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Frequency response of film casting process

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Abstract

The sensitivity of the product to the ongoing sinusoidal disturbances of the process has been investigated in the film casting of viscoelastic polymer fluids using frequency response analysis. As demonstrated for fiber spinning process (Jung *et al.*, 2002; Devereux and Denn, 1994), this frequency response analysis is useful for examining the process sensitivity and the stability of extensional deformation processes including film casting. The results of the present study reveal that the amplification ratios or gains of the process/product variables such as the cross-sectional area at the take-up to disturbances exhibit resonant peaks along the frequency regime as expected for the systems having hyperbolic characteristics with spilt boundary conditions (Friedly, 1972). The effects on the sensitivity results of two important parameters of film casting, i.e., the fluid viscoelasticity and the aspect ratio of the casting equipment have been scrutinized. It turns out that depending on the extension thinning or thickening nature of the fluid, increasing viscoelasticity results in enlargement or reduction of the sensitivity, respectively. As regards the aspect ratio, it has been found that an optimum value exists making the system least sensitive. The present study also confirms that the frequency response method produces results that corroborate well those by other methods like linear stability analysis and transient solutions response. (Iyengar and Co, 1996; Silagy *et al.*, 1996; Lee and Hyun, 2001).

Keywords : draw resonance, extension thickening, extension thinning, film casting, fluid viscoelasticity, frequency response, PTT fluids, sensitivity

1. Introduction

The film casting is a high-speed process widely used in industry for making oriented film for various end uses like tapes and packaging films. (Kanai and Campbell, 1999; Chae *et al.*, 2000). The molten film cast out of the extrusion die is rapidly stretched and cooled before reaching the chill roll on which the film moves with a speed faster than that at the die exit. When this drawdown ratio is increased beyond a critical value, the film casting process can become unstable, i.e., instability called "draw resonance" occurs (Anturkar and Co, 1988; Iyengar and Co, 1996; Silagy *et al.*, 1996; Jung *et al.*, 1999b). This draw resonance instability is an industrially important productivity issue as well as an academically interesting stability topic for other extensional deformation processes as well like fiber spinning and film blowing. It is thus not surprising to see many experimental and theoretical studies conducted on this subject over the last four decades (e.g., for fiber spinning, Pearson and Matovich, 1969; Gelder, 1971; Fisher and Denn, 1976; Beris and Liu, 1988).

Regarding the stability study of any processes, normally linear stability analysis suffices for determining the onset of the instability, while for more detailed sensitivity analysis transient solutions of the governing equations are warranted. These methods are, however, not without shortcomings, i.e., the linear stability analysis lacking full information about the system response to the various frequency spectra of input disturbances, and the transient solutions only possible through solving the full set of partial differential equations with appropriate boundary conditions, which needs high computation loads and time, even if they are ever possible.

As an alternative to these, the frequency response analysis has often been employed to obtain similar information about the system, albeit the linearized system, expending less time and energy due to the fact that only steady state solutions are needed to predict the process dynamics. In the present study, the same frequency response analysis has been conducted on film casting following a similar study on fiber spinning by Jung *et al.* (2002). The nonlinear dynamics of film casting along with its transient solutions was studied by Lee *et al.* (2001).

The linearized perturbation equations of the film casting process in the frequency domain were solved for various input disturbances such as at take-up velocity, extrusion velocity, film thickness and film width at the die. The infor-

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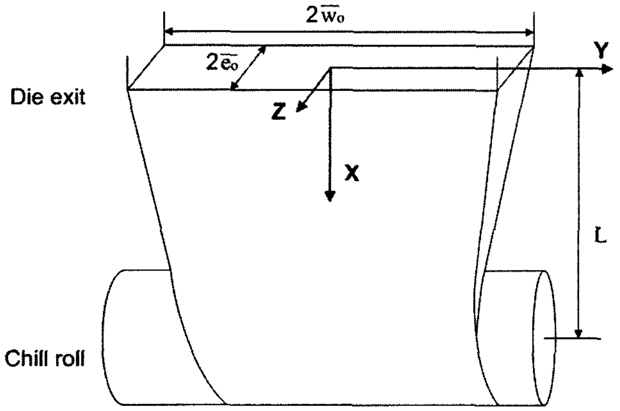

Fig. 1. Schematic of film casting process.

mation garnered then can be displayed in the Bodė diagrams where the amplification ratio and the phase angle of the output response are plotted against the frequency of the input disturbances.

2. Model formulation

As in the previous study about the film casting process in Fig. 1 by Lee *et al.* (2001), an isothermal one-dimensional varying film width model is used here, which Silagy *et al.* (1996) had developed and successfully used to obtain many valuable results. A Phan-Thien-Tanner constitutive equation (Phan-Thien and Tanner, 1977; Phan-Thien, 1978) is employed as a viscoelastic fluid model in the present study. PTT model is known for its robustness and accuracy in describing extensional deformation processes (Khan and Larson, 1987; Kwon and Leonov, 1995) for both extension thickening and thinning fluids.

Continuity equation: $\frac{\partial(ew)}{\partial t}+\frac{\partial(ewv)}{\partial x}=0$ (1)

where, $e=\bar{e}/\bar{e}_0,\ w=\bar{w}/\bar{w}_0,\ v=\bar{v}/\bar{v}_0,\ t=\bar{t}\bar{v}_0/L,\ x=\bar{x}/L$

Equation of motion: $F_x=\sigma_{xx}ew$ (2)

where, $\sigma_{ij}=\bar{\sigma}_{ij}L/\eta_0\bar{v}_0,\ \tau_{ij}=\bar{\tau}_{ij}L/\eta_0\bar{v}_0$

Constitutive equations (PTT fluids):

$$K\tau_{xx}+De\left[\frac{\partial\tau_{xx}}{\partial t}+v\frac{\partial\tau_{xx}}{\partial x}-2(1-\xi)\tau_{xx}\frac{\partial v}{\partial x}\right]=2\frac{\partial v}{\partial x} \quad (3)$$

$$K\sigma_{xx}+De\left[\frac{\partial\sigma_{xx}}{\partial t}+v\frac{\partial\sigma_{xx}}{\partial x}-2(1-\xi)\tau_{xx}\left\{2\frac{\partial v}{\partial x}+\frac{1\partial w}{w\partial t}+\frac{v\partial w}{w\partial x}\right\}\right]$$
$$+2De(1-\xi)\sigma_{xx}\left\{\frac{\partial v}{\partial x}+\frac{1\partial w}{w\partial t}+\frac{v\partial w}{w\partial x}\right\}=2\left[2\frac{\partial v}{\partial x}+\frac{1\partial w}{w\partial t}+\frac{v\partial w}{w\partial x}\right] \quad (4)$$

$$K\sigma_{yy}+De\left[\frac{\partial\sigma_{yy}}{\partial t}+v\frac{\partial\sigma_{yy}}{\partial x}+2(1-\xi)(\sigma_{xx}-\tau_{xx})\left\{\frac{\partial v}{\partial x}+\frac{2\partial w}{w\partial t}+\frac{2v\partial w}{w\partial x}\right\}\right]$$
$$-2De(1-\xi)\sigma_{yy}\left\{\frac{1\partial w}{w\partial t}+\frac{v\partial w}{w\partial x}\right\}=2\left[\frac{\partial v}{\partial x}+\frac{2\partial w}{w\partial t}+\frac{2v\partial w}{w\partial x}\right] \quad (5)$$

where $K=\exp(\varepsilon\, De\, trace(\underline{\underline{\tau}}))$, $De=\lambda_0\bar{v}_0/L$

Film edge conditions: $\sigma_{xx}\left(\frac{\partial w}{\partial x}\right)^2=A_r^2\sigma_{yy},\ \sigma_{zz}=0$ (6)

where, $A_r=L/W_0$

Boundary conditions:
$t=0$: $e=e_s,\ w=w_s,\ v=v_s,\ \tau=\tau_{xx,s},\ \sigma=\sigma_{xx,s},\ \sigma=\sigma_{yy,s}$
$t>0$: $e=e_0=1,\ w=w_0=1,\ v=v_0=1$ at $x=0$
$v=v_L=r$ at $x=1$ (7)

where, e = dimensionless film thickness, w = dimensionless film width, v = dimensionless velocity, x = dimensionless spatial coordinate, L = distance between die exit and chill roll, F_x = tension applied on the molten film, σ_{ij} = dimensionless total stress in ij-direction, τ_{ij} = dimensionless extra stress in ij-direction, t = dimensionless time, λ = material relaxation time, De = Deborah number, ε, ξ = PTT model parameters, A_r = aspect ratio, r = drawdown ratio, and subscripts 0, L, S denote die exit, take-up, and steady state conditions, respectively. Overbars denote dimensional values. Several assumptions have been incorporated in the model. The edge effects (i.e., edge beads) and thermal effects are neglected in this isothermal one-dimensional model. The secondary forces on the film such as gravity, inertia, air drag and surface tension are also neglected because they are usually small in film casting process.

3. Sensitivity analysis

Frequency response exhibits the information about sensitivity of the linearized process to ongoing disturbances (Kase and Araki, 1982; Devereux and Denn, 1994). The partial differential equations (The reduced vector form $\underline{R}(\underline{y},\dot{\underline{y}},p)=\underline{0}$ is simply used here.) of film casting process described in Eq. (1)-(6) were linearized about the steady state solutions using the following perturbation forms of solution vector and input parameter to be disturbed in Eq. (8) (Detailed description is presented by Jung *et al.*, 2002).

$$\underline{y}=\underline{y}_s+\zeta\underline{z}\exp(i\omega t),\quad p=p_s+\zeta\exp(i\omega t) \quad (8)$$

where, $\underline{y}$ is the vector of dependent variables such as film thickness, film width, velocity, and stresses, $\underline{y}_s$ represents $\underline{y}$ at steady states, p is any parameter set to be perturbed, p_s is p at steady states, ω is the frequency of ongoing disturbance, $\underline{z}$ is the complex value representing the amplitude and phase lag of solutions relative to the imposed disturbance, ζ is the complex value of the amplitude of the imposed disturbance, and $i=\sqrt{-1}$.

The linearized perturbation equations are then Laplace transformed, leading to the set of linear ordinary differential equations. For convenience, thus obtained equations are described as follows.

$$(i\omega\underline{\underline{M}}+\underline{\underline{J}})\underline{z}=-\underline{F} \tag{9}$$

where, $\underline{\underline{J}}\equiv\left(\frac{\partial \underline{R}}{\partial \underline{y}}\right)_{\underline{y}_s,p_s}$ is Jacobian matrix at steady states, $\underline{\underline{M}}\equiv\left(\frac{\partial \underline{R}}{\partial \dot{\underline{y}}}\right)_{\underline{y}_s,p_s}$ is mass matrix, and $\underline{F}\equiv\left(\frac{\partial \underline{R}}{\partial p}\right)_{\underline{y}_s,p_s}$ is forcing vector of residuals to the parameter p evaluated at steady state.

These ordinary differential equations in frequency domain are solved using Runge-Kutta 4th-order method. The response to sinusoidal disturbances then is a complex function of frequency. The amplification ratio or gain of the i-component of $\underline{z}$, G_i, and the corresponding phase angle ϕ_i, to a disturbance can be defined from the complex response of Eq. (9). Since the amplification ratio contains the information of sensitivity (Jung, 1999; Jung *et al.*, 2002), the phase lag is not particularly considered in this study.

$$G_i=|z_i|\equiv\{(Re(z_i))^2+(Im(z_i))^2\}^{1/2},\quad \phi_i\equiv\tan^{-1}\frac{Im(z_i)}{Re(z_i)} \tag{10}$$

where, z_i denotes i-component of the solution vector $\underline{z}$.

The frequency response of an example process to four different ongoing disturbances is shown in Fig. 2 to exhibit many peaks for the gain in the frequency domain. It is found that each peak occurs at the same frequency of the disturbances regardless of the origin of disturbances, meaning that the effect of the disturbances on the system is immaterial of their origin or location but only depends on their frequency. Frequencies corresponding to resonant peaks are the same to the imaginary parts of successive leading eigenmodes from the linear stability analysis, which are related to the wave propagation. Moreover, process sensitivity is directly related to the amplification ratio G_i. That is, the more sensitive process is, the higher the peak of amplification ratio to sinusoidal disturbances. This amplification ratio on the frequency response can also be confirmed by the transient response to the sinusoidal disturbance that is available only if the solutions of governing equations can be indeed obtained.

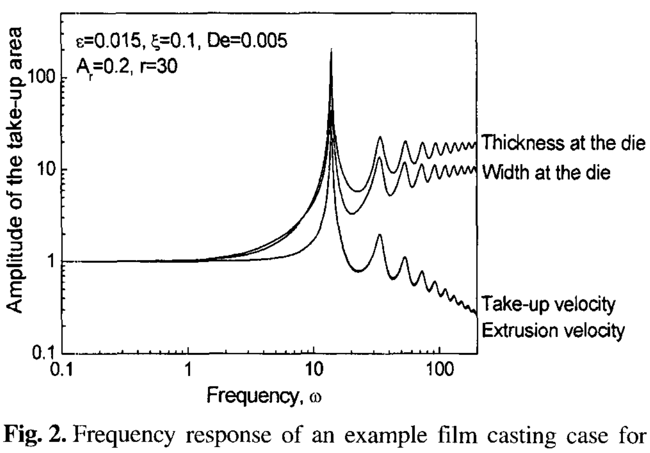

Fig. 2. Frequency response of an example film casting case for four different ongoing disturbances.

4. Effect of process variables

Fig. 3 clearly shows the opposing trends in the effect of fluid viscoelasticity on film casting sensitivity, depending on whether the fluids are extension thickening or thinning. Such two example cases having the corresponding parameters for PTT constitutive models intended to portray LDPE as an extension thickening fluid and HDPE extension thinning one have been studied to produce the fre-

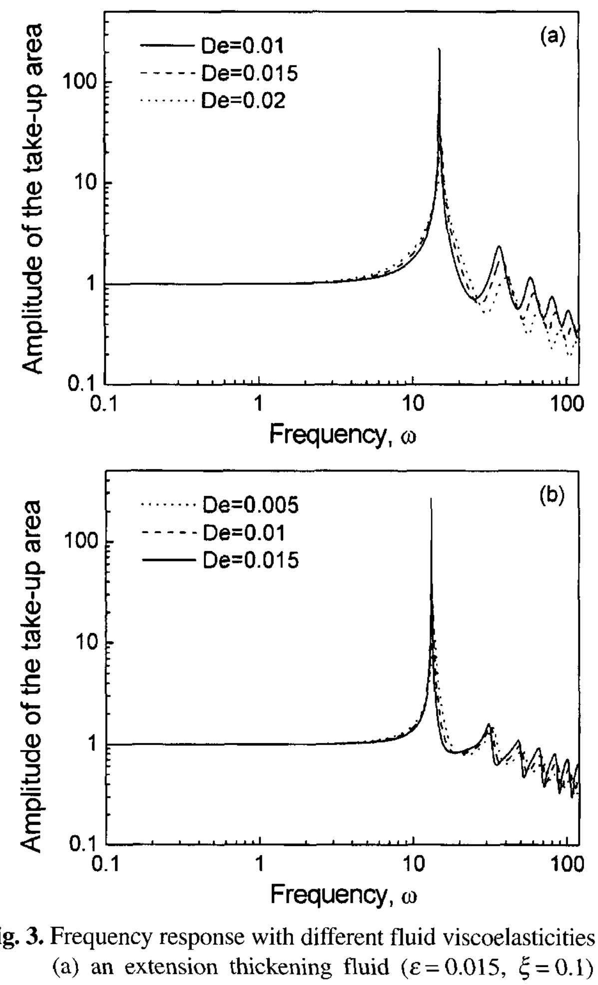

Fig. 3. Frequency response with different fluid viscoelasticities of (a) an extension thickening fluid ($\varepsilon=0.015$, $\xi=0.1$) at $r=25$ and (b) an extension thinning fluid ($\varepsilon=0.015$, $\xi=0.7$) at $r=20$.

quency response results of Fig. 3. It is distinctly clear that for the extension thickening fluids like LDPE the increasing fluid viscoelasticity, i.e., Deborah number, results in reduced sensitivity whereas for extension thinning like HDPE the opposite is the case.

The above findings are consistent with other researchers results, experimental and theoretical. Lee *et al.* (1995) reported the same dichotomous behavior of viscoelastic fluids through both experimental and theoretical studies in fiber spinning while Iyengar and Co (1996) did the same thing for film casting. Lee and Hyun (2001) obtained the transient solutions of film casting and confirmed the same opposing sensitivity behavior of viscoelastic fluids in the film casting process.

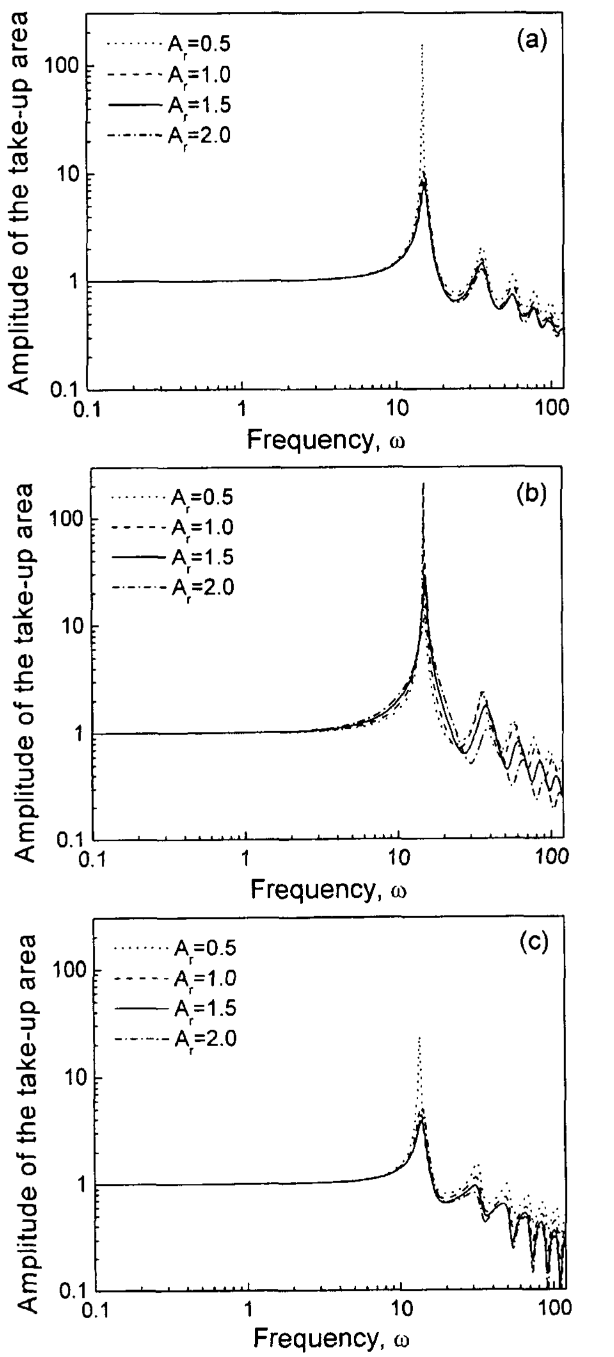

Fig. 4. Frequency response with different aspect ratio of (a) a Newtonian fluid at $r=25$, (b) an extension thickening fluid ($\varepsilon=0.015$, $\xi=0.1$, $De=0.01$) at $r=25$ and (c) an extension thinning fluid ($\varepsilon=0.015$, $\xi=0.7$, $De=0.01$) at $r=20$.

As for the aspect ratio, defined as the casting distance divided by the film width at the die, the sensitivity results show that there exists an optimal value of the aspect ratio A_r, at which the film casting displays the lowest sensitivity. Of interesting here is the fact of the minimum sensitivity being exhibited when the aspect ratio takes on the optimal value, equally holds for all kinds of fluids including Newtonian (a), extension thickening (b) and thinning (c) fluids as illustrated in Fig. 4. The optimal value is 1.5 for Newtonian fluids exactly confirming the results by Silagy *et al.* (1996). This minimum sensitivity also corresponds to maximum stability of the process as further corroborated by Fig. 5 where the same value of 1.5 for A_r is obtained from the stability results of the film casting cases.

The effects of fluid viscoelasticity and aspect ratio on the sensitivity of the process as discussed in Figs. 3 to 5 are further confirmed by the transient results in Fig. 6 obtained from the transient solutions of the governing equations of the system (Jung *et al.*, 1999a). The opposing trends exhibited by extension thickening and thinning fluids as related to the sensitivity and stability are illustrated in (a) and (b) whereas the existence of the optimal aspect ratio value is illustrated by (c) and (d).

Finally the results obtained from the linear stability analysis are shown in Fig. 7 confirming the same conclusion about the viscoelasticity and aspect ratio as explained above. The transfer function approach as reported by Kase and Araki (1984) for fiber spinning can also be introduced to film casting.

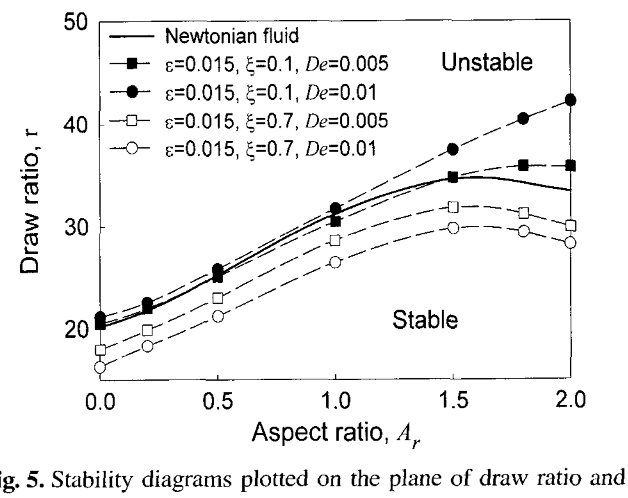

Fig. 5. Stability diagrams plotted on the plane of draw ratio and aspect ratio.

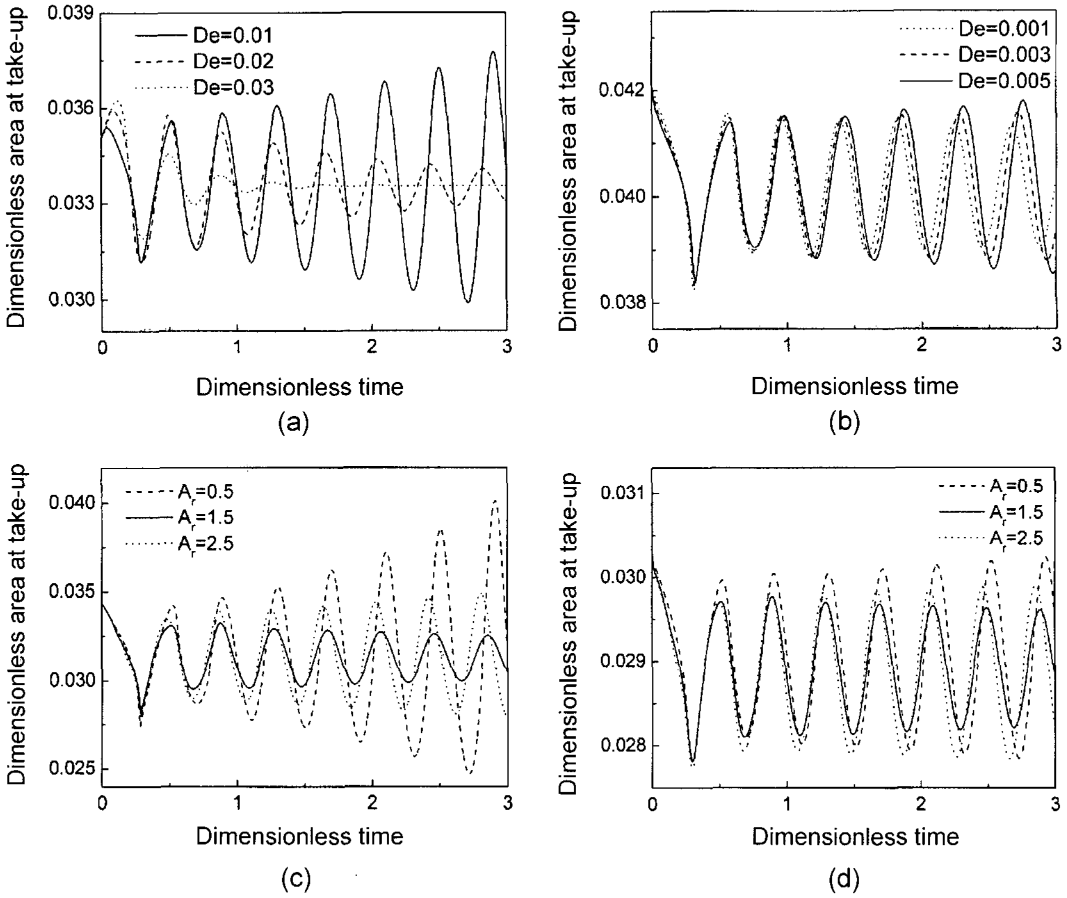

Fig. 6. Transient response of film casting of (a) an extension thickening fluid with different Deborah number ($r = 30$ and $A_r = 0.5$), (b) an extension thinning fluid with different Deborah number ($r = 25$ and $A_r = 0.5$), (c) an extension thickening fluid with different aspect ratio ($r = 30$ and $De = 0.005$) and (d) an extension thinning fluid with different aspect ratio ($r = 25$ and $De = 0.005$).

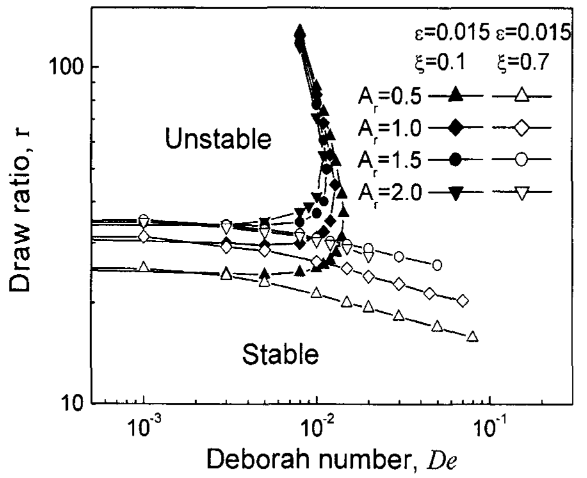

Fig. 7. Stability diagrams plotted on the plane of draw ratio and Deborah number.

5. Conclusions

The frequency response analysis of film casting process has been studied using an isothermal one-dimensional varying width model employing a Phan-Thien-Tanner constitutive equation for both extension thickening and extension thinning fluids to produce the sensitivity of the final product to the various ongoing sinusoidal disturbances. Due to the hyperbolic characteristics of the system, amplitudes of the cross-sectional area at the take-up show multiple resonant peaks along the disturbance frequency regime. The process sensitivity is then predicted by the amplitude ratio of the output to that of the disturbance.

The effects of fluid viscoelasticity and aspect ratio on the process sensitivity have been determined from this frequency response analysis. Depending on whether the fluid is extension thickening or thinning, the viscoelasticity decreases or increases the sensitivity, respectively, whereas the aspect ratio turns out to have an optimal value to minimize the sensitivity for all fluids including Newtonian, extension thickening and thinning fluids. These trends exhibited by the fluid viscoelasticity and aspect ratio are well corroborated by the results reported by other studies using transient solutions and linear stability analysis.

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