Journal of the Korea Institute of Applied Superconductivity and Cryogenics
Vol.5, No.1, (2003), pp. 81~86

1-Dimensional simulation of nonlinear magnetic diffusion in high-Tc superconductor

K. S. Heo, S. Y. Seol
Department of Mechanical Engineering, Chonnam National University, Gwang-ju, Korea
Tornado3@hitel.net

***Abstract*-- This paper presents a numerical analysis of transient magnetic diffusion in a melt-cast-processed BSCCO-2212 tube subjected to sinusoidal applied fields. The nonlinear *E-J* relation obtained by experiments in liquefied N_2 is used to find the magnetic diffusion coefficient. The magnetic flux density, shield current density and ρ-J^2 loss are considered. According to the result of this study, the shielding current density is varied with external applied field and coordinate in the superconductor tube. The result of analysis can be used to explain the response of a Supercon -ductor-Shield-Core-Reactor subjected to sinusoidal applied fields.**

1. Introduction

SSCR(Superconductor Shield Core Reactor) is one of the leading candidate of FCL(Fault Current Limiter) that protects electric power system from an accidental current. SSCR consists mainly of a closed iron core inside a superconductor tube and a copper coil wound on the outside of superconductor tube. Under fault condition, the high current in the copper coil exceeds the shielding capability of superconductor tube and the magnetic field of copper coil penetrate into the superconductor tube. Therefore, in order to understand the behavior of SSCR, it is necessary to study about the magnetic diffusion in superconductor [1-4].

However, we do not understand how the superconductor tube behaves both magnetically and thermally during a fault. Furthermore, it has been known that the Bean's critical state model is not approached subjected to a periodic applied field with a frequency of 50~60 Hz [4-5]. In recent years, a study has focused on the linear magnetic diffusion to qualitatively explain the magnetic diffusion under AC steady state field. However, the linear diffusion model cannot predict the actual situation because of the constant magnetic diffusion coefficient [6-7].

In this paper, we use the nonlinear *E-J* relation obtained by experiments on 77 K of liquefied N_2 to find the shield current density and magnetic diffusion coefficient. The nonlinear magnetic diffusion equation is solved by finite difference method. The results of analyses are compared with the experimental observation of Ref. [5]. After verifying the numerical method, the shield current density, penetration time, and time delay are considered. Finally, we discuss the limitations of this study and the effect of ρ-J^2 loss.

2. Governing Equation

A magnetic diffusion equation of a magneto-quasi-static state is developed with an assumption that the ρ-J^2 loss is quite smaller than the hysteresis loss in fault condition [6].

$$\nabla\times[D_m(\mathbf{J})\nabla\times\mathbf{B}]=\frac{\partial\mathbf{B}}{\partial t} \tag{1}$$

A magnetic diffucivity D_m is a function of an electric resistivity as below.

$$D_m=\frac{\rho(J)}{\mu_0} \tag{2}$$

Although the electric resistivity of superconductor is a function of the current density, temperature, and magnetic field density, we assume the electric resistivity as a function of current density only [5].

$$E=3.2\times10^{-22}|J|^{7.356} \tag{3}$$

In (3), unit of current density J is [A/㎠] and [μV-㎝] for electric field intensity E. The current density J are obtained by Ampere's law.

$$J=-\frac{1}{\mu_0}\frac{\partial B}{\partial x} \tag{4}$$

The electric resistivity of superconductor is obtained as following by ρ=dE/dJ relation.

$$\rho=2.354\times10^{-21}|J|^{6.356}. \tag{5}$$

The superconductor tube is heated significantly during fault condition, and its recovery is usually much longer than that of an inductive device because heat must be removed after fault is ended. Therefore, in order to understand the behavior of SSCR in fault condition, it is necessary to consider the effect of the temperature. The effect of coolant temperature on the *E-J* relation has been studied extensively. However, the influences of temperature in superconductor tube needs more study. In this study, we assume that the SSCR as an isothermal system.

3. NUMERICAL ANALYSIS

The analysis model consists of copper coil and BSCCO-2212 cylindrical tube made by a melt-cast process. The copper coil is 70.5 mm long and 13.4 mm inside radius, 24.3 mm out side radius. And the superconductor tube is 74.8 mm long and has a wall thickness of 5.5 mm and 25.6 mm outside diameter.

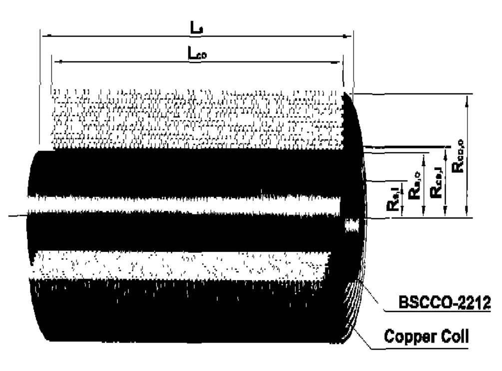

Fig.1. Schematic diagram of BSCCO-2212 tube and exciting coil.

Fig. 1 shows the schematic of the analysis model. The superconductor tube and exciting coil are cooled by the liquefied N_2 to be 77 K. The gap between copper coil and superconductor tube is maintained to allow coolant flow so the temperature of superconductor is minimally affected by resistive heating in the copper coil. It is assumed as axisymmetric system. The magnetic diffusion equation can be simplify as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD_m(J)\frac{\partial B}{\partial r}\right)=\frac{\partial B}{\partial t} \quad (6)$$

The sinusoidal magnetic field with a frequency of 60Hz are applied on the outside of a superconductor tube.

$$B = B_0 \sin(\omega t) \quad (7)$$

The amplitude of magnetic field, B_0 at the central point of copper coil is obtained by (8) and (9). In (8) and (9), $\alpha=R_{co,o}/R_{co,i}$, $\beta=L_{co}/(2R_{co,I})$ and $\gamma=L_{co}/2(R_{co,o}-R_{co,i})$ [8].

$$B_0 = \mu_0 \cdot F(\alpha,\beta,\gamma)\cdot\frac{NI_{max}}{L_{co}} \quad (8)$$

$$F(\alpha,\beta,\gamma)=\gamma\ln\left(\frac{\alpha+\left(\alpha^2+\beta^2\right)^{1/2}}{1+\left(1+\beta^2\right)^{1/2}}\right) \quad (9)$$

In numerical analysis, the discretization equation with fully implicity FDM method is used. Under AC applied field, internal magnetic field density shows rapid change. Therefore, we should use very small grid space and time step: 0.0275 mm grid space and about 0.02 ns minimum time step.

4. RESULTS OF ANALYSIS

4.1. Comparison with the experiment

To verify the validity of nonlinear magnetic diffusion model, we compare the numerical results with experiment of Ref. [9].

Fig. 2 is the result of experiment Ref. [9], and Fig. 3 is the numerical results of nonlinear magnetic diffusion model with the same condition. *H* is a penetration field density, measured by a hall probe inside the superconductor tube, and I_s is the induced current, measured by Rogowski coil.

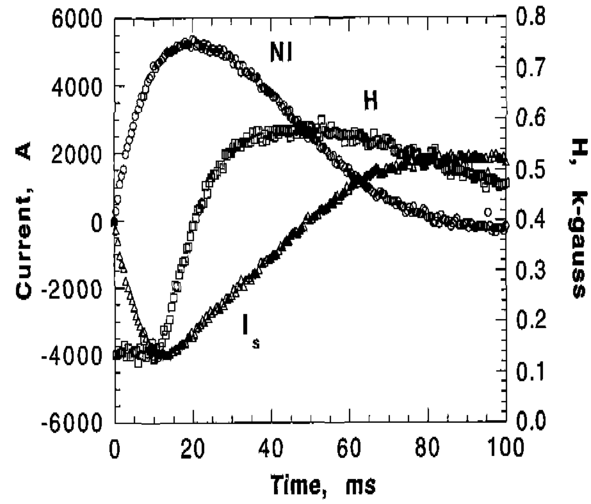

Fig.2. Test result of magnetic field *H*, induced current I_s with NI_{max}=5383A.

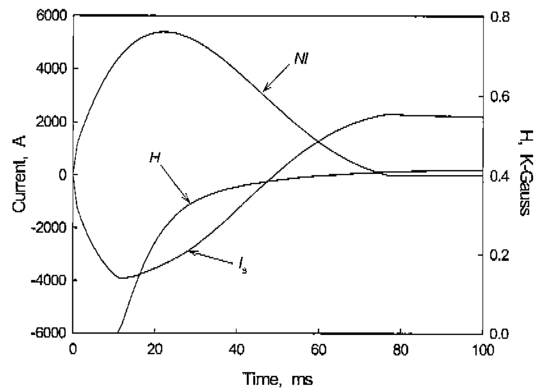

Fig.3. Analysis result of magnetic field H, induced current I_s with NI_{max}= 5383A.

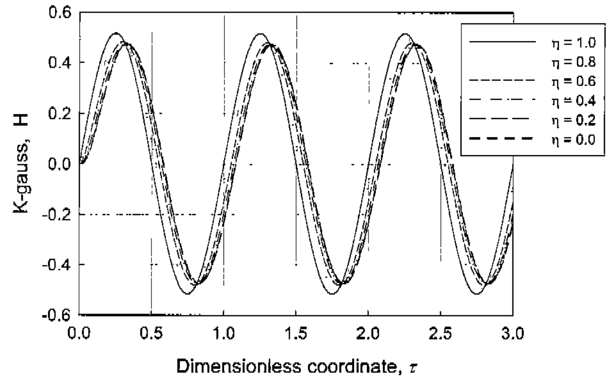

Fig.4. Analysis result of linear magnetic diffusion model for J=Jc, NImax=3000 A.

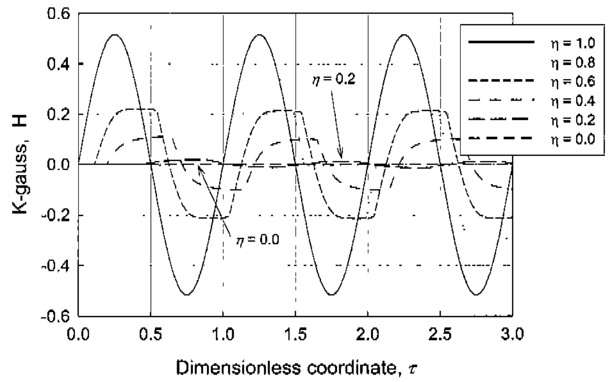

Fig.5. Waveform of magnetic flux density at different value of η with NI_{max}=3000 A.

Induced current I_s is decrease to -4000 A until 10 ms and then increase to 2000 A and after 80 ms there is no remarkable change. Those phenomena are well predicted by numerical work as shown in Fig. 3. Consequently, Fig. 3 tells us that nonlinear magnetic diffusion model is well describes the variation of shield current density in fault condition.

The penetration flux density H that is measured by 0.47 k-Gauss at 100 ms, however, the actual flux density is about 0.35 k-Gauss because of 0.12 k-Gauss background flux. The numerical work calculates it as 0.414 k-Gauss. According to experimental result, penetration flux density has maximum value at 50 ms and is dwindling. But, those tendencies do not appear in Fig. 3. One possible interpretation of this difference is the ignorance of heating effect. The thermal effect and ρ-J^2 loss will be examined later again.

4.2. Flux density

Fig. 4 presents the result of linear diffusion model that is proposed by Ref. [7], and Fig. 5 shows the result of nonlinear diffusion model. Fig. 4 and Fig. 5 are the results of the same condition with NI_{max}=3000 A. The linear diffusion model by Ref. [7] assumes a constant magnetic diffusion coefficient. The dimensionless coordinate η and dimensionless time τ are defined as below.

$$\eta = \frac{r - R_{s,i}}{R_{s,o} - R_{s,i}} \quad \text{(10)}$$

$$\tau = \frac{2\pi t}{\omega} \quad \text{(11)}$$

There is a distinguishable difference of Fig. 4 and 5. The flux wave of Fig. 5 is deformed to a saw-tooth shape near the inside of tube. However, the linear diffusion model shows the sinusoidal shape through the tube as shown in Fig. 4. The major reason of the wave deformation is considered that ignorance of ρ-J^2 loss. It is impossible to explain the ρ-J^2 loss without including the shield current density.

Fig. 6 shows that the nonlinear analysis result for high exciting current NI_{max}=25000 A. In high exciting current, nonlinear and linear diffusion model show the similar results. It means that shield current density is lower than critical current density when exciting current NI_{max}=3000 A, and as exciting current increases, shield current density and diffusion coefficient increases. Therefore, contrary to Bean's critical state model, the shield current density is not constant value.

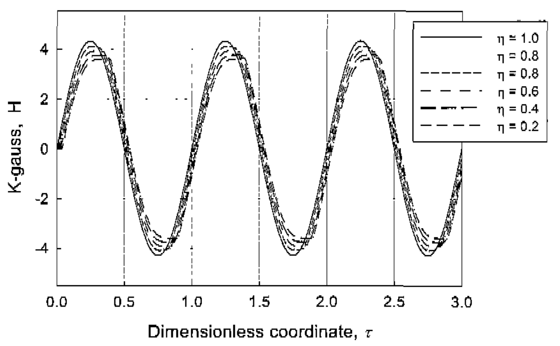

Fig. 6. Waveform of magnetic flux density at different value of η with NI_{max}=25000 A.

4.3. Shield current density

The Bean's model assumes that current density in penetration range as a critical current density. According to Been model, the amount of induced current has a fixed value that multiply critical current density and cross sectional area after penetration is completed. However, it was pointed out in the previous section that the induced current changes after penetration, and the shield current density are also varied with the exciting current.

Fig. 7 represents the amplitude of shield current density for different NI_{max} and η.

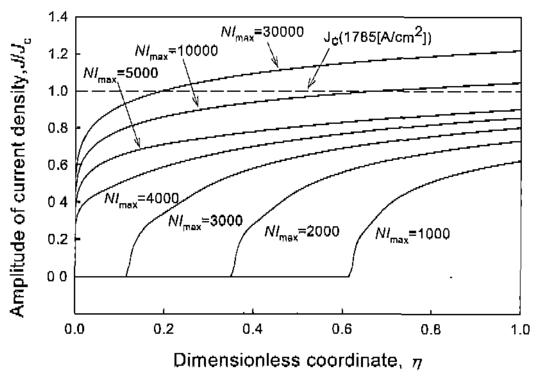

Fig. 7. Amplitude of shielding current density for different NI_{max}.

The amplitude of shield current density increases as exciting current. For the exciting current below 5000 A, the shield current density of whole superconductor is smaller than the critical current density. For NI_{max}=30000 A, about 80 percent of superconductor is in over critical state of which shield current density is greater than critical current density. Also, Fig. 7 shows that the maximum shield current density appears on the outer surface of superconductor tube, and the minimum value inside surface.

In actual condition, the superconductor tube surface is cooled by liquefied N_2. Therefore, the maximum temperature may not happen on the surface but interior of superconductor. The elevated temperature increases the magnetic diffusion coefficient. Therefore, it is supposed that the magnetic diffusion coefficient of superconductor interior is higher than the analysis result.

4.4. Penetration time and time delay

The penetration time is varying with the out side magnetic field size and it's change rate. Fig. 8 shows the relation of the dimensionless penetration time ($2\pi t_p/\omega$) and the exciting current. In Fig. 8, the notation J=J_c represents the penetration time when the shield current density is critical current density.

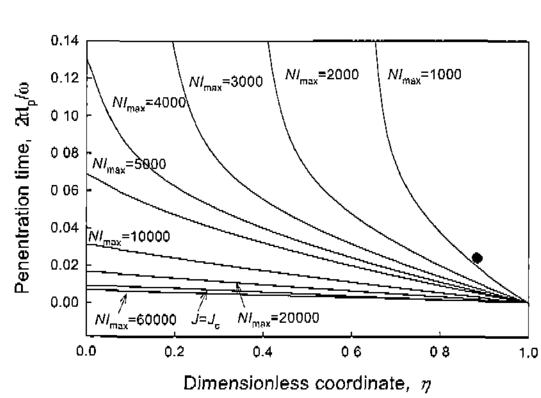

Fig. 8. Variation of penetration time with dimensionless coordinate η for different NI_{max}.

In small exciting current (NI_{max}<3000 A), the penetration is not completed to inside surface. The penetration time increase exponentially with penetration depth. The penetration time decreases and approaches to the case of J=J_c as increasing of NI_{max}.

Through this figure, two interesting features are investigated. One is that the penetration depth is finite even if time passes infinitely. The other one is that the penetration time of high exciting current is similar to that of J=J_c case.

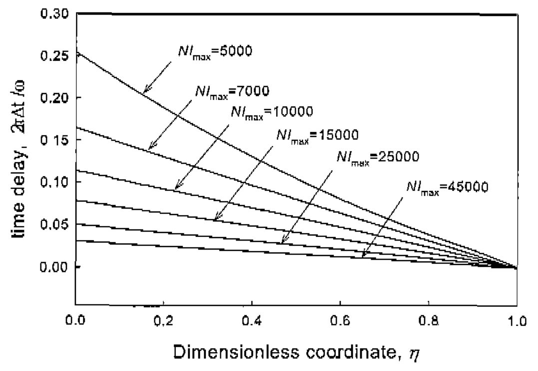

Fig. 9. Variation of time delay with dimensionless coordinate η for six NI_{max}.

Fig. 9 shows the dimensionless time delay, $2\pi\Delta t/\omega$, at various locations in the superconductor for six values of NI_{max}. The time delay increases with penetration depth. According to linear model, the time delay proportionally increases with penetration depth in the small exciting current. But in nonlinear model, Fig. 9 shows that the time delay and penetration depth have not linear relation.

4.5. Joule heat generation

The ρ-J^2 loss per unit length of tube is calculated as the following equation.

$$Q^* = \int_{R_{s,i}}^{R_{s,o}} 2\pi r \rho J^2 dr \quad (12)$$

Fig. 10 shows that the loss as a function of time. The maximum loss occurs when the change rate of outside magnetic field is high. The time delay appears for the small exciting current. As exciting current increases, time delay decreases rapidly and can be neglected for high exciting current.

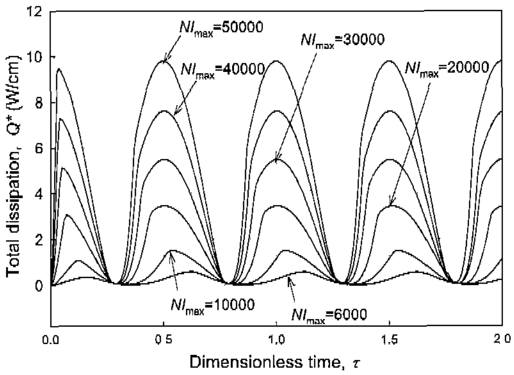

Fig. 10. Variation of total ρ-J^2 loss, Q^*, with dimensionless time τ for six NI_{max}.

The ρ-J^2 loss; i.e. Joule heating, rises temperature and magnetic diffusion coefficient. The volumetric heating makes rapid increase of temperature, however, it's recovery time is much longer than rising time because the heat must be extracted via surface of tube. Therefore, the actual change of Joule heat generation is somewhat different from Fig. 10. More research is needed to include the Joule heat generation and temperature variation inside supercon -ductor.

5. SUMMARY AND CONCLUSION

In this study, we analyze a magnetic diffusion of high Tc superconductor under AC fault condition with a n onlinear E-J relation obtained by experiment result of Ref. [5] and FDM method. The conclusion of our study is as following.

(1) Penetration magnetic flux and the behavior of induced current on experiment result of Ref. [5] are successfully described with nonlinear diffusion model.

(2) When outside magnetic field permeate into the inside of superconductor tube, the waveform of flux density does not keep a sinusoidal wave form but changes in gradually to saw-tooth form.

(3) Shield current density distribution is not uniform in the superconductor, and outer surface current density are greater than that of the inside surface. As the exciting current increases, the current density in the tube also increases.

(4) The penetration depth of magnetic field in superconductor tube is finite even if time passes infinitely. And, as increase the exciting current, penetration time is decreased. The time delay of the peak B is gradually increases with penetration depth.

(5) The ρ-J^2 loss shows the maximum value at outer surface of superconductor tube, and it shows the minimum value at inner surface of tube. Total ρ -J2 loss increase as the outside magnetic flux change rate, and it's time delay can be neglected in high exciting current.

(6) In this study, we do not include thermal effect, so future study is needed to include the effect of inside high-Tc superconductor temperature variation.

ACKNOWLEDGMENT

This study has been supported by "Korea Science and Engineering Foundation", as a part of "mechanical analysis of HTS fault current limiter", under contract R01-2000 -000-00308-0(2002).

REFERENCE

[1] P. Bauman, M. Friesen, D. Phillips, "On the periodic behavior of solution to diffusion problem describing current in high -temperature superconductor", Physica, Vol.D, No.137, pp.172-191, 2000

[2] A. V. Eremin, O. S. Esikov, V. A. Kashurnikov, A. E. Khodot, Y. N. Pirogov, E. A. Protasov, I. A. Rudnev, M. V. Zubin, "Numerical simulation and experimental observation of magnetic flux distri bution in high-temperature superconductor", Supe -rcond. Sci. Technol. Vol.14, pp. 690-694, 2001

[3] H. Ueda, A. Ishiyama, H. Kado, M. Ichikawa, "Thermal and magnetic characteristic of bulk superconductor and performance analysis of magnetic shielding type of superconducting fault current limiter", IEEE Transaction on Applied Superconductivity, Vol.11, No.1, March.2001

[4] M. G. Ennis, T. J. Tobin, Y. S. Cha, "Fault current limiter -predominantly resistive behavior of a BSCCO-Shielded-Core

Reactor", IEEE Transaction on Applied Superconductivity, Vol.11, No.1, March.2001

[5] Y. S. Cha, T. R. Askew, "Transient characteristics of high-Tc superconductor tube subjected to internal and external magnetic field", IEEE Transaction on Applied Superconductivity, Vol.11, No.1, March.2001

[6] V. Sokolovsky, V. Meerovich, S. Goren, G. Jung, "Analytical approach to AC loss calculation in high-Tc superconductors", Physica, Vol.C, No.306, pp.154-162, 1998

[7] Y. S. Cha, "Magnetic diffusion and dissipation in high-Tc superconductor subjected to sinusoidal applied field", Physica, Vol.C, No.301, pp.1-12, 2001

[8] M. N. Wilson, "Superconducting magnets", Chapter 3, Clarendon Press. Oxford, 1983

[9] Y. S. Cha, T. R. Askew, "Transient response of a high-temperature supwrconductor tube to pulsed magnetic fields ", Physica, Vol.C, No.302, pp.57-66, 1998