Design of an Optimal Planar Array Structure with Uniform Spacing for Side-Lobe Reduction

Ji-Hoon Bae · Nak-Seon Seong · Cheol-Sig Pyo · Jae-Ick Choi · Jong-Suk Chae

Abstract

In this paper, we design an optimal planar array geometry for maximum side-lobe reduction. The concept of thinned array is applied to obtain an optimal two dimensional(2-D) planar array structure. First, a 2-D rectangular array with uniform spacing is used as an initial planar array structure. Next, we modify the initial planar array geometry with the aid of thinned array theory in order to reduce the maximum side-lobe level. This is implemented by a genetic algorithm under some constraint, minimizing the maximum side-lobe level of the 2-D planar array. It is shown that the optimized planar array structure can achieve low side-lobe level without optimizing the excitations of the array antennas.

Key words: Antenna Array Pattern Synthesis, Genetic Algorithm, Side-Lobe Reduction, Optimization.

I. Introduction

Act ve phased array antennas(APAA) for GEO (geostationary orbit) or LEO(low earth orbit) have been studied and developed for more than ten years. In this paper, we present the analysis and result of a basic study on a planar array structure with microstrip patch antennas for side-lobe reduction. A desired low sidelobe evel(SLL) can be accomplished by forming a proper array geometry with uniform excitation of each array antenna. The excitation includes the amplitudes and phases of array elements. An optimal non-uniform spacing can be also available for reducing the SLL of a given planar array [1],[2]. However, it is difficult to construct sub-array modules and to implement a simple array antenna system for the non-uniformly spaced planar array(NUSPA). In this study, thinned array theory is applied to form an optimal planar array structure of uniform spacing, under some constraints. It is illustrated in [3] that thinning an array means turning off some elements in a uniformly spaced or periodic array to create a desired amplitude density across the aperture. Thinned arrays have been investigated for several decades in many array antenna fields since Skolnik et al. [4] applied dynamic programming to the design of thinned array. Recently, derivative-free optimizat on methods, such as simulated annealing(SA) and genet c algorithm(GA), have drawn much attention for this area^{[3],[5],[6]}. While SA use a single agent in search for an optimal solution, GA has a multi-agent system. GA is composed of natural evolution mechanism of SA, random search mechanism, and biological mechanism. Therefore, we make use of the GA in order to implement thinned array structure. The resulting uniformly spaced planar array(USPA) under some constraints can accomplish low SLL without optimizing the excitations of each array element. An element pattern is obtained by circularly polarized microstrip patch antenna.

This paper is organized as follows. In Section II, We describe a field pattern for a circularly polarized rectangular single patch antenna. In Section III, we formulate the problem of interest for a USPA. In Section IV, we describe the pattern synthesis method of USPA in detail. In Section V, we show a simulation example. Finally, we draw our conclusions in Section VI.

Fig. 1 (a) shows a structure of a rectangular microstrip patch antenna and Fig. 1 (b) the top view of the Fig. 1 (a) with circular polarization. Circular polarization can be obtained if two orthogonal modes are excited with a 90° time-phase difference between

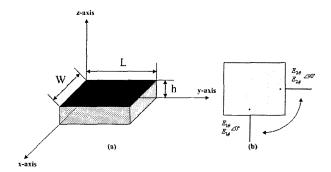


Fig. 1. Rectangular microstrip patch antenna.

them^[7]. As shown in Fig. 1 (b), two radiation field patterns for the two orthogonal modes, TM_{100}^Z and TM_{010}^Z can be calculated respectively as follows^[7]:

$$E_{1}(\theta, \phi) = E_{1\theta}(\theta, \phi) \stackrel{\wedge}{\theta} + E_{1\phi}(\theta, \phi) \stackrel{\wedge}{\phi}$$

$$E_{2}(\theta, \phi) = E_{2\theta}(\theta, \phi) \stackrel{\wedge}{\theta} + E_{2\phi}(\theta, \phi) \stackrel{\wedge}{\phi}$$
(1)

where,

$$\begin{split} E_{1\theta}(\theta,\phi) &= -jCLh\cos\phi\Big(\frac{\sin Y}{Y}\Big)\Big(\frac{\sin Z}{Z}\Big)\cos\Big(\frac{\kappa}{2}\frac{W}{\sin\theta}\cos\phi\Big) \\ E_{1\theta}(\theta,\phi) &= -jCLh\cos\theta\sin\phi\Big(\frac{\sin Y}{Y}\Big)\Big(\frac{\sin Z}{Z}\Big)\cos\Big(\frac{\kappa W}{2}\sin\theta\cos\phi\Big) \\ E_{2\theta}(\theta,\phi) &= -jCWh\sin\phi\Big(\frac{\sin X}{X}\Big)\Big(\frac{\sin Z}{Z}\Big)\cos\Big(\frac{\kappa L}{2}\sin\theta\sin\phi\Big) \\ E_{2\theta}(\theta,\phi) &= -jCWh\cos\theta\cos\phi\Big(\frac{\sin X}{X}\Big)\Big(\frac{\sin Z}{Z}\Big)\cos\Big(\frac{\kappa L}{2}\sin\theta\sin\phi\Big) \\ C &= \frac{\kappa}{R}\frac{E_0\exp(-jkr)}{\pi r}\,, \quad X &= \frac{\kappa}{2}\frac{W}{2}\sin\theta\cos\phi\,, \quad Y &= \frac{\kappa L}{2}\sin\theta\sin\phi, \\ \text{and} \quad Z &= \frac{\kappa}{2}\frac{h}{2}\cos\theta\,. \end{split}$$

The magnitude of the circularly polarized patch antenna is obtained by

$$\sqrt{[E_1(\theta,\phi)+jE_2(\theta,\phi)][E_1*(\theta,\phi)-jE_2*(\theta,\phi)]}$$
.

III. Problem Formulation for Uniform Planar Array

For an odd number of elements, if isotropic array elements are uniformly distributed along the x-axis and are symmetric about the array center, the array radiation pattern over the set of angles $\theta_1, \theta_2, \dots, \theta_L$ can be described as follows:

$$P^{1D}(\theta_i) = \frac{1}{N} \sum_{m=1}^{N} \exp\left[j(m-1) \left(\kappa \, dx (\sin \theta_i - \sin \theta_0) \right) \right]$$
$$= \frac{2}{M} \sum_{m=1}^{M} \cos(\kappa \, m dx (\sin \theta_i - \sin \theta_0)) + \frac{1}{N} \quad (2)$$

where, N is the total number of element antennas, κ is free space propagation constant, dx is inter-element spacing, θ_0 is the maximum radiation angle, and M is given by (N-1)/2. If the linear array is extended to

2-D rectangular array lattice along the row and column directions according to the uniformly located positions of the linear array elements, the USPA pattern can be written as follows:

$$P^{2D}(u, v) = \frac{1}{N^2} \left[2 \sum_{m=1}^{M} \cos(\kappa \, m dy \cdot (v - v_0)) + 1 \right] \times \left[2 \sum_{m=1}^{M} \cos(\kappa \, n dx \cdot (u - u_0)) + 1 \right]$$
(3)

where, $u = \sin \theta \cos \phi$, $v = \sin \theta \sin \phi$, and dy = dx.

In the following section, a pattern synthesis method to find the most suitable planar array geometry is derived from the initial USPA given in eqn. (3).

IV. Uniform Planar Array Pattern Synthesis

In this section, to achieve low SLL from the initial USPA of eqn. (3), a GA is applied to a modification of the initial rectangular array structure.

GA is stochastic search procedures modeled on the Darwinian concepts of natural selection and evolution^[8]. The basic process of the GA is shown in Fig. 2.

The general concepts and applications of the GA in EM problems have been presented in detail^{[3],[9]~[11]}. In our case, to formulate a pattern synthesis of the USPA using the GA, we start with Fig. 3. Fig. 3 shows the initial $N \times N$ rectangular array arrangement.

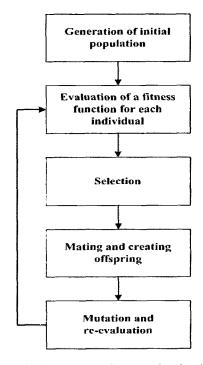


Fig. 2. The basic process of a genetic algorithm.

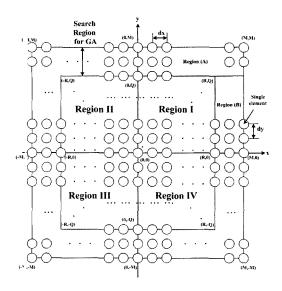


Fig. 3. 2-D rectangular array geometry.

As shown in Fig. 3, each array element is symmetrically positioned along the uniformly spaced rectangular grid with respect to x-axis and y-axis. Due to this symmetry, we consider only the outer elements under the region (A) and (B) belonging to the first quadrant, not whole spaces, to minimize the computational complexity of the 2-D planar array and to obtain the symmetrical array structure. The USPA pattern of eqn. (3) can be written as follows:

$$P^{2D_{zz}} = \frac{1}{N^{2}} \left[2 \sum_{n=1}^{R} \cos(\kappa \, n dx \cdot (u - u_{0})) + 1 \right]$$

$$\cdot \left[2 \sum_{m=1}^{Q} \cos(\kappa \, m dy \cdot (v - v_{0})) + 1 \right]$$

$$+ \frac{1}{\Lambda^{2}} \left[4 \sum_{n=1}^{R} \sum_{m=Q+1}^{M} A_{mn} \cos(\kappa \, n dx \cdot (u - u_{0})) \cdot \cos(\kappa \, m dy \cdot (v - v_{0})) \right]$$

$$+ 2 \sum_{m=Q+1}^{M} \cos(\kappa \, m dy \cdot (v - v_{0}))$$

$$+ 2 \sum_{m=R+1}^{M} \cos(\kappa \, m dy \cdot (v - v_{0}))$$

$$+ 2 \sum_{m=R+1}^{M} \cos(\kappa \, n dx \cdot (u - u_{0})) \cdot \cos(\kappa \, m dy \cdot (v - v_{0}))$$

$$+ 2 \sum_{m=R+1}^{M} \cos(\kappa \, n dx \cdot (u - u_{0})) \right]$$

$$(4)$$

where A_{mn} is amplitude weight of element = $\{1 \text{ or } 0\}$, $u_0 = \sin \theta_0 \cos \phi_0$, $v_0 = \sin \theta_0 \sin \phi_0$, and M, Q, and R are given in Fig. 3. $A_{mn} = 1$ represents the element status as "on", whereas $A_{mn} = 0$ "off". Values for the parameters of the GA can be represented by a binary string or real valued string. In this study, we adopt a binary string because A_{mn} have discrete values, 1 or 0. The cost function, F, to evaluate the fitness value of given individuals is defined as follows:

$$F = \max_{\text{(SLL)}} \{ 20 \log(|P^{2D}|) \}$$
 for $\theta = \text{side-lobe}$ region, and $\phi = 0^{\circ} \sim 180^{\circ}$. (5)

The procedure for the pattern synthesis of the USPA using the GA is summarized as follows:

- Step 1: Randomly generate an initial population for A_{mn} which represent a chromosome consisting of binary string.
- Step 2 : Calculate the maximum SLL(MSLL) using eqn. (5).
- Step 3: Rank chromosomes from best to worst, according to their fitness values obtained by Step 2, and discard bottom 50 %.
- Step 4: Create new offspring settings from the selected top 50 % using the crossover operator.
- Step 5: Mutate the new offspring based on probability of mutation.
- Step 6: Iterate Step $2 \sim$ Step 5 until the fitness value F is less than a predefined threshold value.

V. Simulation Result

Suppose we have a 23×23 rectangular array of microstrip patch sub-arrays with 2.2λ spacing. Each sub-array is 2×2 rectangular patch antennas with interelement spacing of 1λ . For the circularly polarized rectangular patch antenna, its overall dimensions are given in Table 1. To generate an optimal USPA with low SLL, we modify the initial rectangular array structure using the planar array pattern synthesis method in Section IV.

In order to optimize the initial rectangular array with uniform spacing, we set the R(=Q) value to 7 for the 23×23 USPA. Fig. 4 shows the resulting USPA with the MSLL of -22.53 dB for all the azimuth planes ($\phi = 0^{\circ} \sim 180^{\circ}$). From the result of the Fig. 4, we observe that status variations of only the outer elements, namely 1 or 0, far from the array center can achieve the improved array radiation pattern with low SLL.

We focus on the constitution of an array shape from the result of Fig. 4. If a modification is applied to the optimized USPA of Fig. 4, maintaining the form of the array as much as possible, the resulting array shape can be obtained as shown in Fig. 5. From Fig. 5, we

Table 1. Overall dimensions of the rectangular patch.

Dimensions	W_{-}	0.3904 cm
	L	0.3904 cm
	h	0.1588 cm
	Dielectric constant of $\varepsilon_{\rm r}$	2.2
	Center frequency	20 GHz

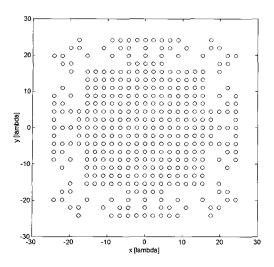


Fig. 4. Optimized USPA.

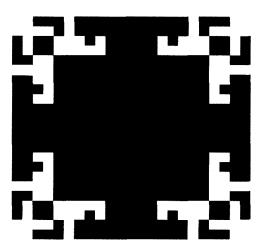


Fig. 5. USPA shape with rectangular lattice.

observe that the resulting array shape is quite different from an expected circular one.

Side view of the 2-D planar array pattern for the array shape given in Fig. 5 is plotted in Fig. 6 as a function of $u = \sin\theta \cos \phi$ and $v = \sin\theta \cos \phi$. The MSLL of -20.57 dB is achieved for the array shape of Fig. 5.

Fig. 7 and Fig. 8 show the total radiation patterns for the optimized USPA shape at $\phi = 0^{\circ}$ and $\phi = 90^{\circ}$, when the mainbeam is scanned to $(\theta_0, \phi_0) = (-5^{\circ}, 0^{\circ})$, $(0^{\circ}, 0^{\circ})$, and $(5^{\circ}, 0^{\circ})$, respectively. Finally, MSLLs, main-lobe levels, and 3 dB beamwidths for the initial 23×23 rectangular array and the optimized USPA are compared and summarized in Table 2 when the main beams are steered to the three different directions. As shown in Table 2, it should be pointed out that the optimized USPA shape can provide low SLL at the

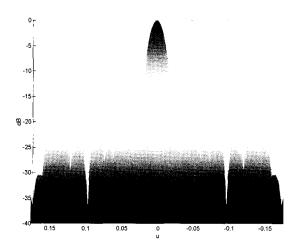


Fig. 6. Side view of the 2-D planar array pattern.

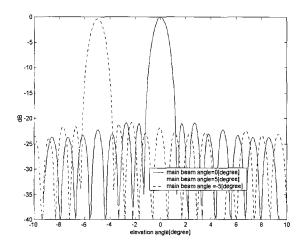


Fig. 7. Total radiation pattern for the optimized USPA shape at $\phi = 0^{\circ}$.

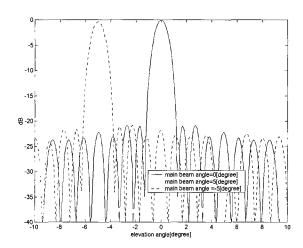


Fig. 8. Total radiation pattern for the optimized USPA shape at $\phi = 90^{\circ}$.

Table 2. Comparison of the two planar arrays.

Class	Items	Maximum radiation angle $(\phi_0=0^\circ)$		
		$\theta_0 = -5^{\circ}$	θ 0 =0°	$\theta_0 = 5^{\circ}$
The initial 23×23 rectangular array	Maximum SLL [dB]	- 13.39	- 13.25	- 13.39
	Main-lobe level [dB]	- 0.37	0	- 0.37
	3 dB beamwidth [°]	0.94	1.0	0.94
The optimized USPA	Maximum SLL [dB]	- 20.83	- 20.86	- 20.83
	Main-lobe level [dB]	- 0.37	0	- 0.37
	3 dB beamwidth [°]	1.02	1.08	1.02

expense of a slight beam broadening, compared to the initial 23×23 rectangular array.

VI. Conclusion

In this paper, an optimization approach is used to synthesize a planar array pattern with uniform spacing. In order to reduce the maximum SLL of the initial planar array, we applied the thinned array theory to obtain an optimal planar array structure, instead of optimizing the excitations of the array elements. To minim ze the degradation of the aperture power efficiency and the computational complexity, a constraint optimization using the genetic algorithm is applied to a construction of an optimal array geometry. In comparison of the initial 23 × 23 USPA, an amount of about 7.5 dB reduction of MSLL is achieved with the optimized USPA, leading to an increase of approximately 8 % over the 3 dB main-lobe beamwidth of the initial USPA. The result shows that the optimized planar array is not similar to a circular array shape and can provide lower SLL than the initial rectangular arrangement, maintaining the uniform amplitudes or phases of the excited array antennas.

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Ji-Hoon Bae



He received the B. S. degree in electronic engineering from Kyungpook National University in 2000, and the M. S. degree in electrical and computer engineering from Pohang University of Science and Technology(POSTECH) in 2002. Since 2002, he has been a member of engineering staff at Electr-

onics and Telecommunications Research Institute (ETRI). His research interests include array antenna system, array signal processing, and radar target imaging and recognition.

Jae-Ick Choi



He received the B. S., M. S. and Ph. D degree in electronic engineering from Korea University in 1981, 1983, and 1995 respectively. Since 1983 he has been working for Electronics and Telecommunications Research Institute (ETRI). He has been researching Antenna and RF/IF systems for wireless com-

munications. His main interests are RF/ antenna system design technologies for SDR and UWB system.

Nak-Seon Seong



He received the B. S. degree in electronic engineering from Pusan National University in 1985, the M.S. degree in electrical & electronic engineering from Korea Advanced Institute of Science and Technology(KAIST) in 1988. Since 1988 he has been with Electronics and Telecommunications

Research Institute(ETRI) as a researcher. His main research interests are in the area of the earth and space-borne Array Antenna systems and RF/IF systems for satellite communications. He is also interested in Economical Efficiency Analysis for Mobile and Satellite Communications systems.

Jong-Suk Chae



He has been taking sabbatical year at Yonsei University since March 2003. Just before this, he was director of Advanced Radio Technology Department of ETRI. In this position he is responsible for developing advanced radio and antenna technology. Prior to this position he has been taking overall responsibility

for the development of IMT-2000 system as executive director of IMT-2000 Development Div., ETRI since 1999. Also, he has been heavily involved in efforts to develop the first generation Koreasat ground stations, such as a DAMA-SCPC satellite ground station and a Koreasat digital DBS system as a project director or manager at ETRI since 1985. Prior to joining ETRI, he was a research engineer of ADD. He has 25 years experience in the telecommunications and radio engineering. He received a national decoration for his achievements in satellite communication technology development(1996). He holds a B.S. degree from National Aviation College(1977), and M.S.(1979) in electronics and Ph.D.(1989) from Yonsei University.

Cheol-Sig Pyo



radio systems.

He received the B. S. degree in electronic engineering from Yonsei University in 1991, the M.S. degree in electrical engineering from Korea Advanced institute of Science and Technology(KAIST) in 1999. Since 1991, he has been a senior engineer at Electronics and Telecommunications Research Institute(ETRI). His research interests include antenna and