Journal of the Korean Nuclear Society
Volume 35, Number 1, pp.1 ~ 13, February, 2003

Estimation of Beam Mode Frequencies of Co-axial Cylinders Immersed in Fluid by Equivalent Mass Approach

Tae-Wan Kim, Suhn Choi, and Keun-Bae Park
Korea Atomic Energy Research Institute
150 Dukjin-dong, Yuseung-gu, Daejeon 305-353, Korea
twkim3@kaeri.re.kr

(Received June 26, 2002)

Abstract

In this study, an effective method to estimate the fundamental frequencies of co-axial cylinders immersed in fluid is proposed. The proposed method makes use of the equivalent mass or density that is derived from the added mass matrix caused by the fluid-structure interaction (FSI) phenomenon. The equivalent mass is defined from the added mass matrix based on a 2-D potential flow theory. The theory on two co-axial cylinders is extended to the case of three cylinders. To prove the validity of the proposed method, the eigenvalue analyses upon coaxial cylinders coupled with fluid gaps are performed using the equivalent mass. The analyses results upon various fluid gap conditions reveal that the present method could provide accurate frequencies and be suitable for expecting the fundamental frequencies of fluid coupled cylinders in beam mode vibration.

Key Words : fluid-structure interaction, added mass, equivalent mass, fluid element, beam mode vibration, cylindrical shell, ANSYS

1. Introduction

When a structure vibrates in contact with fluid, the fluid shall be displaced to accommodate the motion. Then fluid pressures are generated as a result. Fluid forces on the structure occur due to the integrated effect of these pressures. The force is usually proportional to the relative accelerations of the structure, and gives rise to a hydrodynamic coupling, fluid structure interaction (FSI). Many authors have treated the hydrodynamic coupling as an inertia term, and formulated as an added mass or virtual mass upon a structure [1]. There are number of structures contacting with the coolant in a nuclear power plant such as fuels, internal structures, spent fuel storages, vessels, and so forth. In case of the reactor vessel and internal structures, authors idealize the case as two co-axial cylinders with a fluid gap.

Chen and Jung [2][3] provided the natural frequencies and corresponding added masses of two co-axial cylinders by solving the motion of fluid and structure simultaneously. Au-Yang[4][5] suggested a theoretical and experimental investigation of co-axial cylinders with different lengths. Au-Yang also proposed a procedure to evaluate the added mass matrix of fluid-coupled cylinders, and performed dynamic analysis upon nuclear components. Fritz[6] published a theoretical investigation of two co-axial cylinders based on the two dimensional flow theory. Kim[7] discussed the validity of finite element analysis by comparing the experimental results over a reactor internal model. Perov[8] investigated the vibration mode of the reactor vessel and the internals of a WWER-1000 type reactor with finite element (F.E.) technique.

There are two ways to simulate FSI. One is to adopt the inertia concept, and the other is to insist on a strict structural-fluid formulation. The former derives a mass matrix to accommodate the effect of hydrodynamic coupling of fluid and structure, and the later directly solves the coupled equation of motion using a numerical method. Since there are merits and limitations to both, a preferable method depends upon the goal of analysis. By replacing the fluid elements with mass matrices or equivalent elements, the added mass method can provide a rather simple model and minimize the degree of freedom. In the modeling process, mass matrices replace fluid elements between structures and connect two structural nodes. Although this process seems to be quite a simple one, a lot of effort to connect all structural nodes with the corresponding mass matrices is required. In case of complex structures, the connection of structural nodes with mass matrix often leads to quite a tedious job. Then, needs for more effective methods to minimize the modeling effort or straightforward means are raised.

In this study, an effective method replacing the added matrix with the equivalent amount of mass is proposed. The proposed method simulates the mass matrix in terms of an equivalent amount of mass over a structure. To prove the validity of proposed method, a series of eigenvalue analyses to estimate the fundamental frequencies of cylinders immersed in fluid in the beam mode vibration is performed. In addition, the theory to define the added mass of three co-axial cylinders is proposed.

2. Derivation of the Added Mass for Co-axial Cylinders

2.1 Added Mass of Two Co-axial Cylinders

To define the added mass matrix of two co-axial cylinders, the theory of potential flow is referred[6]. Consider two long concentric cylinders separated by a fluid annulus as shown Fig. 1. The inner radius a is surrounded by an outer concentric cylinder of inner radius b. The length of the annulus is L, where L is assumed to be much greater than b. The outer cylinder is assumed to have a velocity of $\dot{x}_2$ and inner cylinder $\dot{x}_1$. The relative displacement x_1 - x_2 is assumed to be small compared to b - a. A velocity potential ϕ may be defined:

$$V_r = -\frac{\partial \phi}{\partial r}, \quad V_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{1}$$

,where, V_r is radial velocity of fluid, and V_θ is tangential fluid velocity. The fluid is considered

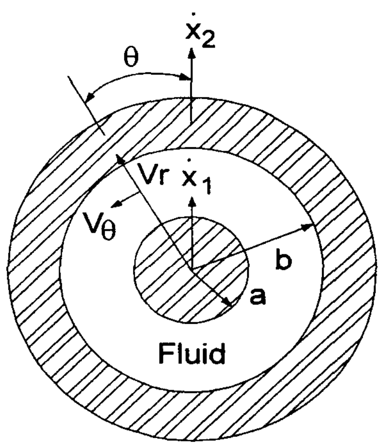

Fig. 1. Two-cylinder Motion with a Fluid Couple

frictionless and to be at stationary when the cylinder is at rest. Under these conditions, the fluid is irrotational and ϕ will be single-valued. The boundary conditions are:

$$-\frac{\partial\phi}{\partial r} = \dot{x}_1 \cdot \cos\theta \quad at \quad r = a \tag{2}$$

$$-\frac{\partial\phi}{\partial r} = \dot{x}_2 \cdot \cos\theta \quad at \quad r = b \tag{3}$$

The continuity equation is given by equation (4), and a form of solution could be assumed:

$$\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2\phi}{\partial\theta^2} = 0 \tag{4}$$

$$\phi = f(r) \cdot \cos\theta \tag{5}$$

From equation (4) and (5), the governing equation could be a form of differential equation.

$$r^2 f'' + rf' - f = 0 \tag{6}$$

,where, the prime denotes differential with respect to r. The solution of equation (6) with the boundary condition of equations (2) and (3) is as follows.

$$V_r = \left(\frac{B}{r^2} - A\right) \cdot \cos\theta \tag{7}$$

$$V_\theta = \left(\frac{B}{r^2} + A\right) \cdot \sin\theta \tag{8}$$

,where, $B = \frac{b^2 a^2}{b^2 - a^2}(\dot{x}_1 - \dot{x}_2)$, $A = \frac{\dot{x}_1 a^2 - \dot{x}_2 b^2}{b^2 - a^2}$

By applying Lagrange's equation of motion, the fluid reaction forces on the cylinder could be depicted as equation (9).

$$F_{fi} = -\frac{d}{dt}\left(\frac{\partial T_f}{\partial \dot{x}_i}\right) + \frac{\partial T_f}{\partial x_i} \tag{9}$$

,where, x_i indicate the generalized coordinate of motion and T_f is the kinetic energy of the fluid. If the motion of cylinders assumed to be small compared to the thickness of fluid gaps, it is reasonable to neglect the last term in equation (9). Then, the kinetic energy of fluid could be expressed as equation (11).

$$F_{fi} = -\frac{d}{dt}\left(\frac{\partial T_f}{\partial \dot{x}_i}\right) \tag{10}$$

$$T_f = \int_a^b \int_0^{2\pi} \frac{1}{2}\rho \cdot r \cdot L \cdot dr \cdot d\theta \left(V_r^2 + V_\theta^2\right) \tag{11}$$

By substituting the equations (7) and (8) for velocity terms in equation (11), following two equations of motion could be derived from equation (10). The forces on the left side indicate reaction forces on the inner and outer cylinder.

$$\begin{Bmatrix} F_{f1} \\ F_{f2} \end{Bmatrix} = -\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} \tag{12}$$

,where, $M_{11} = \pi\rho L a^2\left(\frac{b^2 + a^2}{b^2 - a^2}\right)$, $M_{22} = \pi\rho L b^2\left(\frac{b^2 + a^2}{b^2 - a^2}\right)$,

$$M_{12} = M_{21} = -2\pi\rho L a^2\left(\frac{b^2}{b^2 - a^2}\right),$$

and ρ is the density of fluid. Since the equation (12) indicates an external load on cylinders, the integrated equation of motion on cylinders could be expressed in term of structural motions, equation (13).

$$\left(\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\right)\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{13}$$

,where, m_{ij} and k_{ij} denote the mass and stiffness of cylinders. In equation (13), the effect of the fluid gap is considered as added mass in the equation of motion for cylinders. The added mass matrix consists of diagonal and off-diagonal terms with symmetric element. Although this formulation does not consider the longitudinal mode of the cylinders, it provides effective solutions over various structures in beam mode vibration.

2.2. Added Mass of Three Co-axial Cylinders

The case of three concentric cylinders with two fluid gaps in-between could be considered as a starting point of the general theory on the multi-cylinders immersed in fluid, but studies on the subject are rare. Consider three concentric cylinders with two fluid gaps as shown in Fig. 2. The reaction forces of fluid on each cylinder could be expressed by superposing the equation (12) over two cylinders in reciprocal combination. The boundary conditions for fluid velocity are expressed in equation (14). The boundary condition for the cylinder b is defined at the middle plane for simplicity but the actual dimension includes the thickness of the cylinder.

$$-\frac{\partial\phi_1}{\partial r} = \dot{x}_1\cos\theta,\ at\ r=a,\quad -\frac{\partial\phi_1}{\partial r} = \dot{x}_2\cos\theta,\ at\ r=b$$
$$-\frac{\partial\phi_2}{\partial r} = \dot{x}_2\cos\theta,\ at\ r=b,\quad -\frac{\partial\phi_2}{\partial r} = \dot{x}_3\cos\theta,\ at\ r=c \qquad (14)$$

Since equation (12) provides the reaction forces developed between adjacent cylinders, the corresponding forces for each pair of

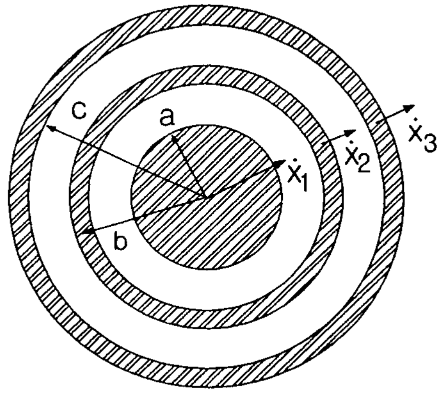

Fig. 2. Three-cylinder Motion with Fluid Couples

cylinders could be expressed as following equations (15) through (17). Then, equation (15) only indicates mutual influence of the inner and outer cylinder without consideration of the middle cylinder.

$$\begin{Bmatrix} F_{f1} \\ F_{f3} \end{Bmatrix} = -\begin{bmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{Bmatrix} \qquad (15)$$

$$\begin{Bmatrix} F_{f2} \\ F_{f3} \end{Bmatrix} = -\begin{bmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{bmatrix} \begin{Bmatrix} \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} \qquad (16)$$

$$\begin{Bmatrix} F_{f1} \\ F_{f2} \end{Bmatrix} = -\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} \qquad (17)$$

The total forces on each cylinder could be defined by summing the reaction forces in equations (15) through (17). By arranging the summed reaction forces with respect to the generalized coordinate, the equation of motion could be expressed as follows.

$$\begin{Bmatrix} F_a \\ F_b \\ F_c \end{Bmatrix} = -\begin{bmatrix} M_{11}^{<a-b>} + M_{11}^{<a-c>} & M_{12}^{<a-b>} & M_{13}^{<a-c>} \\ M_{21}^{<a-b>} & M_{22}^{<a-b>} + M_{22}^{<b-c>} & M_{23}^{<b-c>} \\ M_{31}^{<a-c>} & M_{32}^{<b-c>} & M_{33}^{<a-c>} + M_{33}^{<b-c>} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} \qquad (18)$$

,where, F_i is the resulting force on each cylinder, and $M_{kk}^{<i-j>}$ is the added mass resulting from each fluid gap. $M_{kk}^{<i-j>}$ could be calculated from the equation (12) by replacing the indices with a, b and c.

2.3. Definition of the Equivalent Mass

2.3.1. Case of Two Cylinders

From the equation (12), two equations defining the forces on cylinders could be derived:

$$-F_{f1} = (M_{11}\ddot{x}_1 + M_{12}\ddot{x}_2),\quad -F_{f2} = (M_{21}\ddot{x}_1 + M_{22}\ddot{x}_2) \qquad (19)$$

Previous studies suggest that the lowest frequencies of the inner cylinder in beam mode could be found by considering the outer cylinder to be static, $\ddot{x}_2=0$. And the frequencies relevant to the outer cylinder come out when two cylinders vibrate in the same phase, $\ddot{x}_1 = \ddot{x}_2$. The equation (19) results in the equation (20) after substituting the terms defined in the equation (12) and considering the direction of motion.

$$-F_{f1} = M_{11}\ddot{x}_1 = \pi\rho La^2\left(\frac{b^2+a^2}{b^2-a^2}\right)\ddot{x}_1$$

$$\begin{aligned} -F_{f2} &= (M_{21}\ddot{x}_2 + M_{22}\ddot{x}_2) \\ &= \pi\rho L\left(-2a^2\left(\frac{b^2}{b^2-a^2}\right)+b^2\left(\frac{b^2+a^2}{b^2-a^2}\right)\right)\ddot{x}_2 \\ &= \pi\rho Lb^2\ddot{x}_2 \end{aligned} \tag{20}$$

From equation (20), one can determine the amount of added mass to estimate the lowest frequency of each cylinder in beam mode vibration. Equation (20) also means that the amount of added mass is highly affected by the distance between cylinders, or the fluid gap size. A dimensionless parameter, C_m, indicating the distance between two cylinders, is then introduced as follows.

$$C_m = \left(\frac{b^2+a^2}{b^2-a^2}\right) \tag{21}$$

For example, a large value of C_m denotes that the size of fluid gap is very small.

2.3.2. Case of Three Cylinders

In case of three cylinders, the same idea used for two cylinders is adopted. Since the assumption on the direction of motion controls the amount of added mass, several sets of assumed motion for cylinders are prepared to trace the exact solution.

2.3.2.1. Case of Inner Cylinder

To determine the added mass of inner cylinder, total three cases are considered. The first case considers the outer cylinder to be static, and the middle cylinder is considered to vibrate in out-of-phase with respect to the inner cylinder. From the equation (18), the resulting reaction force on the inner cylinder shall be as follows.

Case 1; $\ddot{x}_3 = 0,\ \ddot{x}_1 = -\ddot{x}_2$,

$$\begin{aligned} -F_a &= \left(M_{11}^{<a-b>} + M_{11}^{<a-c>}\right)\ddot{x}_1 - M_{12}^{<a-b>}\ddot{x}_1 \\ &= \pi a^2 L\rho\left(\frac{b^2+a^2}{b^2-a^2}+\frac{c^2+a^2}{c^2-a^2}\right)\ddot{x}_1 + 2\pi b^2 L\rho\frac{a^2}{b^2-a^2}\ddot{x}_1 \end{aligned} \tag{22}$$

In the next place, the motion of all cylinders is considered and assumed to be in out-of-phase. Then, this case tends to provide more amount of mass than any other case, and the resulting equation is as follows.

Case 2; $\ddot{x}_3 \neq 0,\ \ddot{x}_1 = -\ddot{x}_2,\ \ddot{x}_3 = -\ddot{x}_1$

$$\begin{aligned} -F_a &= \left(M_{11}^{<a-b>} + M_{11}^{<a-c>}\right)\ddot{x}_1 - M_{12}^{<a-b>}\ddot{x}_1 - M_{13}^{<a-c>}\ddot{x}_1 \\ &= \pi a^2 L\rho\left(\frac{b^2+a^2}{b^2-a^2}+\frac{c^2+a^2}{c^2-a^2}\right)\ddot{x}_1 + 2\pi L\rho\left(\frac{b^2a^2}{b^2-a^2}+\frac{c^2a^2}{c^2-a^2}\right)\ddot{x}_1 \end{aligned} \tag{23}$$

The last case considers diagonal terms only as shown in equation (24) for the purpose of providing less conservative value.

Case 3; Consider only diagonal terms

$$\begin{aligned} -F_a &= \left(M_{11}^{<a-b>} + M_{11}^{<a-c>}\right)\ddot{x}_1 \\ &= \pi a^2 L\rho\left(\frac{b^2+a^2}{b^2-a^2}+\frac{c^2+a^2}{c^2-a^2}\right)\ddot{x}_1 \end{aligned} \tag{24}$$

2.3.2.2. Case of the Middle Cylinder

At first, the motion of inner cylinder is assumed to be in in-phase with respect to the middle cylinder, and the motion of the outer cylinder is assumed to be in out-of-phase. Next, the outer cylinder is considered to be static but other conditions are maintained. Finally the diagonal terms are summed to provide a minimal case.

Case 1; $\ddot{x}_3 \neq 0$, $\ddot{x}_1 = \ddot{x}_2$, $\ddot{x}_3 = -\ddot{x}_2$

$$-F_b = M_{21}^{<a-b>}\ddot{x}_2 + \left(M_{22}^{<a-b>} + M_{22}^{<b-c>}\right)\ddot{x}_2 - M_{23}^{<b-c>}\ddot{x}_2$$

$$= -2\pi L\rho\frac{a^2b^2}{b^2-a^2}\ddot{x}_2 + \pi b^2 L\rho\frac{b^2+a^2}{b^2-a^2}\ddot{x}_2 \qquad (25)$$

$$+\pi b^2 L\rho\frac{c^2+b^2}{c^2-b^2}\ddot{x}_2 + 2\pi L\rho\frac{c^2b^2}{c^2-b^2}\ddot{x}_2$$

Case 2; $\ddot{x}_3 = 0$, $\ddot{x}_1 = \ddot{x}_2$

$$-F_b = M_{21}^{<a-b>}\ddot{x}_2 + \left(M_{22}^{<a-b>} + M_{22}^{<b-c>}\right)\ddot{x}_2 \qquad (26)$$

$$= -2\pi L\rho\frac{a^2b^2}{b^2-a^2}\ddot{x}_2 + \pi b^2 L\rho\frac{b^2+a^2}{b^2-a^2}\ddot{x}_2 + \pi b^2 L\rho\frac{c^2+b^2}{c^2-b^2}\ddot{x}_2$$

Case 3; Consider only diagonal terms

$$-F_b = \left(M_{22}^{<a-b>} + M_{22}^{<b-c>}\right)\ddot{x}_2 \qquad (27)$$

$$= \pi b^2 L\rho\frac{b^2+a^2}{b^2-a^2}\ddot{x}_2 + \pi b^2 L\rho\frac{c^2+b^2}{c^2-b^2}\ddot{x}_2$$

2.3.2.3. Case of Outer Cylinder

The direction of motion for all cylinders is assumed to be in the same direction in accordance to the case of two cylinders. The resulting equation (28) indicates that the effect of fluid for this case is double of total mass of the fluid inside the outer cylinder. The second case only considers the diagonal terms to provide a guide for the first case.

Case 1; $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}_3$

$$-F_c = M_{31}^{<a-c>}\ddot{x}_3 + M_{32}^{<b-c>}\ddot{x}_3 + \left(M_{33}^{<a-c>} + M_{33}^{<b-c>}\right)\ddot{x}_3$$

$$= \pi L\rho\left\{-2\left(\frac{c^2a^2}{c^2-a^2}\right) - 2\left(\frac{c^2b^2}{c^2-b^2}\right) + c^2\left(\frac{c^2+a^2}{c^2-a^2} + \frac{c^2+b^2}{c^2-b^2}\right)\right\}\ddot{x}_3$$

$$= \pi L\rho\left\{c^2\left(\frac{c^2-a^2}{c^2-a^2} + \frac{c^2-b^2}{c^2-b^2}\right)\right\}\ddot{x}_3 \qquad (28)$$

$$= 2\pi L\rho c^2\ddot{x}_3$$

Case 2; Consider only diagonal terms

$$-F_c = \left(M_{33}^{<a-c>} + M_{33}^{<b-c>}\right)\ddot{x}_3 \qquad (29)$$

$$= \pi L\rho c^2\left(\frac{c^2+a^2}{c^2-a^2} + \frac{c^2+b^2}{c^2-b^2}\right)\ddot{x}_3$$

3. Eigenvalue Analyses

Dynamic analyses to find the natural frequencies of cylinders coupled with the fluid gaps are performed over the various fluid gap sizes. First, the full analysis model which consists of structural

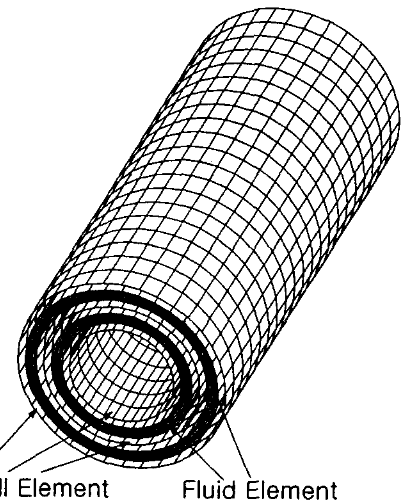

Fig. 3. Typical Full F.E. Model for Three Cylinders

Table 1. Examples of C_m Value Used for Eigenvalue Analyses

b	c	Cm (a-b)	Cm (b-c)	Cm (a-c)
0.15	0.20	6.29	8.78	2.26
0.15	0.25	6.29	2.95	1.63
0.20	0.25	2.26	11.27	1.63
0.20	0.30	2.26	3.64	1.40
0.50	0.55	1.12	26.26	1.10

*) Radius of inner cylinder(a) = 0.1m
Thickness of cylinder(t) = 0.03m
Length of cylinder (L)= 1.0m / 2.0 m

and fluid elements is built using the commercial program ANSYS[9] to constitute reference values. Since many authors provide the validity of ANSYS through numerous studies, the results of the program is believed to be close to the exact solution. The cylinders are modeled with shell elements at the middle plane. Fig. 3 shows a typical finite element model for three cylinders with two fluid gaps. One of the ends of the cylinder is fully fixed to provide a cantilever condition and the property of water is adopted for analyses. The elastic modulus of the cylinders is 195.0E9Pa with density of 7800kg/m^3, and the bulk modulus of 2.07E9Pa [9] is used for the water. The radius of the inner cylinder remains constant to determine the values of C_m, and Table 1 lists the example value of C_m for three cylinders used for analyses. The added mass defined through equations (19) to (27) is transformed into structural density by dividing the mass with the volume of each cylinder, and the eigenvalue analyses over various sizes of fluid gaps are then performed.

3.1. Results of Two Cylinders

Table 2 and 3 list the fundamental frequencies

Table 2. Comparison of Frequencies (a/L=0.1)

C_m	Full F.E. Model		Equivalent Model		Difference (%)	
	1st Mode	2nd Mode	1st Mode	2nd Mode	1st Mode	2nd Mode
12.02	97.6	222.3	89.2	222.1	9.4%	0.1%
6.29	120.0	230.8	112.0	232.2	7.1%	0.6%
2.26	156.3	273.6	146.0	272.8	7.1%	0.3%
1.39	166.7	329.5	158.4	315.8	5.2%	4.3%
1.20	167.4	352.4	161.6	328.8	3.6%	7.2%
1.12	166.3	357.4	162.9	327.3	2.1%	9.2%

Table 3. Comparison of Frequencies (a/L=0.05)

C_m	Full F.E. Model		Equivalent Model		Difference (%)	
	1st Mode	2nd Mode	1st Mode	2nd Mode	1st Mode	2nd Mode
12.02	23.9	59.8	23.3	60.4	2.6%	1.0%
6.29	29.4	62.6	29.3	63.8	0.3%	1.9%
2.26	39.3	78.1	38.2	79.5	2.9%	1.8%
1.39	42.7	105.2	41.4	103.1	3.1%	2.0%
1.20	43.4	124.4	42.2	118.4	2.8%	5.1%
1.12	43.7	137.2	42.6	127.7	2.6%	7.4%

of two cylinders from both full F.E. analyses and analyses with the equivalent mass. In the tables, the 1st mode means the frequencies for the inner cylinder and the 2nd refers the frequencies of the outer cylinder. The general trend shows that the proposed method well pursuits the beam mode frequencies of two cylinders within a 10% of deviation. There is not any prominent trend in deviation from the exact value, but the longer cylinder shows less difference in general.

3.2. Results of Three Cylinders

Fig. 4 through 6 compares the results of each equation with those of full F.E. analyses. Since the equations (21) through (27) provide different amount of the added mass for each cylinder, the validity of each equation is reviewed to develop comprehensive equations. For example, in case of inner cylinder, the equation (22) in Fig. 4 is selected as a representative equation. The equations (25) and (28) are also selected as representative equations after reviewing the coverage of each equation over the F.E. results. Although the representative equations are believed to follow the general trend of F.E. analysis results, much deviation still exist in certain ranges of geometry. The main reason for these deviations resides in the assumption that the motion of each cylinder resembles that of a rigid body with same magnitude. Therefore, the amount of the added mass does not match the exact value in certain region and the representative equations return less or more conservative results. Then, modification of representative equation to catch up with the full F.E. solution is needed.

Case 1; Inner cylinder

The sensitivity of equation (22) is found to be affected by the second term, the interaction of the inner and middle cylinder, as shown in Fig. 4. As the radius of the middle cylinder becomes large, the equation agrees with the finite element results. In addition, any severe transients in frequencies are not monitored according to the variation of fluid gap size between the middle and outer cylinder. Thus, it is believed that the gap between the inner and middle cylinder have major influence upon the case. The modification is done by introducing a gap parameter C_{fa}.

$$m_a = \pi a^2 L\rho\left(\frac{b^2+a^2}{b^2-a^2}+\frac{c^2+a^2}{c^2-a^2}\right)+2\pi b^2 L\rho\frac{a^2}{b^2-a^2}C_{fa} \qquad (30)$$

,where, $C_{fa}=\dfrac{C_m^{<a-b>}-1}{C_m^{<a-b>}}$ and $C_m^{<a-b>}=\dfrac{b^2+a^2}{b^2-a^2}$.

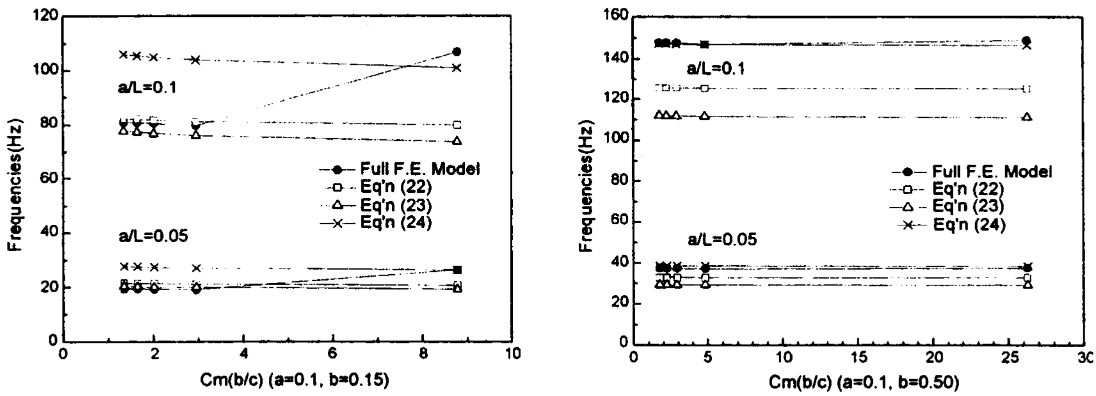

Fig. 4. Typical Results for Inner Cylinder

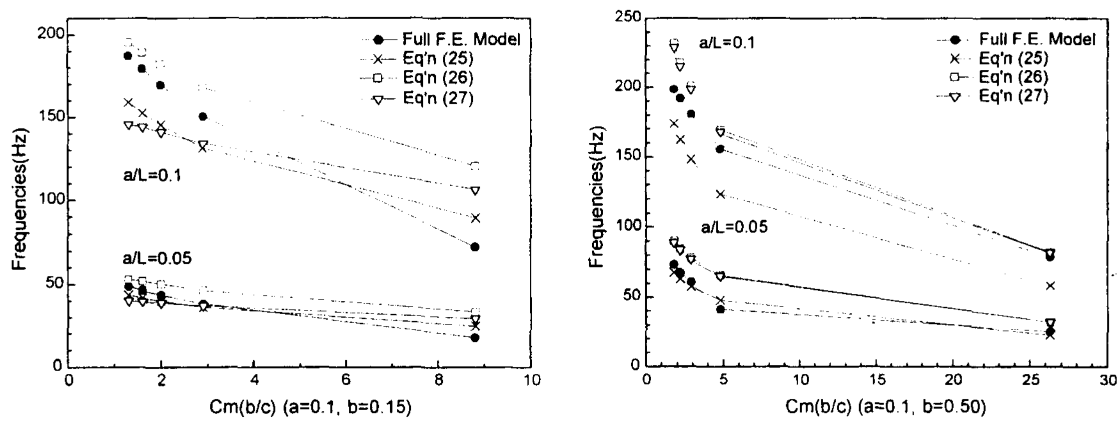

Fig.5. Typical Results for Middle Cylinder

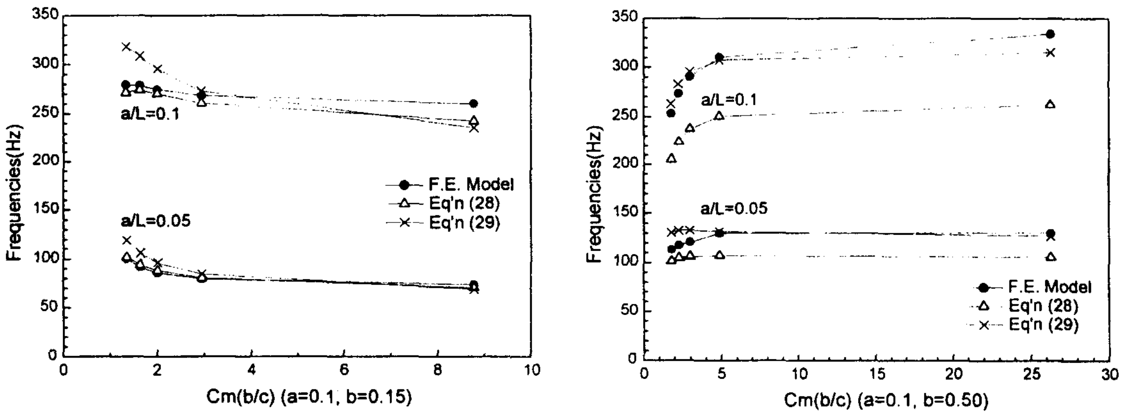

Fig. 6. Typical Results for Outer Cylinder

Case 2; Middle cylinder

The equation (25) is mainly affected by the gap between the middle and outer cylinder as shown in Fig. 5. Since the motion of the middle cylinder is sensitive due to the presence of inner and outer cylinder, the estimated frequencies are rather far from the finite element analysis results. Fig. 5 also reveals that the equation (25) tends to provide more conservatism than the equation (27). Then, the last term of the equation (25), interaction of middle and outer cylinder, shall be adjusted to reduce the conservatism.

$$m_b = -2\pi L\rho\frac{a^2b^2}{b^2-a^2} + \pi b^2 L\rho\frac{b^2+a^2}{b^2-a^2} + \pi b^2 L\rho\frac{c^2+b^2}{c^2-b^2} + 2\pi L\rho\frac{c^2b^2}{c^2-b^2}C_{fb} \quad (31)$$

,where, $C_{fb} = \frac{Cm^{<b-c>}-1}{Cm^{<b-c>}}$ and $C_m^{<b-c>} = \frac{c^2+b^2}{c^2-b^2}$.

Case 3; Outer cylinder

Fig. 6 shows that the equation (28) is less affected by the gap between the middle and outer cylinder. The equation provides more conservatism when the distance of inner and middle cylinder is large. Then, a coefficient to account for the interaction between the inner and middle cylinder is introduced.

$$m_c = (1 + C_{fc})\pi L \rho c^2 \tag{32}$$

,where, $C_{fc} = \dfrac{C_m^{<a-b>} - 1}{C_m^{<a-b>}}$ and $C_m^{<a-b>} = \dfrac{b^2 + a^2}{b^2 - a^2}$.

3.3. Results of the Modified Equations

Fig. 7 through 15 compares the results of modified equations with initial ones. Each figure shows that the modified equations are much closer to the F.E. results than the initial ones. The maximum deviation is found to be less than 30% throughout the whole cases, and the cases with smaller fluid gap show larger deviation in general.

In case of inner cylinder, Fig. 7 through 9 shows that the modified equation well traces the full F.E. results except the case when the gap of all cylinders is small, Fig. 7 and 8. However, this tendency disappears as the gap between the inner and middle cylinder increases, Fig. 9, nevertheless the gap between the middle and outer cylinder decreases. This is because the reaction forces on

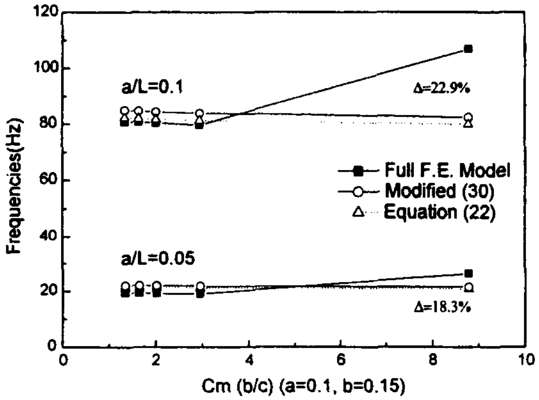

Fig. 7. Results of Inner Cylinder

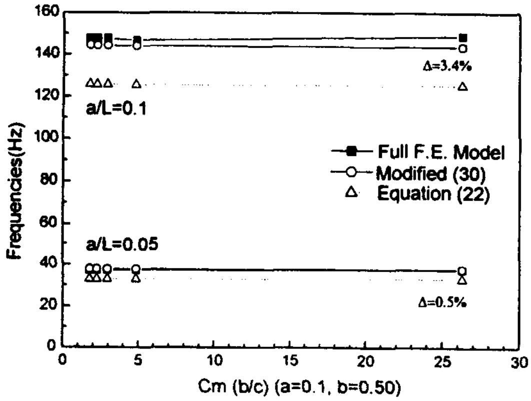

Fig. 9. Results of Inner Cylinder

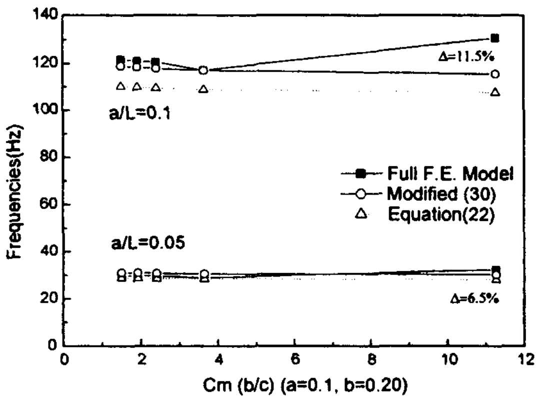

Fig. 8. Results of Inner Cylinder

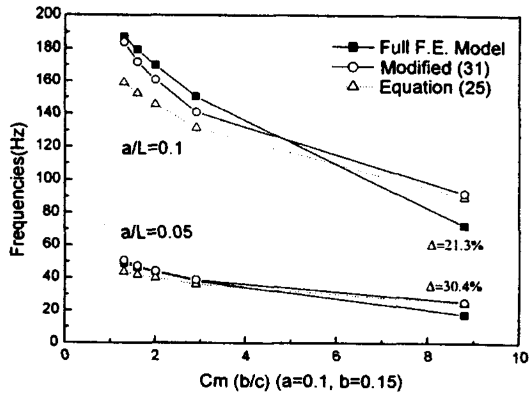

Fig. 10. Results of Middle Cylinder

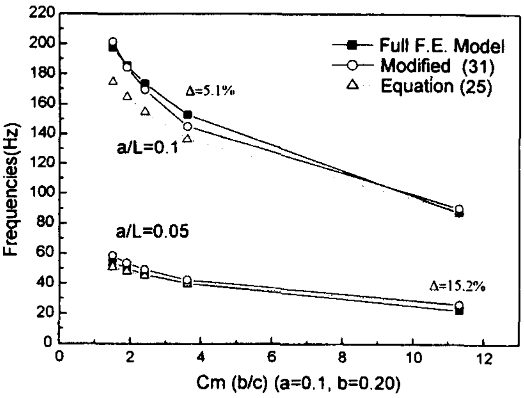

Fig. 11. Results of Middle Cylinder

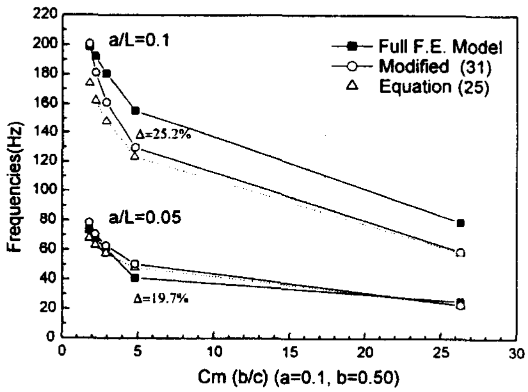

Fig. 12. Results of Middle Cylinder

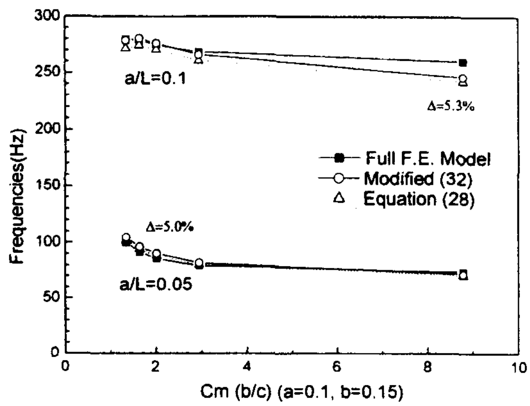

Fig. 13. Results of Outer Cylinder

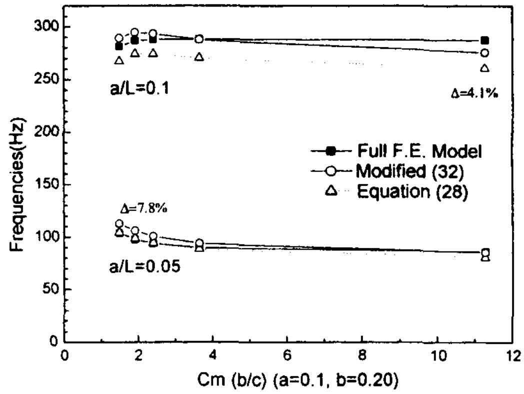

Fig. 14. Results of Outer Cylinder

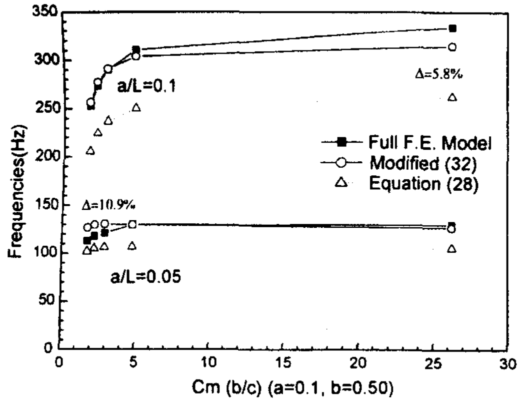

Fig. 15. Results of Outer Cylinder

the inner cylinder lessen due to the interaction of middle and outer cylinder when the gaps between all cylinders are small. Whereas, the reaction forces on the middle cylinder might increase, and Fig. 10 shows the lower frequencies at the corresponding region.

In case of the middle cylinder, the modified equation shows 30% of the maximum deviation. However, the modified equation follows the general trend of F.E. as a whole. In Fig. 12, it is found that the results depend upon the length of cylinder. Because the ratio of the diameter of outer cylinder to the length is greater than 1.0 for this case, the current equations shows a limitation over this region.

In case of the outer cylinder, less than 11% of deviations are monitored. Fig. 13 through 15 shows rather higher frequencies as the gap between the middle and outer cylinder is small. This trend vanishes when the length of each cylinder increases. The appearance of higher frequencies comes from the assumption that all cylinders vibrate in the same phase. As the gap between the middle and outer cylinder decreases, the out-of phase vibration might occur. Then, the same phenomenon as mentioned for the inner cylinder is expectable, but the strength of such a reverse effect is found to be less severe.

4. Conclusions

This study proposes an effective method to replace the added mass matrix due to FSI with the equivalent mass or density in carrying out the eigenvalue analyses for cylinders immersed in fluid. First, the equivalent mass for two cylinders having a fluid gap is introduced, and the theory is extended to three cylinders coupled with two fluid gaps. Since the current method only refers the magnitude of the added mass for eigenvalue analyses, the analysis procedure is highly simplified. In general, the proposed method shows less than 10% of deviation from the full F.E. results, and maximum of 30% deviation is found at specific region due to complicated interaction. When the gap between adjacent cylinders is small, the F.E. results show that the motion of a cylinder could be blocked by the other cylinder due to mutual interaction. Such blocking phenomenon results in rather higher frequencies and is found to decrease as the length of the cylinder increases. Since the current method is devoted to estimation of the beam mode frequencies of cylinders without hydraulic coupling term, direct application to the dynamic response analysis shall be limited.

Acknowledgement

This project has been carried out under Nuclear R&D program by MOST.

References

1. S.J. Brown, "A Survey of Studies into the Hydrodynamic Response of Fluid-Coupled Circular Cylinders", J. of Pressure Vessel Technology, Vol.104, No.3, (1982).
2. S.S. Chen and G.S. Rosenberg, "Dynamics of a Coupled Shell-Fluid System", Nuclear Engineering and Design, Vol. 32, pp.302-310, (1975).
3. S.S. Chen and Ho Jung, "Design Guide for Calculating Hydrodynamic Mass Part I : Circular Cylindrical Structure", ANL-CT-76-45, Argonne National Laboratory, (1976).
4. M.K. Au-Yang, "Free Vibration of Fluid-Coupled Coaxial Cylindrical Shells of Different Lengths", J. of Applied Mechanics, Vol. 43, pp.480-484, (1976).
5. M.K. Au-Yang and J.E. Galford, "A Structural Priority Approach to Fluid-Structure Interaction Problem", Vol. 103, J. of Pressure Vessel Technology, pp.142-150, (1981).
6. R.J. Fritz, "The Effect of Liquids on the Dynamic Motions of Immersed Solids", J. of Engineering for Industry, pp.167-173, (1972).
7. T.R. Kim et al., "The Study on the Change in Dynamic Characteristics of Reactor Internals", KAERI/RR-1267/93, Korea Atomic Energy Research Institute, (1993).

8. S. Perov, E. Altstadt and M. Werner, "Vibration Analysis of the Pressure Vessel Internals of WWER-1000 Type Reactors with Consideration of Fluid-Structure Interaction", Vol. 27, Annals of Nuclear Energy, pp. 1441-1457, (2000).
9. ANSYS, Inc., "ANSYS User's Manual", (2001).
10. ASME, "ASME Boiler and Pressure Vessel Code", Sec III, Division 1, Appendix N Dynamic Analysis, N-1450, (1989).
11. ASCE, "Seismic Analysis of Safety-Related Nuclear Structures and Commentary on Standard for Seismic Analysis of Safety Related Nuclear Structures", (1986).