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# An experimental study on the cooling performance and the phase shift between piston and displacer in the Stirling cryocooler

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*Abstract*--**In the design of the split type free displacer Stirling cryocooler, the motion of the displacer is very important to decide the cooling capacity, which depends upon the working gas pressure, the swept volume in the compression space and the expansion space, operating frequency, the phase shift between piston and displacer, etc. In this study, Stirling cryocooler actuated by the electric force of the dual linear motor is designed and manufactured. Cool down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler is evaluated. The charging pressure was 15kg$_f$/cm$^2$ and operating frequency was 50Hz. Input power and the lowest temperature were about 32W and 67K, respectively. And, displacement of the piston is measured by LVDTs (Linear Variable Differential Transformers), displacement of thedisplacer is measured by laser optic method, and phase shift between piston and displacer is discussed. As the peak-to-peak pressure of the compressor was increased, peak-to-peak displacement of the displacer was increased. The peak-to-peak displacement of the displacer increases in the range of 0 – 64.5Hz(resonant frequency of the displacer), but decreases steeply when the operating frequency is bigger than the resonant frequency. Finally, when the phase shift between displacements of the *piston and displacer* is 45°, operating frequency is optimum and is decided by resonant frequency of the expander, mass and cross section area of the displacer and constant by friction and flow resistance.**

**Key words: Charging pressure, Operating frequency, Cooling capacity**

## 2. INTRODUCTION

Over the past decade and a half there has been rapid development of Stirling cryocoolers, mainly for military and space application. The cryocooler working on the Stirling cycle are characterized by high efficiency, fast cool down, small size, light weight, low power consumption and high reliability. The Stirling cryocoolers have been widely used for the cooling of the infrared sensors and high temperature superconducting filters to the liquid nitrogen temperature. Recently, the split-type Stirling cryocoolers driven by linear compressor are used to fit requirements of a long life1,2,3.

Free-piston Stirling engines acting as power systems were invented by William Beale in the early 1960s and have been in continuous development since that time at Sunpower. The first development of linear free-piston Stirling cryocooler was accomplished at the Philips Laboratories, Eindhoven. Haarhius(1978)4. Later De Jonge(1979) presented a classic paper related to the theoretical analysis of free-piston Stirling machines5.

The FPFD(Free Piston Free Displacer) Stirling cryocooler consists of two compressor pistons driven by linear motors, which make pressure waves, and a pneumatically driven displacer piston with regenerator. In general, efficiency of the Stirling cryocooler is mainly affected by the efficiency of the linear motor, resonant frequency of the compressor and the expander, displacement of the piston and the displacer, and phase shift between piston and displacer.

The displacer in the expansion cylinder is free and oscillates simply as a result of the gas force acting on it. The gas force is provided by the actuating piston oscillating in the compression cylinder with a phase shift relative to the displacer motion. The stroke variation of the displacer indicates the volume change in the expansion space, and its pressure-volume diagram illustrates the work done by the gas on the displacer. This positive work shows some refrigeration effect, and cold finger will become cold.

Therefore, in the design of the split type free displacer Stirling cryocooler, the motion of the displacer is very important to decide the cooling capacity[6], the measurement of the displacer's displacement is indispensable for the development of the split-type Stirling cryocooler.

Yang et al.[7] proposed a relative displacement measurement method, which is non-contact by the use of the linear variable-differential transformer(LVDT), to calibrate the static sensitivity of the measurement of the displacer motion inside the cold finger of a split-type Stirling cryocooler. Xiang et al.[8] took the displacer motion signal to identify the movement damping of the displacer, which changes along with the lowering of the refrigeration temperature, in a split-cycle free piston Stirling cryocooler. Zhang et al.[9] conducted an experimental investigation on the dynamic pressure distribution in a split-piston Stirling cryocooler system, where the displacer motion was measured with a Hall-effect displacement transducer.

In this paper, for the given FPFD Stirling cryocooler, laser displacement sensor is used to measure displacement of the displacer and experimental results on the performance and optimum operating frequency of the Stirling cryocooler according to the variations of the phase shift between piston and displacer are presented.

## 3. Design and manufacturing of the Stirling cryocooler

Fig. 1 shows the schematic view of the FPFD Stirling cryocooler. FPFD Stirling cryocooler consists of two major parts; linear compressor module and expander module.

Linear compressor consists of linear motor, inner and outer yoke, permanent magnet, coil, cylinder, piston and spring, and expander module consists of displacer, regenerator in the displacer, displacer cylinder, spring and heat exchanger.

The displacer with regenerator is actuated by pressure difference between the expansion space and compression space. The equation of motion of the displacer is follows[10].

$$m\ddot{x} + c\dot{x} + kx = F \tag{1a}$$

$$f_n = 1/2\pi\sqrt{k/m} \tag{1b}$$

$$\zeta = c/\sqrt{4mk} \tag{1c}$$

with m : moving mass, c : friction coefficient, k : spring constant, F : driving force, x : displacement, $f_n$ : natural frequency, $\zeta$ : relative damping. The Stirling cryocooler operates at cyclically steady state very close to sinusoidal.

Thus the instantaneous pressure and displacement are follows.

$$P = P_0 \sin(\omega t) \tag{2a}$$

$$x = x_0 \sin(\omega t + \varphi) \tag{2b}$$

with P : instantaneous pressure, $P_0$ : amplitude of pressure wave, $x_0$ : amplitude of displacement, $\omega$ : operating angular velocity, $\varphi$ : phase angle. The solutions of the equation (1a) are well known as the dynamic characteristics of the forced vibration system. The operating frequency, the natural frequency of the displacer and relative damping have effects on the dynamics of the displacer.

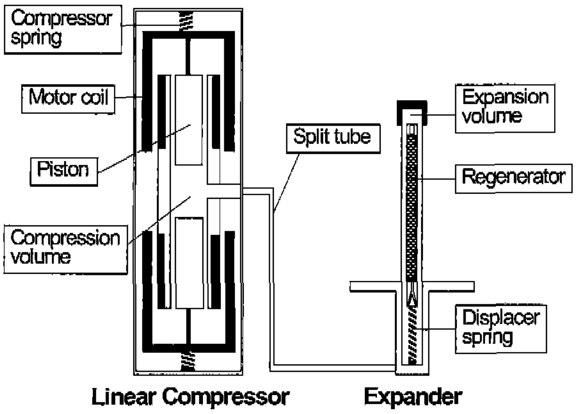

Fig. 1. Schematic diagram of the FPFD Stirling cryocooler

The ideal refrigeration (PV power) of the Stirling cryocooler can be written as following equation.

$$Q_{ideal} = \pi \cdot f \cdot P_o \cdot A \cdot x_o \cdot \sin\varphi \tag{3}$$

with A : cross sectional area of the expansion space, f : operating frequency. As shown equation (3), the ideal refrigeration is greatly affected by the operating frequency, the amplitude of pressure and dynamics of the displacer. The actual cooling capacity should be considered with radiation loss, conduction loss, and etc. If the temperatures at the cold end are same, thermal losses including the radiation and conduction will be the same order of magnitude. And if the amplitudes of pressure wave are same at given displacer, only the operating frequency and dynamic characteristics of the displacer have impact on the cooling capacity.

Table 1 shows dimensions and materials of piston, displacer, regenerator and magnet in the Stirling cryocooler. Helium is used as the working fluid in the Stirling cryocooler cycle because of its ideal gas properties, its high thermal conductivity, and its high ratio of specific heats.

TABLE Ⅰ Dimensions and materials of the FPFD Stirling Cryocooler

| Items | Materials | Dimensions(mm) |
|---|---|---|
| Piston | SUS 304 | $\Phi 12 \times 20$ |
| Displacer | SUS 304 | $\Phi 5.8 \times 70$ |
| Regenerator | SUS 304 | #325 |
| Magnet | NdFeB | $\Phi 35 \times 20$ |

## 4. Experimental procedure

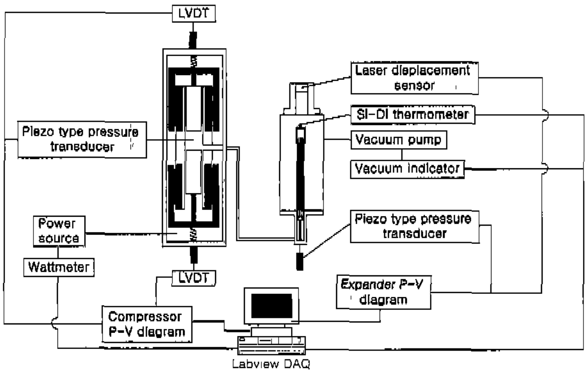

Fig. 2. Experimental apparatus of the FPFD Stirling cryocooler

Fig. 2 shows the schematic diagram of the experimental apparatus of the FPFD Stirling cryocooler. FPFD Stirling cryocooler. Piezo-electric dynamic pressure sensors were used to monitor pressure oscillations at the outlet of the compressor and buffers of each end. LVDTs(Linear Variable Differential Transformers) were provided at each end of the compressor for displacement measurement of the pistons. Laser displacement sensor was used to measure displacement of the displacer. A silicon-diode thermometer was attached to the cold head to measure the temperature of the cold end. After attaching those, the apparatus of the Stirling cryocooler except the component of the room temperature region was connected to the vacuum flange. During the experiment, a vacuum chamber was connected to the high vacuum pump under a pressure of $10^{-5}$ Torr.

The high vacuum pump system consists of a rotary roughing pump, turbo-molecular pump and vacuum gauges[11,12].

The following tests had been undertaken for the experimental analysis about the phase shifts among the pressure, displacements of the piston and displacer in the Stirling cryocooler:

1. The relationship between the output voltage of the laser displacement sensor and position of the displacer
2. Cool down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler
3. Amplitude of the displacer's stroke and phase shift with different temperature
4. The cooling capacity characteristics with natural frequency of the displacer
5. Displacer's amplitude with various charging pressure, amplitude of pressure and operating frequency
6. Phase shifts between displacements of the piston and displacer with different operating frequencies

## 5. Experimental results and discussion

The experimental setup for calibration of displacer's stroke, as shown in Fig. 3, consists of the expander, vernier caliper, optical windows, laser displacement sensor, dewar and digital multimeter. Displacement of the displacer is measured by a caliper vernier with resolution of 0.01mm, and the corresponding voltage output is recorded.

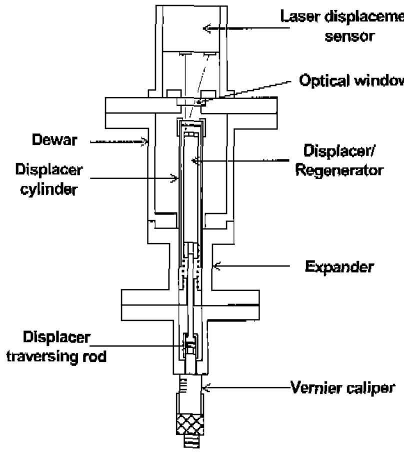

Fig. 3. The experimental setup for calibration of displacer's stroke

Fig. 4 shows that the relationship between the output voltage of the laser displacement sensor and position of the displacer is almost linear.

Fig. 5 shows cool down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler. The charging pressure was $15\mathrm{kg_f/cm^2}$ and operating frequency was 50Hz. Input power and applied voltage were about 32W and 9V, respectively. In this case, the lowest temperature was 67K. Therefore, we were able to analysis the behaviors of the displacer under the condition below the liquid nitrogen temperature(77K) for the cold finger of the expansion space.

Fig. 6 and Fig. 7 present real time waves and phase shifts of the displacer motion measured by laser displacement sensor, piston motion, pressure, current and applied voltage.

Measured data for the peak-to-peak displacement of the displacer with different operating frequency and peak-to-peak pressure of the compressor are presented in Fig. 8. Resonant frequencies of the compressor and expander are 54Hz and 64.5Hz, respectively. As the peak-to-peak pressure of the compressor was increased, peak-to-peak displacement of the displacer was increased. The peak-to-peak displacement of the displacer increases in the range of 0 – 64.5Hz, but decreases steeply when the operating frequency is bigger than the resonant frequency.

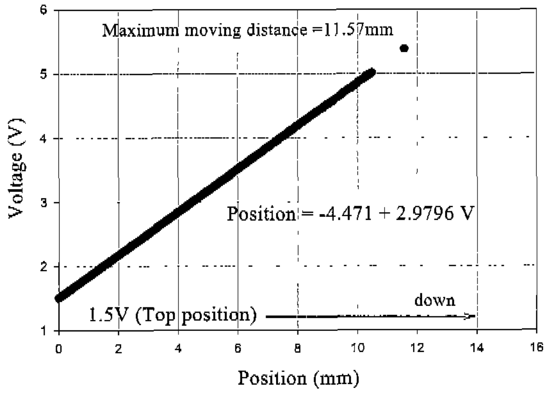

Fig. 4. Calibration of the laser displacement sensor

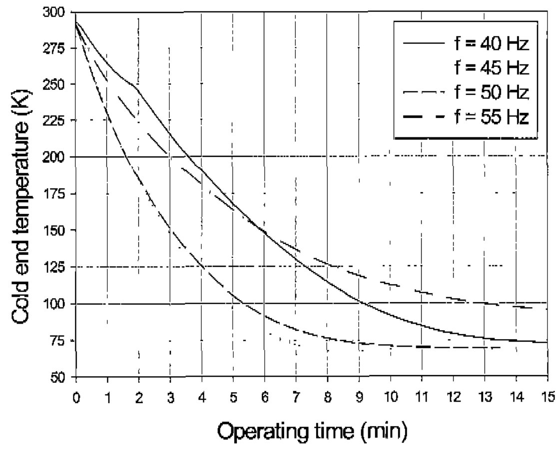

Fig. 5. Cool down characteristics of the cold end

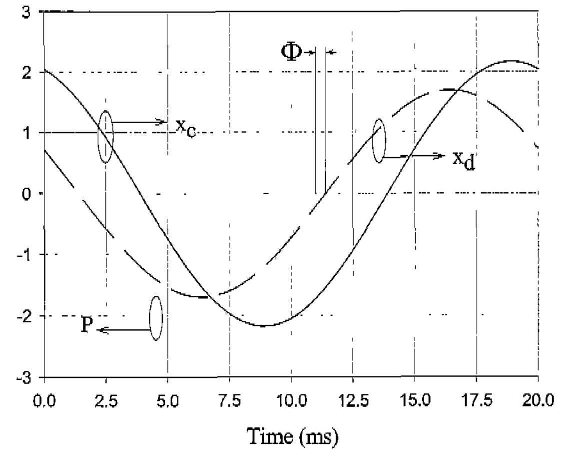

Fig. 6. Real time waves of the pressure, displacer motion, piston motion, current and applied voltage

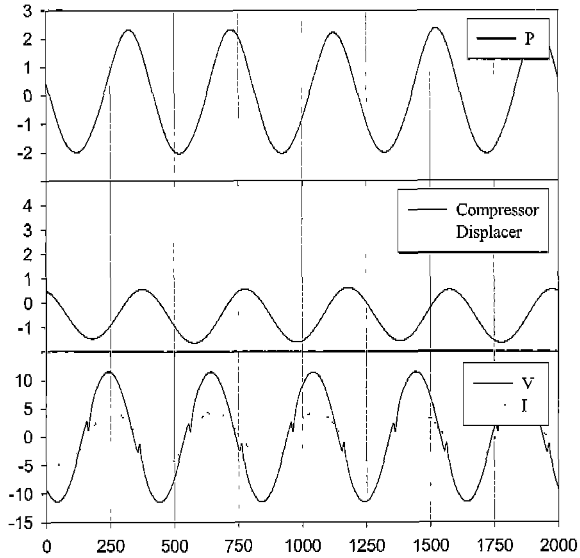

Fig. 7. Real time waves and phase shift among displacer motion, piston motion and pressure

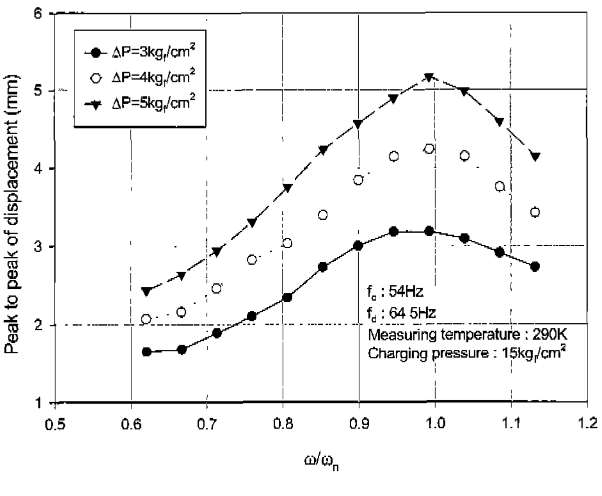

Fig. 8. Peak to peak displacement of the displacer with different operating frequency

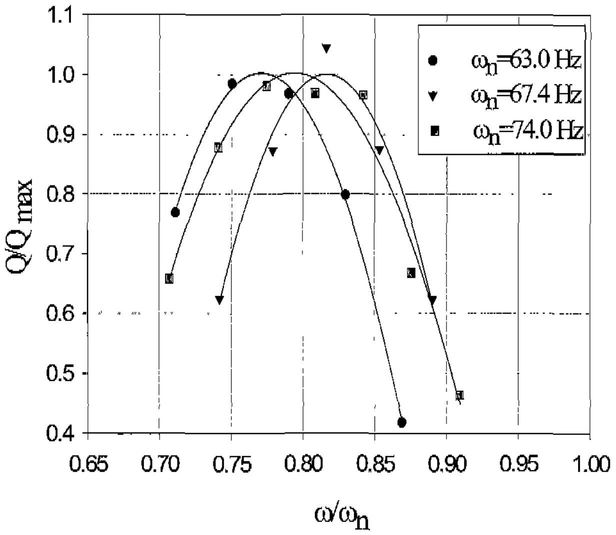

Fig. 9. The cooling capacity characteristics with natural frequency of the displacer

Fig. 9 shows experimental results for normalized cooling capacity at 150 K with different natural frequency of the displacer (63.0, 67.4, 74.0 Hz). The natural frequency of the displacer could be adjusted by changing mass or spring stiffness. In this study, the natural frequency of displacer is adjusted by using springs having different stiffness with the same height. The maximum cooling capacity is occurred in the range of frequency ratio 0.75 - 0.85. Therefore not only the resonance frequency of the linear compressor, but also the natural frequency of the displacer should be considered in the determining the operating frequency of the split-type free displacer Stirling cryocooler. For example, when the natural frequency of the displacer is the range of 60Hz ~ 64Hz, optimum operating frequency of the split-type free displacer Stirling cryocooler is about 50Hz.

Fig. 10 shows displacer's amplitude ratio for normalized result at 40Hz with frequency ratio to the natural frequency of the displacer and temperture of the cold finger. In this experiment, pressure amplitude is constantly 3.5kg/cm$^2$.

The zero-to-peak stroke response x of the displacer driving system to the sinusoidal excitation at the frequency f is given by[13,14]

$$\frac{x}{x_0}=\frac{1}{\sqrt{\left[1-\left(\frac{f}{f_0}\right)^2\right]^2+\left[2\cdot\zeta\left(\frac{f}{f_0}\right)\right]^2}} \qquad (4)$$

where $x_0=F_d/k$ ($F=F_d\cdot\sin(2\pi ft)$) is the static displacement, $\zeta=C/C_c$ is the relative damping, $C_c=2(km)^{1/2}$ is the critical damping coefficient, and $f_0=(k/m)^{1/2}/(2\pi)$ is the natural frequency. The phase $\psi$ of the stroke relative to the applied force is given by

$$\tan\psi=\frac{2\zeta\left(\frac{f}{f_0}\right)}{1-\left(\frac{f}{f_0}\right)^2} \qquad (5)$$

From equation (4), relative damping is predicted as shown in Fig. 10.

As the temperature of the cold finger is decreased, relative damping is increased. It may result from the increase of the friction by increase of the density and viscosity of the helium gas.

Fig. 11 presents the effects of the amplitude of pressure and operating frequency on the displacer's amplitude when temperature of the cold finger is 150K and charging pressure is 20kg/cm$^2$. From equation (4) and Fig. 11, not only the relation between the displacer's amplitude and driving force of the displacer but also the relation between the displacer's amplitude and amplitude of pressure in the compressor is linear. And, as the amplitude of pressure in the compressor and operating frequency are increased, the displacer's amplitude is increased.

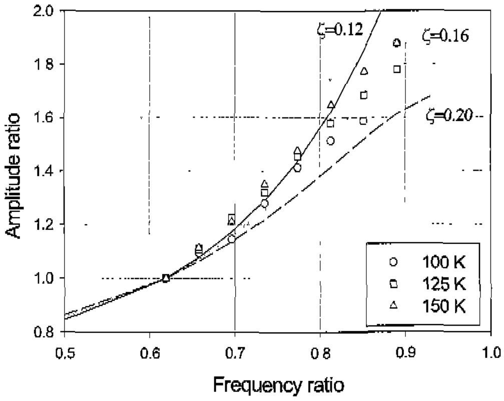

Fig. 10. Displacer's amplitude ratio and predicted damping with frequency ratio and temperture of the cold finger

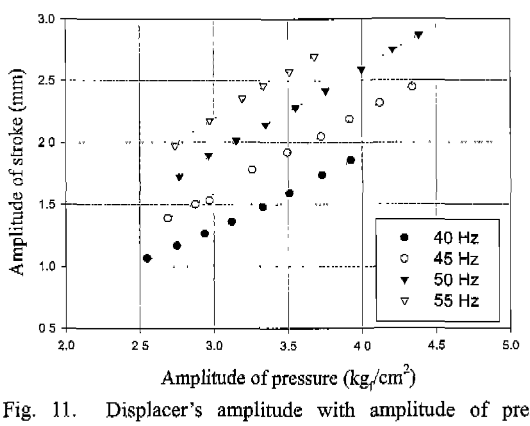

Fig. 11. Displacer's amplitude with amplitude of pressure and operating frequency

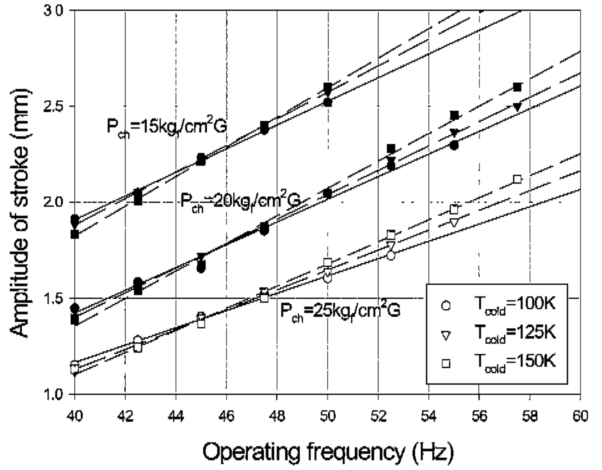

Fig. 12. Displacer's amplitudes for various charging pressure and operating frequency

Displacer's amplitudes for various charging pressure and operating frequency with 3.5kg/cm$^2$ of the constant pressure amplitude are shown in Fig. 12. Displacer's amplitude produced by gas driving force is tend to increase as the charging pressure decreases and operating frequency increases. Although pressure amplitude is constant, if the charging pressure is different, swept volume of the piston decreases, flow rate and pressure drop in the regenerator decrease, finally, driving force decrease. Therefore, operating characteristics of the Stirling cryocooler is changed by the charging pressure. When pressure amplitude is constant, there is few effect of the temperature in the cold finger on the displacer's amplitude

Fig. 13 shows experimental results for amplitude of the displacer's stroke and phase shift between displacements of the piston and displacer with different temperature of the cold end under the condition of constant operating frequency at 50Hz. Amplitude of the displacer's stroke and phase shift between displacements of the piston and displacer decrease as the temperature of cold end decreases.

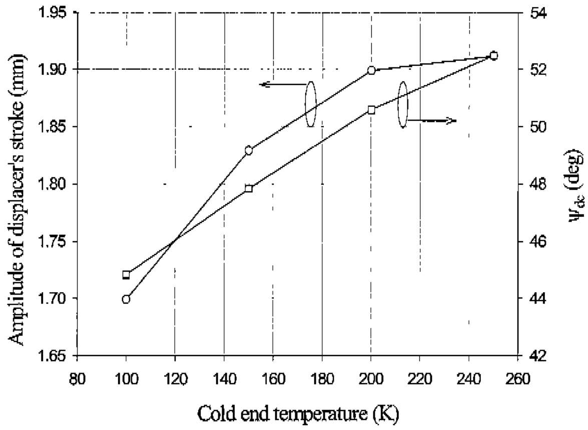

Fig. 13. Amplitude of the displacer's stroke and phase shift with different temperature

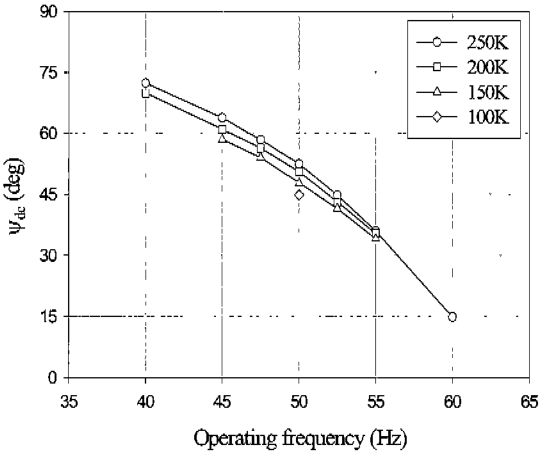

Fig. 14. Phase shifts between displacements of the piston and displacer with different operating frequencies

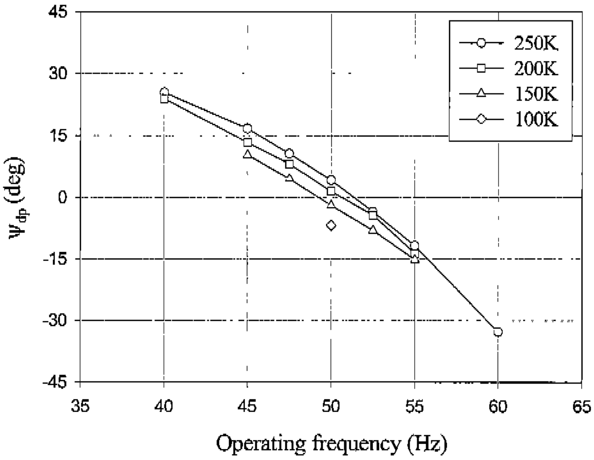

Fig. 15. Phase shifts between displacements of the displacer and pressure of the compression space with different operating frequencies

Fig. 14 shows the phase shifts between displacements($\psi_{dc}$) of the piston and displacer with different operating frequencies. And Fig. 15 shows the phase shifts($\psi_{dp}$) between displacements of the displacer and pressure of the compression space with different operating frequencies. $\psi_{dc}$ and $\psi_{dp}$ decrease as the operating frequency increases and temperature of the cold end decreases. As the temperature of the cold end is lower, $\psi_{dc}$ and $\psi_{dp}$ converge to 45° and -7°, respectively. And it is clear that $\psi_{dc}$ and $\psi_{dp}$ approach 0° and about -40° when operating frequency coincides with resonant frequency of the expander, in other words, refrigeration doesn't occur. Therefore, from the results of Figure 14, 15, when the $\psi_{dc}$ is 45°, operating frequency is optimum and is decided by resonant frequency of the expander, mass and cross section area of the displacer and constant by friction and flow resistance.

## 6. Conclusions

Stirling cryocooler actuated by the electric force of the dual linear motor is designed and manufactured. And, displacement of the piston is measured by LVD Ts(Linear Variable Differential Transformers), displacement of the displacer is measured by laser optic method, and phase shift between piston and displacer is measured and evaluated by experiments.

The relationship between the output voltage of the laser displacement sensor and position of the displacer was almost linear.

Cool down characteristics of the cold end with laser displacement sensor in the expander of the Stirling cryocooler was investigated, in this case, the charging pressure was 15kg$_f$/cm$^2$ and operating frequency was 50Hz. Input power and the lowest temperature were about 32W and 67K, respectively.

The maximum cooling capacity is occurred in the range of frequency ratio 0.75 - 0.85. Therefore not only the resonance frequency of the linear compressor, but also the natural frequency of the displacer should be considered in the determining the operating frequency of the split-type free displacer Stirling cryocooler.

As the temperature of the cold finger is decreased, relative damping is increased. It may result from the increase of the friction by increase of the density and viscosity of the helium gas.

Not only the relation between the displacer's amplitude and driving force of the displacer but also the relation between the displacer's amplitude and amplitude of pressure in the compressor is linear. And, as the amplitude of pressure in the compressor and operating frequency are increased, the displacer's amplitude is increased.

When the pressure amplitude is constant, displacer's amplitude produced by gas driving force is tend to increase as the charging pressure decreases and operating frequency increases.

As the peak-to-peak pressure of the compressor was increased, peak-to-peak displacement of the displacer was increased. The peak-to-peak displacement of the displacer increases in the range of 0 – 64.5Hz(resonant frequency of the displacer), but decreases steeply when the operating frequency is bigger than the resonant frequency.

When the phase shift between displacements of the piston and displacer is 45°, operating frequency is optimum and is decided by resonant frequency of the expander, mass and cross section area of the displacer and constant by friction and flow resistance.

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