a_{12} - a_{21})} \begin{bmatrix} (R_f a_{11} + a_{11} a_{22} - a_{12} a_{21}), (R_f a_{12} + a_{11} a_{22} - a_{21} a_{12}), (R_f a_{13} + a_{22} a_{13} - a_{21} a_{13} + a_{23} a_{11} - a_{23} a_{12}) \\ (R_f a_{21} + a_{11} a_{22} - a_{12} a_{21}), \ (R_f + a_{11} a_{22} - a_{12} a_{21}), \ (R_f a_{23} + a_{11} a_{23} - a_{12} a_{23} - a_{13} a_{21} + a_{13} a_{22}) \\ xx \quad xx \quad xx \end{bmatrix}$$

Eq. (13) and rearrangement can make Eq. (18). Then, a second order polynomial with respect to the distance variable d using expressions of the coefficients as in Eq. (19).

$$\begin{aligned} & d^2(a_r + ja_i) + d(b_r + jb_i) + c_r + jc_i + R_f(d_r + jd_i) = 0 \\ & a_r + ja_i = (A_1 - A_2)D_1 \\ & b_r + jb_i = (A_1 - A_2)(V_{Sa} - V_{Sb} - D_1) + (B_1 - B_2)D_1 \\ & c_r + jc_i = (B_1 - B_2)(V_{Sa} - V_{Sb} - D_1) \\ & d_r + jd_i = (V_{Sa} - V_{Sb} - D_1 - D_2) \\ & D_1 = A_1 I_{Sa} + A_2 I_{Sb} + A_3 I_{Sc} \\ & D_2 = B_1 I_{Sa} + B_2 I_{Sb} + B_3 I_{Sc} \end{aligned} \tag{19}$$

Then, by substituting fault resistance with its imaginary part, the equation gives the following final result in Eq. (20).

$$d^2(a_r - \frac{d_r}{d_i}a_i) + d(b_r - \frac{d_r}{d_i}b_i) + c_r - \frac{d_r}{d_i}c_i = 0 \tag{20}$$

Finally, the fault distance $(1-d)$ can be obtained by solving Eq. (20). Note that this fault location equation based on the direct circuit analysis can be applied to any system-balanced or unbalanced, three-phase or three-/single-phase systems.

3.2 Simulation Results

The suggested direct circuit analysis based algorithm for the line-to-line fault has also been applied to the unbalanced system in Fig. 2 for verification. The line-to-line fault is assumed to have occurred between A and B. The results are compared with those of the conventional method using the distribution factor.

A significant accuracy difference can be observed between the two algorithms. The maximum estimation error is 8% in the case of the conventional algorithm and 0.15% in the case of the proposed algorithm. The error in the proposed algorithm is very small, showing its effectiveness for real applications.

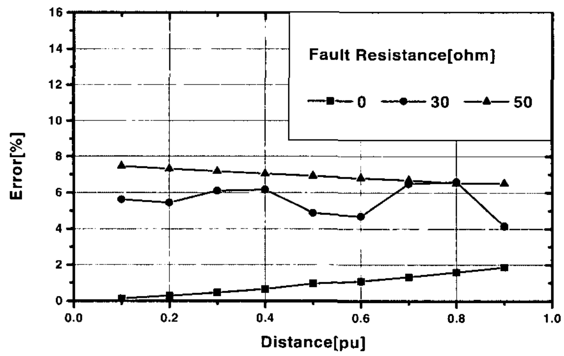

Fig. 6 Errors of conventional method in the case of lin -to-line fault for the unbalanced system

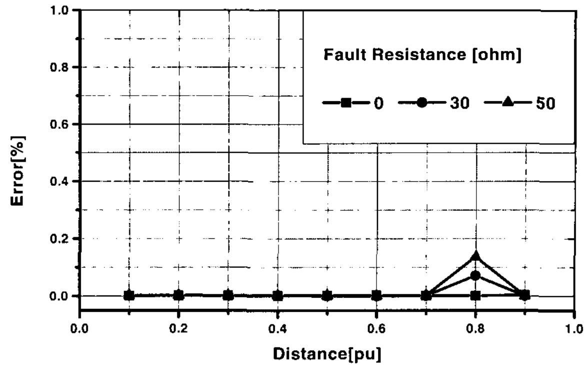

Fig. 7 Errors of the proposed method in the case of line-to-line fault for the unbalanced system

4. Conclusion

New fault location algorithms, based on direct circuit analysis, are suggested in this paper. Application of the matrix inverse lemma has greatly simplified the derivation of the fault location equations that, otherwise, are too complicated to be derived.

The proposed algorithms overcome the limit of the conventional fault location algorithms based on the sequence circuit analysis, which assumes the balanced system requirement. The proposed algorithms are applicable to any power system but are particularly useful for unbalanced distribution systems. Their effectiveness has been proven through many EMTP simulations.

The authors are curretly studying the effects of load variation or uncertainty and multiple loads with radial distribution in practical distribution power systems.

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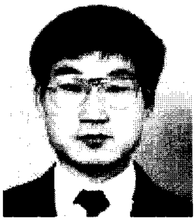

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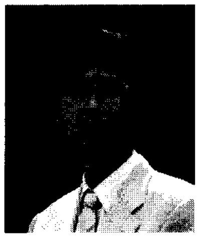

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