

# LPD(Linear Parameter Dependent) System Modeling and Control of Mobile Soccer Robot

Jin-Shik Kang and Chul Woo Rhim\*

**Abstract:** In this paper, a new model for mobile soccer robot, a type of linear system, is presented. A controller, consisting of two loops the one of which is the inner state feedback loop designed for stability and plant be well conditioned and the outer loop is a well-known PI controller designed for tracking the reference input, is suggested. Because the plant, the soccer robot, is parameter dependent, it requires the controller to be insensitive to the parameter variation. To achieve this objective, the pole-sensitivity as a pole-variation with respect to the parameter variation is defined and design algorithms for state-feedback controllers are suggested, consisting of two matrices one of which is for general pole-placement and other for parameter insensitive. This paper shows that the PI controller is equivalent to the state feedback and the cost function for reference tracking is equivalent to the LQ cost. By using these properties, we suggest a tuning procedure for the PI controller. We that the control algorithm in this paper, based on the linear system theory, is well work by simulation, and the LPD system modeling and control are more easy treatment for soccer robot.

**Keywords:** LPD system, modeling, PI, soccer robot, pole placement, pole sensitivity.

## 1. INTRODUCTION

The wheeled mobile robot, especially, the robotic soccer systems and its control schemes have been studied by many researchers with various degrees of application and success [1-6]. Most of these studies are concentrated on the development, control and planning the strategy of mobile robot. But, because of the wheeled mobile robot is modeled and controlled by a nonlinear system framework, its treatment is very complicated and conservative.

In this paper, a new model for mobile soccer robot, a type of linear system, is presented. A controller, consisting of two loops, is suggested. The one of which is the inner state feedback loop designed for stability. And the outer loop is a PI controller designed for tracking the reference input. Because the plant is parameter dependent, it requires the controller to be insensitive to the parameter variation. To achieve this objective, the pole-sensitivity is defined as a pole-variation with respect to the parameter variation and design algorithms for state-feedback

controllers are suggested. The state-feedback gain is consisting of two matrices one of which is designed for general pole-placement and other is designed for parameter insensitive. This paper shows that the PI controller is equivalent to the state feedback and the cost function for reference tracking is equivalent to the LQ cost. By using these properties, we suggest a tuning procedure for the PI controller. The control algorithm and the LPD system modeling of soccer robot, presented in this paper, is more easy then previous works, and which can be applicable to other two wheeled mobile robots.

## 2. LPD SYSTEM MODELING OF SOCCER ROBOT

### 2.1. LPD system

Many of the physical systems can be modeled by a linear parameter dependent system. Before introducing the LPD system, we need to define the set of all admissible parameter trajectories.

**Definition 1[7]:** Given a compact set  $P \subset R^s$ , the parameter set  $F_p$  denotes the set of all piecewise continuous functions mapping  $R^+$  into P with finite number of discontinuities in any interval.

By definition 1, the parameter values  $\rho_i \in F_p$  are differentiable with respect to time.

A state space realization of LPD system is

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + B u(t) \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

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where,  $\rho \in F_p, x(t) \in R^n, u(t) \in R^{n_w}$  and  $y(t) \in R^{n_y}$ . Now, we state the concept of quadratic stability for LPD systems.

**Definition 2:** The system, described by the (1) is quadratically stable over all admissible parameters if a positive definite and symmetric matrix  $P \in R^{n \times n}$  exists, such that for all  $\rho \in F_p$

$$A^T(\rho)P + PA(\rho) < 0. \tag{2}$$

Since the matrix  $A(\rho)$  is a continuous function of parameters  $\rho \in F_p$ , it is clear that the condition (2) implies that the left hand side of the equation is negative definite. In the sense of Lyapunov stability theory, the stability condition is clear and strong because the matrix  $P \in R^{n \times n}$  is real and constant, which is not a function of parameters  $\rho \in F_p$ .

After the notion of quadratic stability, we will now introduce the quadratic stabilizability and controllability.

**Definition 3:** The pair of the matrix function  $[A(\rho(t))B]$  is quadratically stabilizable if there exists a symmetric positive definite matrix  $P_F \in R^{n \times n}$ , not parameter dependent, and a function  $F(\rho) \in C^0(R^s, R^{n_u \times n})$  such that

$$[A(\rho) + BF(\rho)]^T P_F + P_F [A(\rho) + BF(\rho)] < 0 \tag{3}$$

for all admissible parameters.

**Definition 4:** The pair of the matrix function  $[A(\rho(t))B]$  is controllable if

$$\text{rank}[B \ A(\rho) \ B \ A^2(\rho) \ A^3(\rho)B \ \dots \ A^{n-1}(\rho)B] = n. \tag{4}$$

Definition 3 and definition 4 are equivalent notations of the stabilizability and controllability of a well-known linear system. In the Definition 3, the term  $F(\rho) \in C^0(R^s, R^{n_u \times n})$  represents a parameter-dependent state feedback gain which is continuously

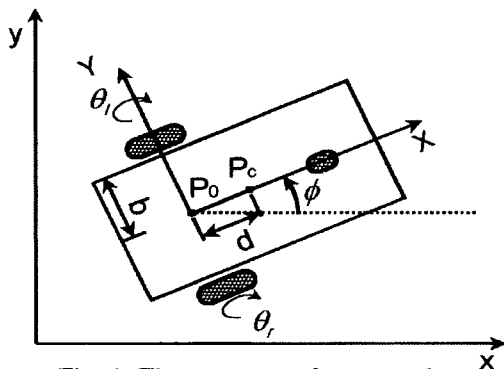


Fig. 1. The structure of soccer robot.

differentiable with respect to parameter value  $\rho$  and time, and has the same dimensions as the general state feedback gain matrix does.

2.2. LPD system modeling of soccer robot

The structure of the mobile soccer robot, considered in this paper, is shown in Fig. 1. The relation between the forward velocity and the wheel angular velocity is described by

$$\begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{5}$$

where,  $v$  and  $\dot{\phi}$  are forward and rotation velocities of the soccer robot, respectively, and  $r$  is the radius of the wheel. And  $b$  is the displacement from center robot to center of wheel. The kinetic equation is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\phi} \end{bmatrix}. \tag{6}$$

In order to derive the dynamic equations, we now define some variables.

- $I_c$  : robot inertia except wheels and rotor
- $I_w$  : motor rotor inertia for wheels and wheel axis
- $I_m$  : motor rotor inertia for wheels and wheel diameter
- $m$  : mass of robot except wheels and motor rotor
- $m_c$  : mass of wheels and motor rotor

The dynamic equation of a soccer of robot is described by

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau - \hat{A}^T(q)\lambda \tag{7}$$

where,  $\lambda$  is Lagrangy multiplier,  $\tau$  is the torque of each wheels, and  $d$  is the displacement from the center of mass to the center of rotation,  $q = [x \ y \ \dot{\theta}_1 \ \dot{\theta}_2]^T$  and

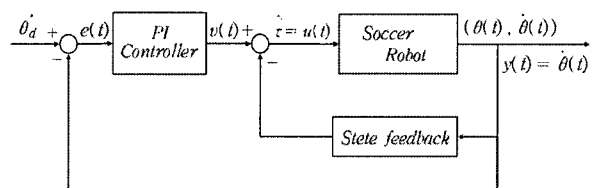


Fig. 2. Controller structure for the soccer robot.

$$M(q) = \begin{bmatrix} m & 0 & -m_c c d \sin \phi & m_c c d \sin \phi \\ 0 & m & m_c c d \cos \phi & -m_c c d \cos \phi \\ -m_c c d \sin \phi & m_c c d \cos \phi & I_c^2 + I_w & -I_c^2 \\ m_c c d \sin \phi & -m_c c d \cos \phi & -I_c^2 & I_c^2 + I_w \end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} 2m_c d \dot{\phi}^2 \cos \phi \\ 2m_c d \dot{\phi}^2 \sin \phi \\ 0 \\ 0 \end{bmatrix}, \hat{A}(q) = \begin{bmatrix} -\sin \phi & \cos \phi & 0 & 0 \\ -\cos \phi & -\sin \phi & cb & cb \end{bmatrix}$$

$$E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

In order to eliminate the Lagrange multiplier, we select the null space of  $\hat{A}(q)$  as

$$S(q) = \begin{bmatrix} cb \cos \phi & cb \cos \phi \\ cb \sin \phi & cb \sin \phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

then, the (7) becomes

$$S^T(q)M(q)(S(q)\ddot{\theta} + \dot{S}(q)\dot{\theta}) + S^T(q)V(q, \dot{q}) = \tau \quad (9)$$

The (9) is a type of nonholonomic equation. This type of system cannot be linearized by using the state feedback.

We now present a LPD system model for the mobile soccer robot. (9) becomes

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 2m_c c b d \dot{\phi}^2 \\ 2m_c c b d \dot{\phi}^2 \end{bmatrix} \quad (10)$$

where,

$$M_{11} = M_{22} = mc^2b^2 + I_c^2 + I_w$$

$$M_{12} = M_{21} = mc^2b^2 - I_c^2$$

$$N_{11} = N_{12} = m_c c^2 b d \dot{\phi}$$

$$N_{21} = N_{22} = -m_c c^2 b d \dot{\phi}$$

In the (10), the variable  $\dot{\phi}$  must be selected as a parameter. Because of the term  $\dot{\phi}^2$ , the dynamic

equation is not linear with respect to the parameter value  $\dot{\phi}$ . To avoid this problem, let us define a new input variable  $\hat{\tau}$  as

$$\begin{bmatrix} \hat{\tau}_1 \\ \hat{\tau}_2 \end{bmatrix} \triangleq \begin{bmatrix} \tau_1 - 2m_c c b d \dot{\phi}^2 \\ \tau_2 - 2m_c c b d \dot{\phi}^2 \end{bmatrix} \quad (11)$$

then, the dynamic equation becomes

$$\begin{bmatrix} \hat{\tau}_1 \\ \hat{\tau}_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (12)$$

and after simple algebraic manipulation, we can obtain the LPD system representation of mobile soccer robot system. Define the state variables, input and the output as

$$x_1 \triangleq \theta_1, x_2 \triangleq \theta_2, x_3 \triangleq \dot{\theta}_1, x_4 \triangleq \dot{\theta}_2$$

$$u = \begin{bmatrix} \hat{\tau}_1 \\ \hat{\tau}_2 \end{bmatrix}, y = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

then, the state space representation of mobile soccer robot is

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1(\dot{\phi}(t))x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (13)$$

where,

$$A_0 = \begin{bmatrix} 0 & 0.01 & 1 & 0 \\ 0 & 0 & 0.01 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & -0.01 & 0 & 0 \\ 0 & 0 & -0.01 & 0 \\ 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_{11}(=a_{12}) = \frac{(2mc^2b^2 + I_w)(m_c c^2 b d \dot{\phi})}{4mc^2b^2I_c^2 + 2I_c I_w}$$

$$a_{21}(=a_{22}) = \frac{-(2mc^2b^2 + I_w)(m_c c^2 b d \dot{\phi})}{4mc^2b^2I_c^2 + 2I_c I_w}$$

$$b_{11}(=b_{22}) = \frac{mc^2b^2 + I_c^2 + I_w}{4mc^2b^2I_c^2 + 2I_c I_w}$$

$$b_{12}(=b_{21}) = \frac{I_c^2 - mc^2b^2}{4mc^2b^2I_c^2 + 2I_c I_w}$$

The (13) show that controllability matrices  $[A_0, B]$  and  $[A_1, B]$  are controllable.

### 3. CONTROL OF MOBILE SOCCER ROBOT

We are state a controller structure and a new control design algorithm presented in this paper for mobile soccer robot.

#### 3.1. Control structure

The controller presented in this paper consists of two loops. One of which is the inner state feedback loop and the outer loop is PI (proportional-integral) control loop. The inner state-feedback loop is designed for minimize the parameter sensitivity of the plant and for make the plant well conditioned. And the outer PI-loop is designed to satisfy the performance requirements, i.e., tracking error, overshoot, etc. The controller schematic is shown in Fig. 2.

#### 3.2. Pole-placement [7]

We are now state the pole placement design via the state feedback. Because the given system dynamic equation is parameter dependent, the constant feedback gain matrix cannot make the system poles lie in the desired location. To hold the system poles in desired location, the state feedback gain matrix must be parameter dependent. We select the select feedback control input as

$$u(t) = -[F_0 + F_1(\dot{\phi}(t))]x(t) + v \tag{14}$$

The matrix  $F_0$  is used for the pole-placement, by which the closed loop poles are located at the desired location. And  $F_1(\dot{\phi}(t))$  is a auxiliary state feedback gain which makes the system is not depended on the parameter variation or attenuate the parameter dependence of the system.

The input matrix  $B$ , the rank of which is  $m$ , is partitioned as

$$B = [U_1 \ U_2] \begin{bmatrix} Z \\ 0 \end{bmatrix} \tag{15}$$

where,  $U_1, U_2$  are unit matrices and  $Z$  is a non-singular matrix with rank  $m$ . Let  $\Lambda_D, V_D$  be desired closed loop poles and right eigenvector matrix, respectively. Then, the following equation is a necessary and sufficient condition for the existence of the state feedback gain matrix, which places the closed loop poles at the desired location.

$$U_2^T (A_0 V_D - V_D \Lambda_D) = 0 \tag{16}$$

If the (16) holds, then the state feedback gain matrix  $F_0$  is

$$F_0 = Z^{-1} U_1^T (A_0 - V_D \Lambda_D V_D^{-1}) \tag{17}$$

The derivation of equations (16) and (17) can be found in many books and papers which treat the linear system control.

Now, we define the pole-sensitivity which can be used in robust pole placement.

**Definition 5:** The pole sensitivity, defined as a ratio of pole displacement with respect to the parameter variation, is described by

$$S_{ij} := \frac{\partial \lambda_i}{\partial \rho_j} = \frac{u_i \frac{\partial A(\rho)}{\partial \rho_j} v_i}{u_i v_i} \tag{18}$$

where,  $u_i, v_i$  is the left and right eigen-vectors of the  $i$ -th system pole, respectively.

By definition 5,  $s_{ij}$  means the  $i$ -th pole displacement with  $j$ -th parameter variation. By using the state feedback input described by the (14), the pole sensitivity for state feedback loop is

$$S_{ij} := \frac{\partial \lambda_i}{\partial \rho_j} = \frac{u_i \frac{\partial [A_1 - B F_1](\rho)}{\partial \rho_j} v_i}{u_i v_i} \tag{19}$$

To make the pole-sensitivity equal to zero or minimized, one of the following equations must hold

$$I_n - (U_1 Z)(Z^{-1} U_1^T) = 0 \tag{20.a}$$

$$A_i - B F_i = V_D^\perp, \tag{20.b}$$

where  $V_D^\perp$  is ortho-normal complement of  $V_D$ , i.e.,

$$V_D V_D^\perp = 0 \text{ or } V_D^\perp V_D = 0.$$

Generally, an (20) does not hold because the second term of (20.a) has rank  $m < n$ , and the matrix  $I_n$  is  $n$  dimensional identity matrix. Rewrite the (19) as

$$S_{ij} = \frac{u_i \left( \begin{bmatrix} A_j^{11} & A_j^{12} \\ A_j^{21} & A_j^{22} \end{bmatrix} - [U_1 U_2] \begin{bmatrix} Z \\ 0 \end{bmatrix} \begin{bmatrix} F_j^1 & F_j^2 \end{bmatrix} \right) v_i}{u_i v_i} \tag{21}$$

By proper selection of the left and right eigenvectors, the pole sensitivity can be minimized. It becomes

$$S_{ij} = u_i^1(A_j^{11} - U_1 Z F_j^1)v_i^1 + u_i^1(A_j^{12} - U_1 Z F_j^2)v_i^2 + u_i^2 A_j^{21} v_i^1 + u_i^2 A_j^{21} v_i^2. \quad (22)$$

Note that the (22) gives us an idea for the selection of the auxiliary state feedback gain matrix which minimizes the pole-sensitivity. The auxiliary state feedback gain matrix is

$$F_1(\rho) = \begin{bmatrix} Z^{-1}U_1^{-1}A_j^{11} & Z^{-1}U_1^{-1}A_j^{12} \end{bmatrix}. \quad (23)$$

By selecting the auxiliary feedback gain matrix described by the (23), the pole sensitivity is minimized and is

$$S_{ij} = u_i^2 A_j^{21} v_i^1 + u_i^2 A_j^{21} v_i^2. \quad (24)$$

The (24) gives us some information. The minimization of pole sensitivity is related to the selection of eigen-vectors and auxiliary state feedback gain matrix. And for some systems, exact cancellation of parameter dependence is possible by proper selection of the auxiliary feedback gain matrix and eigenvectors.

### 3.3. Design of the PI control

For the mobile soccer robot, the reference input signal varies rapidly. The design requirements are: small tracking errors, fast response, and small overshoot, etc.. These requirements are easily satisfied by using the PI controller. The general description of the PI controller is

$$v(t) = K_P e(t) + K_I \int e(t) dt \quad (25)$$

, based on the definition of the state variables, and after simple algebraic manipulation and some modification, the (25) becomes

$$v(t) = \begin{bmatrix} K_I & K_P \end{bmatrix} \begin{bmatrix} \theta_d(t) \\ \dot{\theta}_d(t) \end{bmatrix} - \begin{bmatrix} K_I & K_P \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix}. \quad (26)$$

Note that the (26) is another type of state feedback. The closed loop dynamic equation is described by

$$\dot{x}(t) = \begin{bmatrix} A_0 - BF_0 - B \begin{bmatrix} K_I & K_P \end{bmatrix} \\ + B \begin{bmatrix} K_I & K_P \end{bmatrix} \end{bmatrix} x(t) + d(t) \quad (27)$$

where,

$$d(t) \triangleq (A_1 - BF_1)(\dot{\theta}(t))x(t) + \text{other noise terms}. \quad (28)$$

The PI controller must be designed to guarantee robust performance. For the well tracking result, let us consider the following cost function

$$\min J = \int [e^T(t)Qe(t) + v^T(t)Rv(t)] dt \quad (29)$$

by using the definition of state, error and input and after simple algebraic manipulation, (29) becomes

$$\min J = \int [x^T Q_1 x + x_d^T Q_1 x_d - 2x^T Q_1 x_d] dt \quad (30)$$

where,

$$Q_1 = C^T Q C + [K_I \ K_P]^T R [K_I \ K_P]. \quad (31)$$

The cost function described by the (30) is equivalent to the general LQ cost, and the minimum cost is obtained by using the relationship

$$\min \left\| \begin{bmatrix} C^T & K C \end{bmatrix} \right\|_2 \rightarrow \min J \quad (32)$$

where,  $K$  is the solution of following Riccati equation.

$$KA + A^T K - (KB + Q_1)Q_1^{-1}(B^T K + Q_1^T) + Q_1 = 0 \quad (33)$$

The (32) shows that the cost depends not only on the PI gain but also on the weighting matrices  $Q$  and  $R$ . One possible algorithm for tuning the PI gain is that: first selects the weighting matrices  $Q$  and  $R$ , select PI gain, then compute ARE and cost. If the computed cost is not a desired value, then select a new PI gain and go to the next steps.

## 4. SIMULATION

In simulation, the robot considered is MIROSOT robot, and detailed specifications are summarized in the table 1.

The mass of the robot is 0.0612 Kg m/sec<sup>2</sup> and the mass of wheels is 0.0051 kg m/sec<sup>2</sup>. And other parameters used in this paper were

$$b = 35mm, c = r/2b, d = 10mm.$$

The robot inertia except wheels and rotor is 0.05 Kg cm sec<sup>2</sup> and motor rotor inertia for wheels and wheel axis is 0.0176 Kg cm sec<sup>2</sup>. These parameters were actually measured and computed for MIROSOT robot designed Yujin Robotics corp.. In this paper,

Table 1. The specifications of MIROSOT robot.

Size	70x70x70 mm
Wheel diameter	45 mm
Rpm	8000
Gear ratio	8:1

the maximum velocity of the wheel was computed by the maximum velocity of the motor specification.

By using parameters described above, state space matrices for the mobile soccer robot are

$$A_0 = \begin{bmatrix} 0 & 0.01 & 1 & 0 \\ 0 & 0 & 0.01 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & -0.01 & 0 & 0 \\ 0 & 0 & -0.01 & 0 \\ 0 & 0 & 0.0828\dot{\phi} & 0.0828\dot{\phi} \\ 0 & 0 & -0.0828\dot{\phi} & -0.0828\dot{\phi} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 25.3925 & -19.5119 \\ -19.5119 & 25.3925 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For state feedback design, we selected the desired closed-loop poles as

$$\Delta = [-10+j15 \ -10-j15 \ -17+j17 \ -17-j17]^T.$$

Then, the state feedback gain for pole-placement is obtained by

$$F_0 = \begin{bmatrix} 55.3639 & 24.0262 & 3.2691 & 1.4802 \\ 42.4267 & 31.2610 & 2.5118 & 1.9250 \end{bmatrix},$$

and the additional state feedback gain for guarantee the robust property with respect to the parameter variation is

$$F_1(\dot{\phi}) = \begin{bmatrix} 0.0231 & -0.0250 & 0.0087+0.0018\dot{\phi} & 0.0007+0.0018\dot{\phi} \\ 0.0177 & -0.0192 & 0.0066-0.0018\dot{\phi} & 0.0010-0.0018\dot{\phi} \end{bmatrix}.$$

The state feedback gains obtained here can makes the exact cancellation of parameter and which locate the robot poles in the desired location. The actual robot poles are

$$pol = \begin{bmatrix} -17.0450 + 16.9619i \\ -17.0450 - 16.9619i \\ -10.0050 + 14.9967i \\ -10.0050 - 14.9967i \end{bmatrix}.$$

The PI controller gain is selected by

$$K_I = 210 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad K_P = 100 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where, the integrator and proportional gains are selected by considering the tracking error be small.

Simulation results by using designed controller for

various possible input signals are shown in Fig. 3 to Fig. 14. The sinusoidal and pulse signal was selected as a test signal because these signals are frequently used. Fig. 6, Fig. 10 and Fig. 14 are the trajectory-following results for these test signals. Each trajectories followed the A-B-A form, i.e., start from A(100,100), move to B and return to A.

Fig. 3 to Fig. 6 are simulation results for the sinusoidal reference inputs. In the Fig. 3, the desired and actual velocities are shown. Tracking errors and control inputs are shown in Fig. 4 and Fig. 5, respectively. And the desired robot trajectory and simulated trajectory are shown in the Fig. 6.

Fig. 7 to Fig. 10 are simulation results for the pulse command inputs. It is shown in the Fig. 8 that tracking errors jump at  $t=0$  and  $t=5$  because the signs of the command signals are jump. Also, it is shown in the Fig. 8 that these abrupt changes in the reference signal also could be overcome by the controller presented.

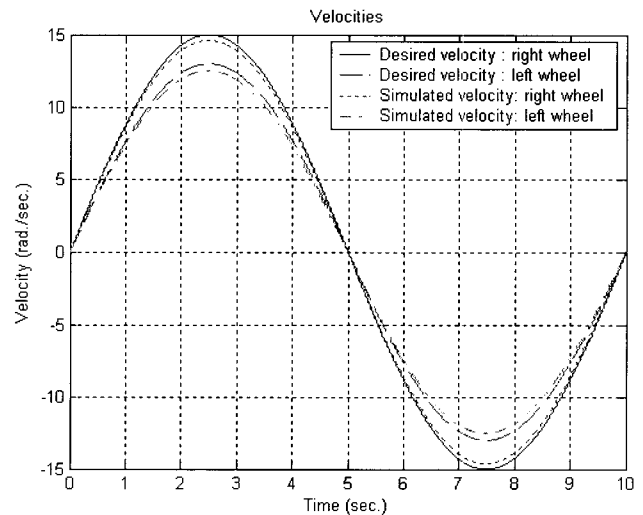


Fig.3. Velocities for sinusoidal reference inputs.

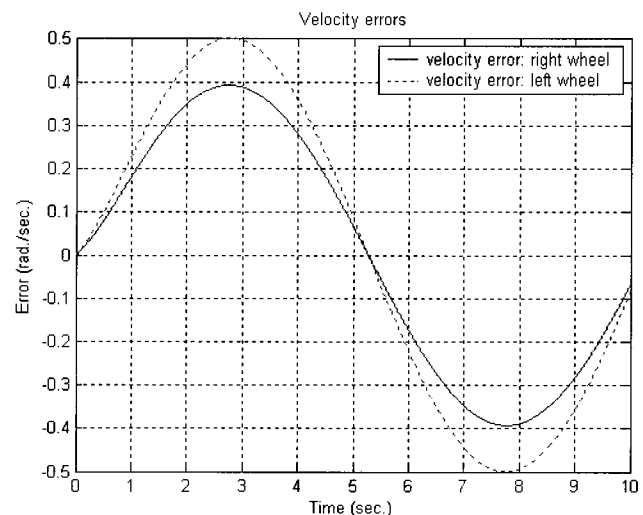


Fig.4. Velocity errors for sinusoidal inputs

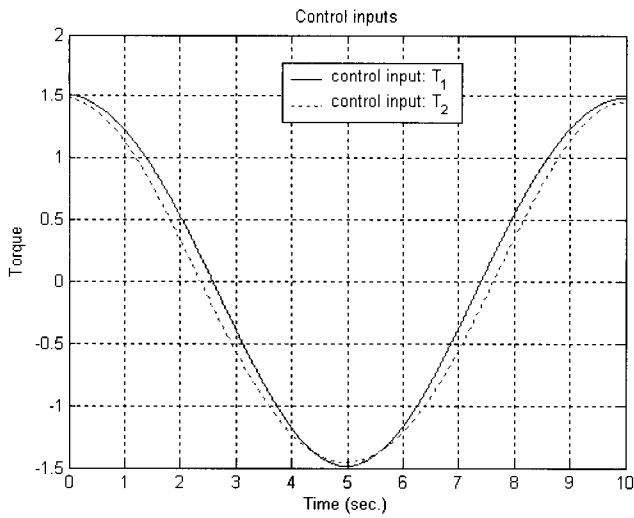


Fig.5. Control inputs for sinusoidal inputs

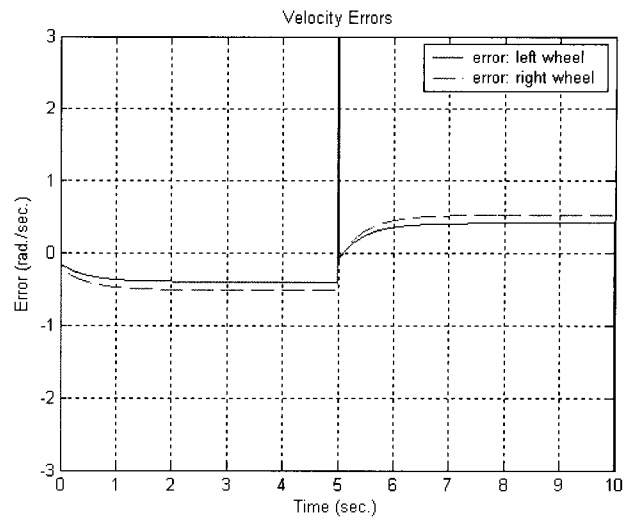


Fig.8. Velocity errors for pulse inputs

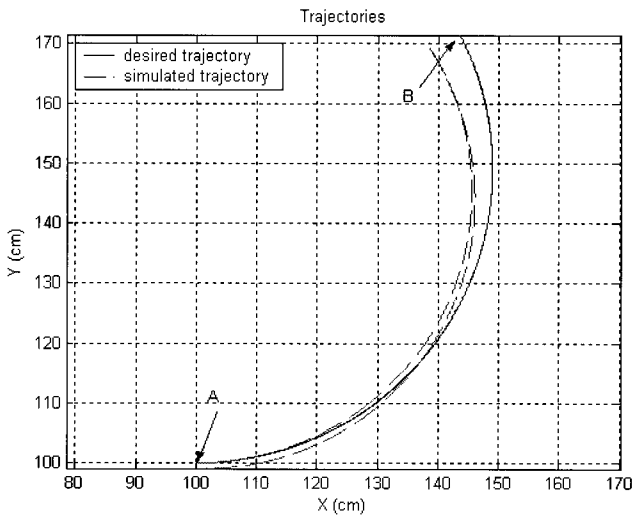


Fig.6. Trajectories for sinusoidal reference inputs

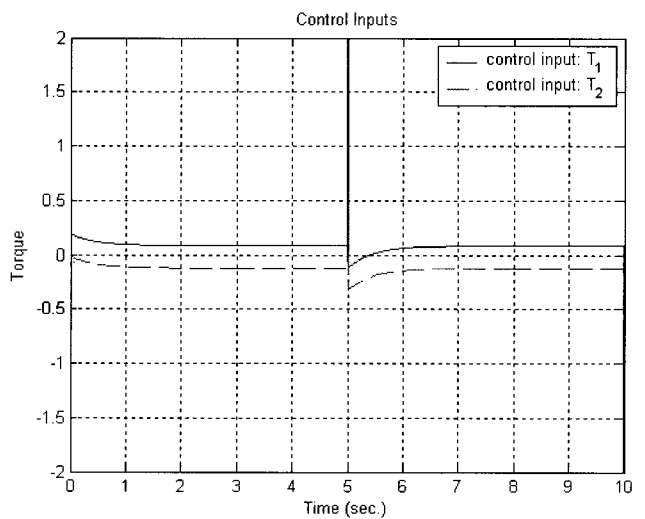


Fig.9. Control inputs for pulse reference inputs

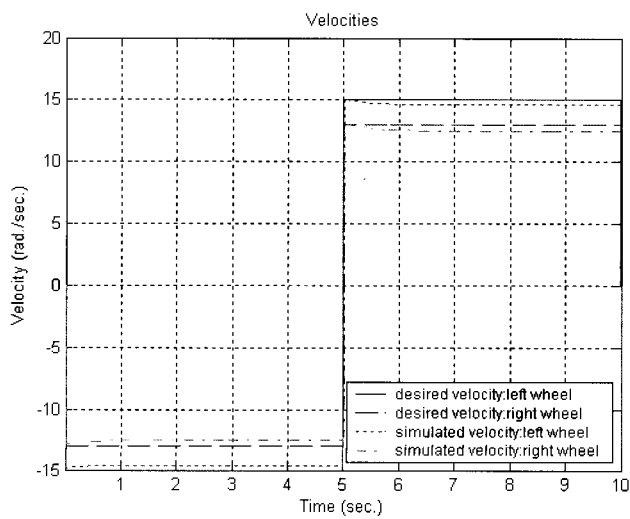


Fig.7. Velocities for pulse reference inputs

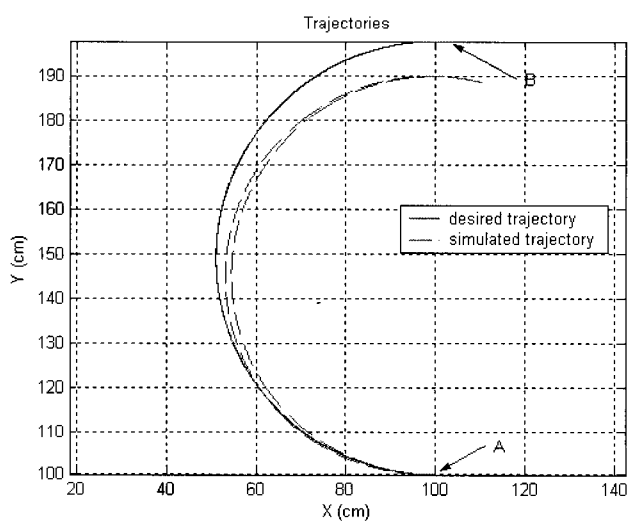


Fig.10. Trajectories for pulse reference inputs

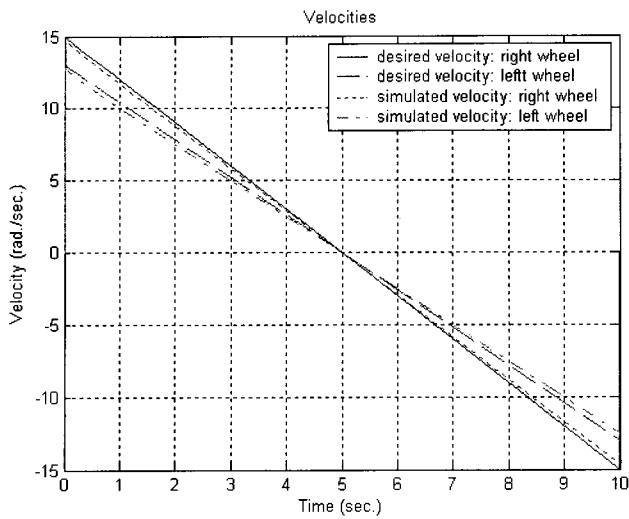


Fig. 11. Velocities for saw-tooth reference inputs

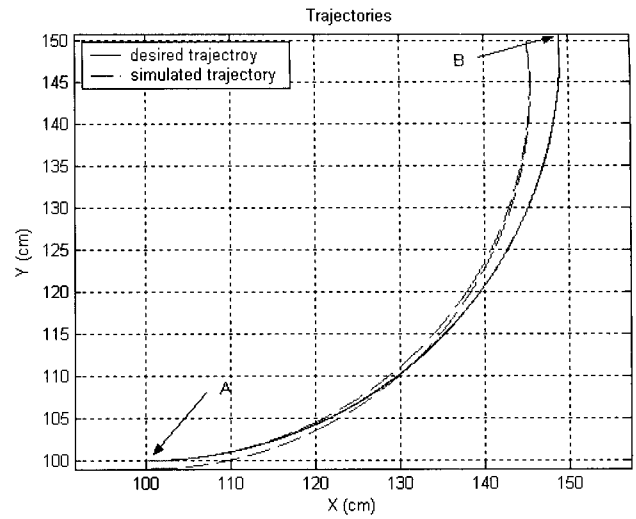


Fig.14. Trajectories for saw-tooth reference inputs

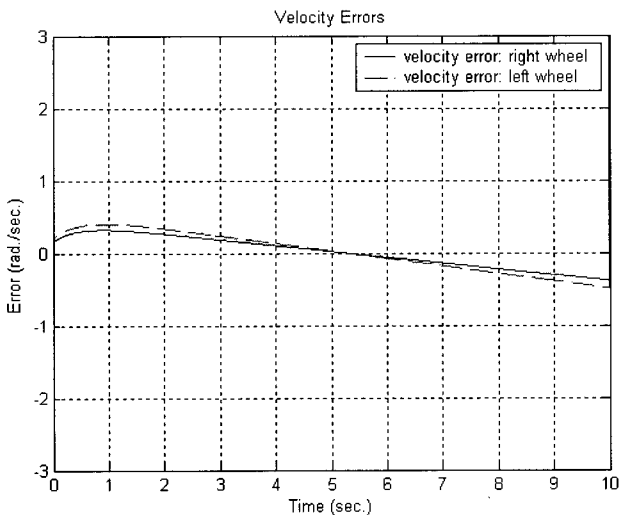


Fig.12. Velocity errors for saw-tooth inputs

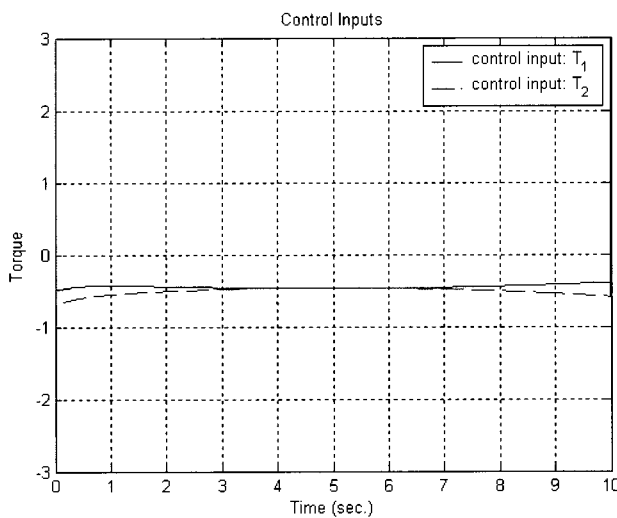


Fig.13. Control inputs for saw-tooth reference inputs

Fig. 11 to Fig. 14 are simulation results for the sinusoidal reference inputs.

Tracking errors in Fig. 6 and Fig. 10 are relatively large because test signals used in this paper were not computed value from desired trajectory but selected ones from a general function of time as reference velocities. We are definitely sure that if reference velocities are computed from desired trajectories then trajectory- following results are more exact.

### 5. CONCLUSION

In this paper, we studied the modeling and control of a soccer robot via LPD system. The control structure presented in this paper consists of two loops. The first one is the state feedback loop and the other is PI control loop. The state-feedback is designed for the transfer function of robot is well conditioned and parameter insensitive. For this purpose, the pole-sensitivity is defined and state feedback is designed.

By using auxiliary state feedback which is a parameter dependent, the presented controller makes the plant well conditioned and minimizes the pole-sensitivity. Finally, it is shown that the PI control loop is equivalent to a type of state-feedback, and the cost function, which minimizes the tracking error, is equal to the LQ (Linear Quadratic Optimization) cost. These properties give us to the tuning ideas for the PI controller gain which described in section 3.3.

It is shown in this paper that the soccer robot can be treated more easily via the LPD framework, which is a type of linear system. And the results of this paper are applicable to other wheeled mobile robots.

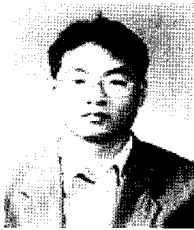
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