International Journal of the Korean Society of Precision Engineering, Vol. 4, No. 3, May 2003.

Compensation of Probe Radius in Measuring Free-Formed Curves and Surfaces

Lisheng Li[1], Jong-Yun Jung[2,#], Choon-Man Lee[3] and Won-Jee Chung[3]

[1] Shanghai Machine Tool Research Institute, Shanghai, P.R. China
[2] Dept. of Industrial & Systems Engineering, Changwon National University, Changwon, S. Korea
[3] Dept. of Mechanical Design & Manufacturing Engineering, Changwon National University, Changwon, S. Korea

ABSTRACT

Compensation of probe radius is required for accurate measurement in metal working industry. Compensation involves correctly measuring data on the surface in the amount of radius of the touch probe with a Coordinate Measuring Machine (CMM). Mechanical parts with free-formed curves and surfaces are complex enough so that accurate measurement and compensation are indispensable. This paper presents necessary algorithms involved in the compensation of the probe radius for free-formed curves and surfaces. Application of pillar curve is the focus for the compensation.

Key Words : Measurement, Compensation of Probe Radius, B-Spline, Pillar curve

1. Introduction

According to Kim [1], measurement with a Coordinate Measuring Machine (CMM) informs users of the position of the probe center when it contacts work-pieces. The radius of the probe is of the same length as the distance from the center of the probe to the touch point. Lee [2] identifies that it is necessary to compensate for the probe radius when data are measured. According to ANSI-CAM-I [3], the direction of the vector from contact point to the probe center is of the same direction as the normal vector of a measuring point on a surface, which however, is not known. This research presents the procedure to identify the method of finding the normal vector for the compensation of the probe radius.

B-spline can be found in Shi [4] and Choi [5]. Least square According to Li [6], B-spline fitting for free-formed curves and surfaces are used for the compensation. One of the properties of B-spline is that it is differentiable in the parameter segment between knot points, and (k-r) degree differentiable on the knots, that is, C^{k-r} — (k-r) parameter continuity. Here k is the degree of B-spline, and the r is the multiplicity of knots. In the reverse calculation of B-spline, especially in application to metrology, the r is equal to 1. In general, a cubic B-Spline is the most commonly used, i.e. k is equal to 3. In other words, even at the knot points, the second partial derivatives can be calculated. As it is known, the first partial derivatives are the tangent vectors homologous to the relative parameter direction, which is perpendicular to normal the vector of the surface. Figure 1 shows a normal vector and tangent vectors on the point of a surface.

Corresponding Author :
Email : jyjung@sarim.changwon.ac.kr
Tel. +82-55-279-7545

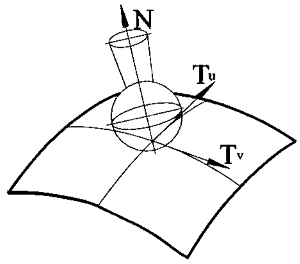

Fig. 1 Normal vector & tangent vector

This property shows an idea to compensate for the probe radius on free-formed curves and surfaces. The locus of probe center in measuring the free-formed surface or curve is different from the one in measuring a convex shape or a convex curve. Examples of convex ones are spheres, cylinders, cones planes, circles, and lines. Their locus is off as much as probe radius to the normal direction than their surface. Thus, the modification of tangent vectors should be made for accurate calculation of a normal vector. In such cases, Xiong [7] calculates Hesse matrix that is a second partial derivatives matrix is to be calculated. Even in this situation, the Cubic B-spline curve or bi-cubic B-spline surface where r is equal to 1 satisfies the requirements of differentiation.

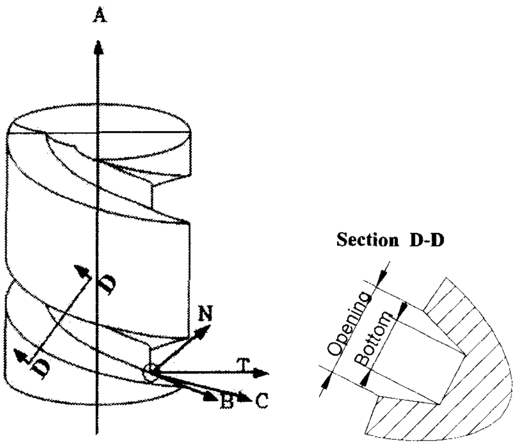

Fig. 2 Vectors in a pillar curve and cam

Pillar curve is a special type of spatial curve, which lies on a cylinder surface. When the pillar curve cannot be expressed with a definite mathematical expression, it belongs to a free curve. The pillar cam is a typical application of a spatial curve. The pillar cam has an angular contour, which is shown in Figure 2. Since the depth of contour in the cam is too narrow to be measured, special consideration should be taken into for compensation of the probe radius for measuring pillar curves.

Since every direction vector must be a unit vector, its magnitude should be 1. That is $\sqrt{p^2+q^2+r^2}=1$, where (p, q, r) is the components of a unit vector or simply $p^2+q^2+r^2=1$.

2. B-Spline and its Tangent Vectors

2.1 Curve

B-spline curve is expressed by the following equation:

$$P(u)=\sum_{i=0}^{n} d_i N_{i,k}(u) \tag{1}$$

where $d_i, i=0,1,\cdots,n$ are control vertices, or de Boor points; $N_{i,k}(u), i=0,1,\cdots,n$ are k degree standard B-spline basis functions. *P(u)* is the point on the curve, which is determined by the parameter value *u*. The de Boor recursive formula can be found in Shi [4] and it is re-written in the following equation:

$$P(u)=\sum_{j=i-k+1}^{i} d_j^l N_{j,k-l}(u)=\cdots=d_j^k$$

$$d_j^l=\begin{cases} d_j & l=0 \\ (1-\alpha_j^l)d_{j-1}^{l-1}+\alpha_j^l d_j^{l-1} & l=1,2,\cdots,k \\ & j=i-k+l,\cdots,i \end{cases}$$

$$\alpha_j^l=\frac{u-u_j}{u_{j+k+1-l}-u_j} \tag{2}$$

where *u* with a subscript is a knot point in the parameter domain and *u* without a subscript is the parameter value in *i-th* segment. The point on the curve can be expressed as a function of the parameter $u\in[u_i,u_{i+1})\subset[u_k,u_{n+1}]$.

Suppose the B-spline is formulated in equation (2) with data points. With the similar de Boor formula in Piegl [8],

$$p^{(r)}(u)=\sum_{j=i-k+r}^{i} d_j^l N_{j,i-r}(u) \quad u\in[u_i,u_{i+1}]$$

$$d_j^l=\begin{cases} d_j & l=0 \\ (k-l+1)\dfrac{d_j^{l-1}-d_{j-1}^{l-1}}{u_{j+k+1-l}-u_j} & l=1,2,\cdots,r \\ & j=i-k+l,\cdots,i \end{cases} \tag{3}$$

From the above equation we can calculate the *r* degree derivative vectors of the B-spline curve. For compensation of a probe radius, only derivative vectors on knot points are utilized for calculation.

2.2 Surface

The control vertices of a B-spline surface lie on a control grid, topologically homologous to the rectangular matrix of a data point, instead of the control polygon in curve fitting. It is expressed as

$$P(u,v) = \sum_{i=0}^{m+k-1} (\sum_{j=0}^{n+l-1} d_{i,j} N_{j,l}(v)) N_{i,k}(u) \tag{4}$$

When fixing either v or u, we can calculate the partial derivatives $P_u^{(r)}(u,v)$ and $P_v^{(r)}(u,v)$, respectively. The first partial derivatives are the tangent vectors T_u and T_v in Figure 1. The normal vector of a free surface can be calculated if the surface is fitted. The normal vector is the vector product of T_u and T_v.

$$N = T_u \times T_v \tag{5}$$

3. Planar Curve

However, for any curve in equation (1), only one tangent vector can be obtained on every data point. In order to get the normal vector of the curve, another vector that is perpendicular both to the normal vector and the tangent vector is required. With a Frenet moving frame in Farin [9] that is a local coordinate sub-system, it becomes easier to constructing this vector. The origin of the frame is located at the current point on the curve and the coordinate axes are the unit vector ***T***, ***N*** and ***B***, which are perpendicular to one another. When a parameter continuously changes, this sub-coordinate should continuously translate and/or rotate. The ***T***, ***N*** and ***B*** are the tangent vector, normal vector and bi-normal vector on the current point of the curve, respectively. They are shown in Figure 3. In a right-handed system, the following equation for vector product is satisfied as follows:

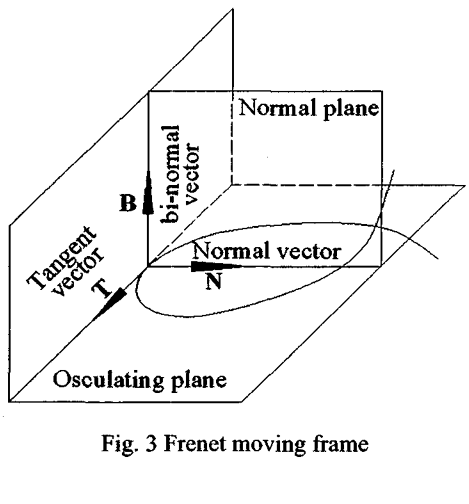

Fig. 3 Frenet moving frame

$$\boldsymbol{N} = \boldsymbol{B} \times \boldsymbol{T} \tag{6}$$

For planar curves, ***B*** is the normal vector of the plane on which the curve lies. ***T*** is the tangent vector calculated according to equation (3) when the r is equal to 1. This is shown in Figure 4.

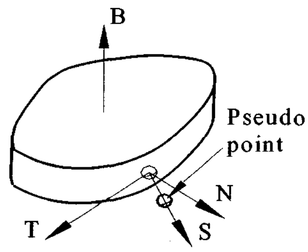

Fig. 4 Compensation on a planar curve

In Figure 4, suppose that there is another vector ***S*** which is pointing from a contact point to one point outside the body. The outside point is the pseudo point that can be sampled in the air (not on the measured surface). The scalar a is calculated as :

$$a = \boldsymbol{B} \times \boldsymbol{T} \bullet \boldsymbol{S} \tag{7}$$

If $a > 0$, the angle θ between the ***S*** and ***N*** is smaller than 90° ; otherwise larger than 90° . One assumption is that we look at the curve from the top of the arrow ***B***, if $\theta < 90°$, the probe is located on the left hand side of the curve; otherwise it is on the right side of the curve. This research sets the following equations for directional position.

$$\begin{cases} f = +1, & a > 0 \\ f = -1, & a < 0 \end{cases} \tag{8}$$

The physical meaning of equation (8) is that if the direction of the normal vector N calculated from equation (6) does not point outward from the real body, the direction of the vector is reversed. The contact point is calculated from the center of the probe according to the following equation:

$$P_{contact} = P_{probe} - f * r * N \quad (9)$$

where the r is the radius of the probe. Mortenson [10] shows that if the curve is not self-intersected, only one judgment from equation (7) and (8) is required for examining the curve.

4. Pillar Curve with Angular Contour

4.1 Convention

In Figure 2 the following notations are used to identify the problem:

T: tangent vector of a point;

A: axis vector of the cylinder;

B: bi-normal vector, indicating the inclination of the angular contour;

C: center line of cutting tool or symmetry line of both sides of the groove in a normal section;

N: normal vector of a point.

The vectors **B**, **C** and **N** are located in the plane perpendicular to vector **T**. Vector **C** is perpendicular to vector **A** and γ is the angle between vector **B** and **C**. The direction of this angle is defined from vector **C** to vector **B**. The normal width of the groove on the cylindrical surface is wider than the bottom shown in Figure 2. The direction of vectors **C** and **B** must point outward from the axis of cylinder.

4.2 Vector relationship

Because $\boldsymbol{C} \perp \boldsymbol{A}$ and $\boldsymbol{C} \perp \boldsymbol{T}$, the following equation is satisfied:

$$\boldsymbol{C} = \boldsymbol{T} \times \boldsymbol{A} \quad (10)$$

If **C** does not point outward from the axis, its direction should be reversed. The relationship between vectors **C**, **B** and **A** are not changed as long as the measuring path remains in one direction without being reversed. Thus only one confirmation is required for a whole path of measurement. It could be on the first or the last point. To keep this relationship constant, if the direction of C is reversed at the first point, A must be also reversed. The **C** vector is normalized, its components are (p_c, q_c, r_c). Because **B** and **C** are located in the plane perpendicular to **T**, the following equation is satisfied:

$$\boldsymbol{T} = \boldsymbol{C} \times \boldsymbol{B} \quad (11)$$

4.3 Calculation of the vectors

Equation (10) is transformed into the following form.

$$\begin{cases} p_t \sin\gamma = q_c r_b - q_b r_c & (12) \\ q_t \sin\gamma = r_c p_b - r_b p_c & (13) \\ r_t \sin\gamma = p_c q_b - p_b q_c & (14) \end{cases}$$

Since only two equations of (12), (13) and (14) are linearly independent of each other, an additional equation in the following form is required.

$$p_b^2 + q_b^2 + r_b^2 = 1 \quad (15)$$

For example, (p_b, q_b, r_b) can be derived with the following formula by combining equations (12), (13) and (15):

$$\begin{cases} r_b = (q_c p_t - q_t p_c)\sin\gamma \\ \pm\sqrt{[(q_c p_t - q_t p_c)^2 - (p_t^2 + q_t^2)]\sin^2\gamma + r_c^2} \\ p_b = \dfrac{(r_b p_c + q_t \sin\gamma)}{r_c} \\ q_b = \dfrac{(r_b q_c - p_t \sin\gamma)}{r_c} \end{cases} \quad (16)$$

The two equations that produce maximum absolute value for (p_c, q_c, r_c) should be selected from equations (12), (13), and (14) in order to avoid dividing by zero or reducing accuracy in calculation.

The sign, $\pm$, in front of the square root in equation (16) should be selected according to the sign of r_c. That is, we select positive sign if $r_c > 0$, and negative otherwise. Normally the angle between **C** and **B** is quite small. The scalar product of vector **C** and **B** thus must be positive.

Similarly to the case of the planar curve, we set a pseudo point outside the physical body and near the first point. Vector **S** is constructed from the first contact point to this pseudo point. The scalar $\boldsymbol{a}$ is calculated according to equation (7). However during the procedure of determining, $\sin\gamma$, the bi-normal vector $\boldsymbol{B}$ is unknown. We therefore calculate the scalar $\boldsymbol{a}$ with the following equation instead of equation (7).

$$a = C \times T \bullet S \tag{17}$$

Similarly, we look at the curve from the top of the bi-normal vector $\boldsymbol{B}$. If $\alpha > 0$, then the probe is located on the left hand side of the curve. Otherwise, it is on the right side of the curve. Equations (8) and (9) are also applied to pillar curves.

Equation (16) means the rotation of $\boldsymbol{C}$ with angle of γ degrees around $\boldsymbol{T}$ to obtain the bi-normal vector $\boldsymbol{B}$, of with the rotating direction not known. Because the normal width of the groove is always wider on the cylindrical surface than at the bottom, as shown in Figure 2, the angular contour must be rotated $|\gamma|$ degrees inside the cylindrical body. This is the same principle as the one used in compensation of the probe radius. Equation (16) is rewritten in the following form:

$$\begin{cases} r_b = f(q_c p_t - q_t p_c)\sin\gamma \\ \pm\sqrt{[q_c p_t - q_t p_c)^2 - (p_t^{\,2} + q_t^{\,2})]\sin^2\gamma + r_c^{\,2}} \\ p_b = \dfrac{(r_b p_c + q_t f\ \sin\gamma)}{r_c} \\ q_b = \dfrac{(r_b q_c - p_t f\ \sin\gamma)}{r_c} \end{cases} \tag{18}$$

The physical meaning of such change is rotating $\boldsymbol{C}$ by γ degrees around $\boldsymbol{T}$ in the counter-clockwise direction to get the bi-normal vector $\boldsymbol{B}$. It is seen from the top of the bi-normal vector $\boldsymbol{B}$, and vector $\boldsymbol{S}$ faces to the direction of vector N. Otherwise, $\boldsymbol{C}$ must be rotated clockwise.

4.4 Procedure for compensation

The following paragraph contains the procedural steps for the compensation of the probe radius.

1) Input the axis vector A via keyboard or measure it on the work piece.

2) Input rotating angle γ via keyboard or measure it on the work piece.

3) Sample data along the curve.

4) Fit them into B-spline utilizing equation (2) and (3).

5) Calculate the *rth* derivatives with the data points measured.

6) Construct vector $\boldsymbol{S}$ by sampling a pseudo point near the first or the last point automatically with a measuring machine.

7) Construct vector $\boldsymbol{C}$ perpendicularly to vector $\boldsymbol{A}$ through each measured point.

8) Calculate scalar $\boldsymbol{a}$ and determine f from equation (17) and (8).

9) Identify an applicable equation according to the value of (p_c, q_c, r_c) and calculate (p_b, q_b, r_b).

10) Calculate the normal vector (p_n, q_n, r_n) through equation (6).

11) Calculate the components of compensation with equation (9).

The procedure for the compensation of a probe radius for pillar curves requires some more notable consideration in real application. The following paragraph is an additional statement about that compensation.

4.5 Path for compensation

In measuring a pillar curve, we sample data from a definite distant on the axis of cylinder. For example it can be on the mid diameter. In this way, all data points are located on an identical cylinder. But after being compensated through equation (9), the contact points are not in the identical cylinder anymore since the normal vector N changes its direction at a different position. This is shown in Figure 5. It is better to have the axis vector $\boldsymbol{A}$ be a measuring path instead of the normal vector N.

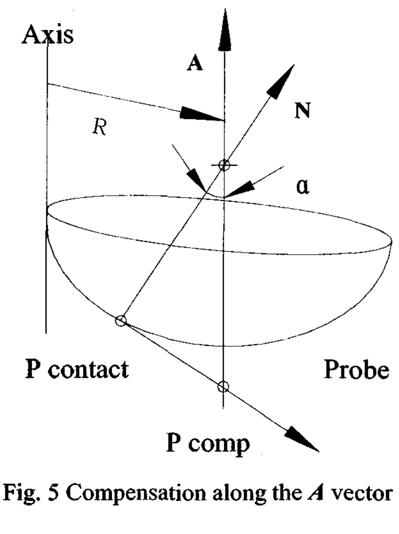

Fig. 5 Compensation along the A vector

In Figure 5 $\boldsymbol{R}$ is the distance from the measuring point to the axis of the cylinder. Equation (9) is rewritten in the following form:

$$P_{comp} = P_{probe} - \frac{f * r}{\sqrt{1-\cos^2 \alpha}} * A \tag{19}$$

where α is the angle between vectors $\boldsymbol{A}$ and $\boldsymbol{N}$. It can be calculated via the dot product of vectors $\boldsymbol{A}$ and $\boldsymbol{N}$, i.e.,

$$\cos\alpha = p_a p_n + q_a q_n + r_a r_n \tag{20}$$

5. Simulation Test

This research constructs a pillar curve in the following equations for a simulation test:

$$\begin{cases} x = 30 \cos \alpha \\ y = 30 \sin \alpha \\ z = 25 \dfrac{\alpha}{\pi} + 102 \end{cases}, \quad \alpha \in [0, \ \pi] \tag{21}$$

This is a spiral curve with a radius of 30 and its lead is 50. The angle of the angular contour γ is 7.5°. The radius of the probe is 2 and the axis vector $\boldsymbol{A}$ is (0, 0, 1).

Total data measured is 40 points (0-39th), and they are fitted with a pillar B-spline. Compensation of the probe radius is conducted along the axis that is parallel to the $\boldsymbol{A}$ axis and along the normal vector respectively. 37 points (from 0th to 36th) are uniformly interpolated, 5° of angular distance between adjacent points. They are fitted with least square spirals. The independent variable is the angle between interpolated points and the first point. The dependent variable is the height z of interpolated points from the first point. The final result compensated along the axis vector is $z = 7.957747 * \alpha + 99.9130$. When the α is replaced with 2π, z becomes 50, which is the lead length. The constant 99.9130 is just the height of the first point after conducting the compensation of probe radius, which includes the noise of influence of both the tangent vector and the angular contour γ. The maximum and minimum residual errors are less than 1×10^{-5} and the standard error of residual errors is 3.3×10^{-6}. The residual error, which evaluates the quality of regression or best fitting, is the error between measured data and the regressive curve. The distance from the interpolated points to the axis is 30. This result means all considerations mentioned above are suitable for a pillar curve with angular contour fitting and probe radius compensation. Compensation along the normal vector gets a similar result. But the distances from the interpolated points to the axis are not the same as the previous one.

The original data measured, control vertices, and compensated data are listed in the appendix. From this data we see following notable statement in order to reduce influence of residual errors.

1) By combining the influences of angular contour and the lead angle of the spiral curve, the angle between the axis vector and the normal vector becomes 16.604°. This makes the value of compensation along the axis vector 2.087 from equation (19).

2) From the uniformly interpolated data, the angular distance and height between any adjacent points are constant.

These two statements imply that the compensation is correct. The spiral curve applied for simulation in the previous paragraph is shown in a developed state in Figure 6. This explains in more detail the relationship of vector $\boldsymbol{B}$ to vector $\boldsymbol{T}$ and $\boldsymbol{N}$.

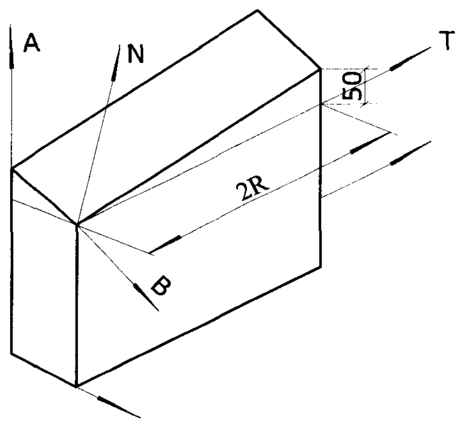

Fig. 6 Developed spiral curve

6. Concluding Remarks

B-spline is a recursive algorithm of linear interpolation. With the translated form of B-spline, the expression of free-formed curves and surfaces becomes easy. It makes the calculation fast and convergent. With the expression of the B-spline, this paper introduces the processing of surfaces and curves. Probe compensation is an example of application in this area. This research presents a procedural algorithm for the compensation of a 3D probe radius. This research finds a normal vector since we should know it for compensating a probe radius. This paper takes a spiral curve for the simulation test and shows the calculation with B-spline fitting is accurate. From the simulation results (appendix) this paper concludes that the algorithm presented reduces residual errors by a notable amount.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the Machine Tool Research Center at Changwon National University.

References

1. Kim, Sung Chung, et. al., "A Study of an OMM System for Machined Spherical form Using the Volumetric Error Calibration of Machining Center," Journal of the Korean Society of Precision Engineering, Vol. 18, No. 7, pp. 98-105, 2001.
2. Lee, Sang-Yoon and Lim, Ssang-Gun, "Three Dimensional Measurements Using Machine Vision," Journal of the Korean Society of Precision Engineering, Vol. 18, No. 3, pp. 18-22, 2001.
3. ANSI/CAM-I 101-1995, Dimensional Measuring Interface Standard (DMIS), Revision 3.0, 1995.
4. Shi, Fazhong, Computer Aided Geometric Design and Non-Uniform Rational B-spline (CAGD & NURBS), (In Chinese), Beijing University of Aeronautics and Astronautics Press, p. 266, 1994.
5. Choi, Byoung K., Surface Modeling for CAD/CAM, Elsevier, pp. 49-54, 1991.
6. Li, Lisheng, et. al., "Least Square B-Spline Fitting For Surface Measurement," submitted to the International Journal of the Korean Society of Precision Engineering.
7. Xiong, Youlen, Mathematical Methods Used in Precision Measurement (In Chinese), China Metrology press, pp. 227-230, May 1989.
8. Piegl, Les and Tiller, Wayne, The NUBRS BOOK, Springer, pp. 48-79, 1995.
9. Farin, Gerald, CAGD A Practical Guide, 4th Edition, Academic Press, pp. 173-175, 1988.
10. Mortenson, M. E., Mathematics for Computer Graphics Applications, 2nd ed., Industrial Press, pp. 83-150, 1999.

Appendix: Partially selected original data and simulation data

Original data (in part of 39 points)			Control vertices before compensation (in part of 41 points)			Compensated Data (in part of 39 Points)		
X	Y	Z	X	Y	Z	X	Y	Z
30.0000	0.0000	102.0000	30.0000	0.0000	102.0000	30.0000	0.0000	99.9130
29.9954	0.5236	102.1389	29.9985	0.1745	102.0463	29.9954	0.5236	100.0519
29.8858	2.6147	102.6944	29.9953	1.0471	102.2777	29.8858	2.6147	100.6074
29.5442	5.2094	103.3889	29.8995	2.7911	102.7407	29.5442	5.2094	101.3019
28.9778	7.7646	104.0833	29.5822	5.2161	103.3889	28.9778	7.7646	101.9963
28.1908	10.2606	104.7778	29.0145	7.7744	104.0833	28.1908	10.2606	102.6908
27.1892	12.6785	105.4722	28.2266	10.2736	104.7778	27.1892	12.6785	103.3852
25.9808	15.0000	106.1667	27.2238	12.6946	105.4722	25.9808	15.0000	104.0796
24.5746	17.2073	106.8611	26.0138	15.0191	106.1667	24.5746	17.2073	104.7741
22.9813	19.2836	107.5556	24.6058	17.2291	106.8611	22.9813	19.2836	105.4685
21.2132	21.2132	108.2500	23.0105	19.3081	107.5556	21.2132	21.2132	106.1630
19.2836	22.9813	108.9444	21.2401	21.2401	108.2500	19.2836	22.9813	106.8574
17.2073	24.5746	109.6389	19.3081	23.0105	108.9444	17.2073	24.5746	107.5519
15.0000	25.9808	110.3333	17.2291	24.6058	109.6389	15.0000	25.9808	108.2463
12.6785	27.1892	111.0278	15.0191	26.0138	110.3333	12.6785	27.1892	108.9408

Control vertices after compensation (in part of 41 points)			Interpolated Data (in part of 37 Points)		
X	Y	Z	X	Y	Z
30.0000	0.0000	99.9130	30.0000	0.0000	99.9130
29.9985	0.1745	99.9593	29.8859	2.6143	100.6073
29.9953	1.0471	100.1907	29.5443	5.2091	101.3018
29.8995	2.7911	100.6537	28.9779	7.7643	101.9962
29.5822	5.2161	101.3019	28.1909	10.2603	102.6907
29.0145	7.7744	101.9963	27.1893	12.6783	103.3851
28.2266	10.2736	102.6908	25.9809	14.9998	104.0796
27.2238	12.6946	103.3852	24.5747	17.2071	104.7740
26.0138	15.0191	104.0796	22.9815	19.2835	105.4685
24.6058	17.2291	104.7741	21.2133	21.2131	106.1629
23.0105	19.3081	105.4685	19.2838	22.9812	106.8574
21.2401	21.2401	106.1630	17.2074	24.5745	107.5518
19.3081	23.0105	106.8574	15.0001	25.9807	108.2463
17.2291	24.6058	107.5519	12.6786	27.1892	108.9407
15.0191	26.0138	108.2463	10.2607	28.1908	109.6352