Design of an Adaptive Obsever for a Class of Nonlinear Systems

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Abstract: In this paper, the problem of designing an adaptive observer for a class of nonlinear systems with linear unknown parameters is studied. The nonlinear system to be considered consists of two blocks, only one of which has measurable states. Assuming the minimum-phase property of the error dynamics obtained after a change of coordinates and imposing some conditions on the functions multiplied by unknown parameters, an adaptive observer is constructed using an existing observer design method.

Keywords: Observer, Adaptive observer, nonlinear system, minimum phase.

1. INTRODUCTION

The observer design problem arises when all of the states of a system are unknown. Besides, if the system has unknown parameters, design effort may be focused on the different objectives. One of them is that those uncertainties are attenuated or rejected: on the other hand, an on-line parameter estimator can be constructed to approximate the parameters, estimating the system states simultaneously. The design of adaptive observers concerns the latter case. Observer or adaptive observer, the state estimation error goes to zero asymptotically, while parameter estimation errors have only to be bounded for all time. When all of the states or even only partial states are available and the system contains uncertain parameters, the parameter identifier can be thought, which guarantees the convergence of parameter estimation error to zero. Parameter convergence is often achieved by the persistency of excitation (PE) condition of signals or, in some cases, only the input signal.

For a linear system, an adaptive observer is constructed in the linear parametric model. Using the concept of transfer function of the linear time-invariant (LTI) system, the parametric model can be employed so that all the signals multiplied by unknown parameters are composed of the input/output signals and/or their filtered signals, which are always available. Then, various optimization techniques or specific structures of the system are exploited to estimate states and parameters.

In the nonlinear case, systems with uncertain parameters are transformed into the form whose adaptive observer design method is known, which usually uses the error linearization or the SPR (Strictly Positive Real) condition according to the structure. The Lipschitz condition of nonlinear terms also is sometimes utilized.

This paper deals with the design of an adaptive observer for the nonlinear systems with the particular structures. The system to be considered has uncertain parameters in the dynamics whose states are unavailable as well as in the dynamics with known states. After a change of coordinates, the uncertain parameters disappear in the dynamics whose states are unmeasurable by virtue of the structure of the system assumed. By utilizing the Lipschitz condition, the adaptive observer is constructed.

Before leaving this introductory section, the organization of this paper is presented. In Section 2, several notations and definitions are given, and a brief review of results on observer and adaptive observer is described. Section 3 constructs an adaptive observer, and an example is shown in Section 4, concluding in Section 5.

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2. NOTATIONS AND BRIEF REVIEW OF PREVIOUS RESULTS

2.1. Notations and definitions

The symbol $\|\cdot\|$ denotes the Euclidean norm. Given $u(t)\in {}^m$, a $(j\times m)$-dimensional vector $\bar{u}_j$ is defined as

$$\bar{u}_j = \left[u^T \dot{u}^T \cdots (u^{(j-1)})^T\right]^T$$

which consists of u and its derivatives up to *(j-1)* times. If *f(x,u)* is a function, a difference function F is defined as

$$F(e;x,u) := f(x+e,u) - f(x,u).$$

A function $g(x)$ is said to satisfy the *locally Lipschitz condition* [1] if there exists a constant L such that

$$\|f(x) - f(y)\| \le L\|x-y\|$$

for all x and y in some neighborhood of $x_0 \in D(f)$ where $D(f)$ denotes the domain of the function f. A function $\psi(z)$ is a *positive function* if $\psi(z) > 0$ for all z. For a differentiable function $V(x,y,z)$, V is said to be *quadratic in y with z* if there are positive functions $\psi_1(z)$, $\psi_2(z)$, and $\psi_3(z)$ such that

$$\begin{aligned} &\psi_1(z)\|y\|^2 \le V(x,y,z) \le \psi_2(z)\|y\|^2, \\ &\|D_y V(x,y,z)\| \le \psi_3(z)\|y\|. \end{aligned} \tag{1}$$

The space of functions $\mathcal{L}_p$, with $1 \le p < \infty$, is the set of function h from $[0,\infty)$ to $\mathbb{R}^m$ satisfying

$$(\int_0^\infty \|h(t)\|^p\, dt)^{1/p} < \infty.$$

When $p = \infty$, a function $h(t)$ in $\mathcal{L}_\infty$ satisfies

$$ess.\sup_{t\in[0,\infty)} \|h(t)\| < \infty.$$

To define the zero dynamics and the minimum-phase property, consider the following system:

$$\begin{aligned} \dot{z} &= q(z,\xi) + \gamma(z,\xi)u, \\ \dot{\xi} &= a(z,\xi) + b(z,\xi)u, \\ y &= \xi, \end{aligned} \tag{2}$$

where $b(z,\xi)$ is locally invertible near the equilibrium point, and states z and ξ and functions have reasonable dimensions. The *zero dynamics* for the above system is defined as the dynamics that satisfies the constraint $y(t) \equiv 0$.

Definition 1: [2] The system (2) is *minimum phase* if the equilibrium point $z = z_e$ of its zero dynamics subsystem defined above is asymptotically stable.

Finally, we define a (global) observer and a (global) adaptive observer given a nonlinear system.

Definition 2: [3] Given a multivariable nonlinear system

$$\begin{aligned} &\dot{x} = f(x,u),\ x(0) = x_0,\ x \in {}^n, u \in {}^m \\ &y = h(x),\ y \in {}^s, \end{aligned} \tag{3}$$

a global *observer* is a dynamic system

$$\begin{aligned} &\dot{\omega} = \alpha_1(\omega, y, u), \omega(0) = \omega_0, \omega \in \mathbb{R}^r, r \ge n \\ &x = \alpha_2(\omega, y, u), \hat{x} \in \mathbb{R}^n \end{aligned} \tag{4}$$

such that for any $x_0 \in \mathbb{R}^n, \omega_0 \in \mathbb{R}^r$ and for any bounded $\|x(t)\|, \|u(t)\|, \forall t \ge 0$, the error $\|x(t) - \hat{x}(t)\|$ is bounded for all $t \ge 0$ and

$$\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0.$$

Definition 3: [3] For the following system,

$$\begin{aligned} &\dot{x} = f(x) + g(x,u) + \sum_{i=1}^{p} \theta_i q_i(x,u), \\ &y = h(x), \end{aligned} \tag{5}$$

in which $x \in \mathbb{R}^n, u \in \mathbb{R}^m, \theta = [\theta_1, \cdots, \theta_p]^T \in \mathbb{R}^p$, $y \in \mathbb{R}, f(0) = 0, h(0) = 0, g(x,0) = 0, \forall x \in \mathbb{R}^n$, a global *adaptive observer* is a finite dimensional system,

$$\begin{aligned} &\dot{\omega} = \alpha_1(\omega, \hat{\theta}, y(t), u(t)), \omega(0) = \omega_0, \omega \in \mathbb{R}^r, r \ge n \\ &\dot{\hat{\theta}} = \alpha_2(\omega, \hat{\theta}, y(t), u(t)),\ \hat{\theta}(0) = \hat{\theta}_0,\ \hat{\theta} \in \mathbb{R}^p \\ &\hat{x} = \alpha_3(\omega, \hat{\theta}, y(t), u(t)),\ \hat{x} \in \mathbb{R}^n, \end{aligned} \tag{6}$$

driven by the inputs $u(t)$, $y(t)$ such that for every $x_0 \in \mathbb{R}^n, \omega_0 \in \mathbb{R}^r, \hat{\theta}_0$, for any value of the unknown parameter θ and for any bounded $\|x(t)\|, \|u(t)\|$, $\forall t \ge 0$,

$\|\omega(t)\|, \|\hat{\theta}(t)\|$ and $\|x(t) - \hat{x}(t)\|$ are bounded, $\forall t \ge 0$,

$\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0$.

2.2. Brief review of previous results

A lot of results on observers have been produced so far. Krener and Isidori [4] provided conditions for a class of nonlinear systems to be transformed into the linear observable system by coordinate transformation and output injection. In Krener and Respondek's work [5], for a class of nonlinear system transformable into the dual Brunovsky canonical form, whose error dynamics is linear, the design method of asymptotic observers is explored. Rajamani [6] suggested observers for the systems whose nonlinear term satisfies the Lipschitz condition through investigating the algebraic Riccati equation. In Shim and Seo's work [7], a brand-new recursive design method for the class of partly lower triangular nonlinear systems, which is general enough to include non-uniformly observable and/or detectable multi-output systems, is proposed.

In comparison to the (non-adaptive) observer design, the class of systems considered in the research of an adaptive observer design has been restricted in system structures and nonlinearities. The work on the nonlinear adaptive observer design dates from 1983, when Bestle and Zeitz [8] attempted to transform the nonlinear system into a canonical form convenient for the observer design. In the linear case, the adaptive observer design of time-invariant system is dealt with considerably [9]. Vargas and Hemerly [10] dealt with the nonlinear system having disturbance as well as uncertain parameters. As Krener and Isidori did in the observer design, Marino and Tomei [3,11] give the equivalent geometric conditions for a system with linear uncertain parameters to be transformed into the special adaptive observer form and use the SPR condition to prove the stability of error dynamics and parameter convergence.

The adaptive observer form [3] is based on the Brunovsky observer form with nonlinear input and output injection, and the more general form of the system, still based on the Brunovsky form, is transformed into the adaptive observer form using the filtered transformation [12]. Furthermore, the geometric conditions are given for a nonlinear system with linear uncertain parameter,

$$\dot{x} = f(x) + q_0(x,u) + \sum_{i=1}^{p} \theta_i q_i(x,u), x \in \mathbb{R}^n, u \in \mathbb{R}^m,$$

to be transformed into the adaptive observer form. Then, the state estimation error converges to zero without persistency of excitation. If the persistency of excitation holds, the parameter estimation error also converges to zero exponentially. Then, the MKY Lemma [3] is applicable to the augmented error dynamics to derive an adaptive law.

Besancon[13] proposed a more general nonlinear adaptive observer form and unified a existing results so far of [3]. The nonlinear adaptive observer form,

$$\begin{aligned} \dot{y} &= \alpha(y,\zeta,u,t) + \beta(y,\zeta,u,t)\theta, \\ \dot{\zeta} &= Z(y,\zeta,u,t),, \end{aligned} \tag{7}$$

where y is the measured output, has a linear uncertain parameter in the dynamics of the measured state only. Moreover, by assuming the detectability of the subsystem consisting of unknown states, a reduced order observer is constructed.

Motivated by this work and by that of Shim and Seo [7], a nonlinear adaptive observer is proposed for a class of systems with linear unknown parameter.

3. NONLINEAR ADAPTIVE OBSERVER

First of all, a simple motivational example is presented.

3.1. Simple motivational example

The adaptive observer proposed is motivated by an example in Shim and Seo [7],

$$\begin{aligned} \dot{x}_1 &= ux_2, \\ \dot{x}_2 &= (u^2-1)x_2, \\ y &= x_1, \end{aligned} \tag{8}$$

where $x_1, x_2 \in \mathbb{R}$ are states, $y \in \mathbb{R}$ is the output, and $u \in \mathbb{R}$ is the input. For the above system, an observer was designed with change of coordinates:

$$\xi_1 = x_1, \xi_2 = x_2 - ux_1,$$

so that ξ-dynamics becomes

$$\begin{aligned} \dot{\xi}_1 &= \dot{x}_1 = u(\xi_2 + u\xi_1), \\ \dot{\xi}_2 &= \dot{x}_2 - \dot{u}x_1 - u\dot{x}_1 \\ &= -x_2 - \dot{u}x_1 \\ &= -\xi_2 - (u+\dot{u})\xi_1. \end{aligned} \tag{9}$$

Here, we consider the system with an uncertain parameter:

$$\begin{aligned} \dot{x}_1 &= ux_2 + \theta, \\ \dot{x}_2 &= (u^2-1)x_2 + u\theta, \\ y &= x_1, \end{aligned} \tag{10}$$

in which $\theta \in \mathbb{R}$.

The above system has an unknown parameter θ and we are going to construct an adaptive observer for the above system. With the same change of coordinates, the system is transformed as

$$\begin{aligned} \dot{\xi}_1 &= \dot{x}_1 + \theta \\ &= u\xi_2 - u^2\xi_1 + \theta, \\ \dot{\xi}_2 &= \dot{x}_2 - \dot{u}x_1 - u\dot{x}_1 \\ &= (u^2-1)x_2 + u\theta - \dot{u}x_1 - u^2x_2 - u\theta \\ &= -x_2 - \dot{u}x_1 \\ &= -\xi_2 - (u+\dot{u})\xi_1. \end{aligned} \tag{11}$$

Then, the adaptive observer for the transformed system is

$$\begin{aligned} \dot{z}_1 &= -u^2z_1 + uz_2 + \hat{\theta} + \gamma(*), \\ \dot{z}_2 &= -z_2 - (u+\dot{u})z_1, \end{aligned} \tag{12}$$

where $\gamma(*)$ is a function with arguments of known signals to be determined. Note that, after the change of coordinates, the unknown parameter shows up only in ξ_2 or z_2-dynamics. Next, we find the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -u^2e_1 + ue_2 - \tilde{\theta} + \gamma(*), \\ \dot{e}_2 &= -e_2 - (u+\dot{u})e_1, \end{aligned} \tag{13}$$

where $e = z - \xi$. If we choose a Lyapunov-like function,

$$V(e_1,e_2,u,\tilde{\theta}) = \frac{1}{2}e_1{}^2 + \frac{1}{2}e_2{}^2 + \frac{1}{2}\tilde{\theta}^2$$

then, the time-derivative of V is

$$\begin{aligned}\dot{V} &= e_1(-u^2e_1 + ue_2 - \tilde{\theta} + \gamma(*)) \\ &+ e_2(-e_2 - (u + \dot{u})e_1) + \dot{\tilde{\theta}}\tilde{\theta} \\ &= -e_2^2 - \dot{u}e_1e_2 - u^2e_1^2 + e_1\gamma(*) - e_1\tilde{\theta} + \dot{\tilde{\theta}}\tilde{\theta} \\ &\le -e_2^2 + \frac{1}{2}e_2^2 + (\dot{u} - u^2)e_1^2 + e_1\gamma(*) - e_1\tilde{\theta} + \dot{\tilde{\theta}}\tilde{\theta}\end{aligned} \tag{14}$$

Selecting an adaptive law as $\dot{\tilde{\theta}} = e_1$ and $\gamma(*) = [-1(\dot{u}^2 - u^2) - 1]e_1$,

$$\dot{V} \le -\frac{1}{2}e_2^2 - e_1^2.$$

Hence, an adaptive observer for the system (10) has been constructed.

3.2. Adaptive observer design

The nonlinear system of interest is given by

$$\begin{aligned}\dot{x}_1 &= f_1(x,u) + g_1(x,u)\theta, \\ \dot{x}_2 &= f_2(x,u) + g_2(x,u)\theta, \\ y &= x_1,\end{aligned} \tag{15}$$

where $x_1 \in \mathbb{R}^{r_1}$ and $x_2 \in \mathbb{R}^{r_2}$ are states; $\theta \in \mathbb{R}^j$ is an unknown constant vector; $g_1 \in \mathbb{R}^{r_1 \times j}$ and $g_2 \in \mathbb{R}^{r_2 \times j}$ are matrix-valued functions and $u \in \mathbb{R}^m$ is the input. Suppose that the functions f_1, f_2, g_1, and g_2 are Lipschitz in x and the input is smooth with respect to time.

Assumption 1: *There exist a nonnegative integer q, a continuous matrix-valued function, $K(x_1,\bar{u}_q)$, a C^1 function $L(x_1,\bar{u}_q)$, a C^1 function $V(x,e_2,\bar{u}_q)$, , which is quadratic in e_2 with $\bar{u}_q$, and a positive function $\alpha_0(u)$ such that*

$$\begin{aligned}&D_xV\cdot[f(x,u) + g(x,u)\theta] \\ &+D_{e_2}V\cdot\left[F_2(0,e_2;x,u) - K(x_1,\bar{u}_q)F_1(0,e_2;x,u)\right] \\ &+D_{\bar{u}_q}V\cdot\dot{\bar{u}}_q \le -\alpha_0(\bar{u}_q)\|e_2\|^2,\end{aligned} \tag{16}$$

$$g_2(x,u) - K(x_1,\bar{u}_q)\cdot g_1(x,u) = 0, \tag{17}$$

and

$$\frac{\partial L}{\partial x_1}(x_1,\bar{u}_q) = K(x_1,\bar{u}_q). \tag{18}$$

Before proposing an adaptive observer, a nonlinear but simple change of coordinates is performed on the system state:

$$\xi_1 = x_1, \xi_2 = x_2 - L(x_1,\bar{u}_q).$$

Thus, the system (15) is represented in the transformed state ξ:

$$\dot{\xi}_1 = f_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u) + g_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u)\theta,$$

$$\begin{aligned}\dot{\xi}_2 &= f_2(x_1,\xi_2 + L(x_1,\bar{u}_q),u) + g_2(x_1,\xi_2 + L(x_1,\bar{u}_q),u)\theta \\ &-\frac{\partial L}{\partial x_1}\cdot\left[f_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u)\right] \\ &-\frac{\partial L}{\partial x_1}\cdot\left[g_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u)\theta\right] - \frac{\partial L}{\partial \bar{u}_q}\cdot\dot{\bar{u}}_q.\end{aligned}$$

With the assumption (17), the above transformed system is simplified as

$$\begin{aligned}\dot{\xi}_1 &= f_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u) + g_1(x_1,\xi_2 + L(x_1,\bar{u}_q),u)\theta, \\ \dot{\xi}_2 &= f_2(x_1,\xi_2 + L(x_1,\bar{u}_q),u) \\ &-K(x_1,\bar{u}_q)f_1(x_1,\xi_2 + L(x_1,\bar{u}_q,u) - \frac{\partial L}{\partial \bar{u}_q}\cdot\dot{\bar{u}}_q.\end{aligned} \tag{19}$$

Then, we consider the adaptive observer for the system (19). The following system is proposed as an adaptive observer for the above system (19):

$$\begin{aligned}\dot{z}_1 &= f_1(x_1,z_2 + L(x_1,\bar{u}_q),u) + \\ &g_1(x_1,z_2 + L(x_1,\bar{u}_q),u)\hat{\theta} - k(z_1 - \xi_1), \\ \dot{z}_2 &= f_2(x_1,z_2 + L(x_1,\bar{u}_q),u) \\ &-K(x_1,\bar{u}_q)f_1(x_1,z_2 + L(x_1,\bar{u}_q),u) - \frac{\partial L}{\partial \bar{u}_q}\cdot\dot{\bar{u}}_q, \\ \dot{\hat{\theta}} &= -S(*),\end{aligned} \tag{20}$$

where z is the estimate for ξ, $\hat{\theta}$ is the estimate for θ, and a function $S(*) \in \mathbb{R}^q$ is to be determined. Note that $S(*)$ has arguments as known variables and k has only to be positive. If we know that the sysem (20) is an adaptive observer for the system (19), then the system (20) with

$$\hat{x}_2 = z_2 + L(x_1,\bar{u}_q) \tag{21}$$

is an adaptive observer for the original system (15).

For the system (20) to be an adaptive observer for the system (19), the function $S(*)$ has to be chosen. In the next proposition, with the above arguments, that function is found.

Theorem 1: Under Assumption 1, the system in (20) and (21) is an adaptive observer for the system in (15).

Proof: If we define the state estimation error $e := z - \xi$, the error dynamics becomes

$$\begin{aligned}\dot{e}_1 &= F_1(0,e_2;\xi_1,\xi_2 + L,u) \\ &+g_1(x_1,z_2 + L,u)\hat{\theta} - g_1(x_1,\xi_2 + L,u)\theta - ke_1 \\ \dot{e}_2 &= (F_2 - KF_1)(0,e_2;\xi_1,\xi_2 + L,u).\end{aligned} \tag{22}$$

Let us define a C^1 function W for $\varepsilon > 0$ as

$$W(x,e,\bar{u}_q,\tilde{\theta}) := V(x,e_2,\bar{u}_q) + \frac{\varepsilon}{2}e_1^Te_1 + \frac{\varepsilon}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta},$$

where Γ is a symmetric positive-definite matrix and

$\tilde{\theta} := \theta - \hat{\theta}$ is the parameter estimation error.

Now, let us show that it is the adaptive observer. The time derivative of W along the system (22) is

$$\begin{aligned}
\dot{W} &= \dot{V}(x_2, e_2, \bar{u}_q) + \varepsilon e_1^T \dot{e}_1 + \varepsilon \dot{\tilde{\theta}}^T \Gamma^{-1} \tilde{\theta} \\
&= D_x V \cdot f + D_{e_2} V \cdot [(F_2 - KF_1)(0, e_2; \xi_1, \xi_2 + L, u)] \\
&\quad + D_{\bar{u}_q} V \cdot \bar{u}_q + \varepsilon e_1^T (F_1(0, e_2; \xi_1, \xi_2 + L, u) \\
&\quad + \varepsilon e_1^T (g_1(x_1, z_2 + L, u)\hat{\theta} - g_1(x_1, \xi_2 + L, u)\theta) \\
&\quad - k\varepsilon \|e_1\|^2 + \varepsilon \dot{\tilde{\theta}}^T \Gamma^{-1} \tilde{\theta} \\
&\le -\alpha_0(\bar{u}_q)\|e_2\|^2 + \varepsilon c_1(u)\|e_1\|\|e_2\| \\
&\quad + \varepsilon e_1^T(-g_1(x_1, z_2 + L, u)\tilde{\theta} + G_1(0, e_2; x_1, \xi_2 + L, u)\theta) \\
&\quad - k\varepsilon \|e_1\|^2 + \varepsilon \dot{\tilde{\theta}}^T \Gamma^{-1} \tilde{\theta}.
\end{aligned}$$

If we choose the adaptive law as

$$\dot{\hat{\theta}} = S(*) := \Gamma g_1^T(x_1, z_2, u) e_1,$$

then

$$\begin{aligned}
\dot{W} &\le -\alpha_0(\bar{u}_q)\|e_2\|^2 + \varepsilon c_1(u)\|e_1\|\|e_2\| \\
&\quad + \varepsilon e_1^T G_1(0, e_2; x_1, \xi_2 + L, u)\theta - k\varepsilon \|e_1\|^2 \\
&\le -\alpha_0(\bar{u}_q)\|e_2\|^2 + \varepsilon(c_1 + d_1|\theta|)\|e_1\|\|e_2\| - k\varepsilon\|e_1\|^2.
\end{aligned}$$

Therefore, for a sufficiently small ε, we have

$$\dot{W} \le -\alpha_1(\bar{u}_q)\|e\|^2, \tag{23}$$

where α_1 is some positive function. Hence, the proposed system is qualified as an adaptive observer for system (15). □

4. AN ILLUSTRATIVE EXAMPLE

Let us consider the following system:

$$\begin{aligned}
\dot{x}_1 &= -x_1 + ux_3 + x_4 + x_2\theta, \\
\dot{x}_2 &= -x_2 + u^2 x_4 + x_2\theta, \\
\dot{x}_3 &= (u^2 - 1)x_3 + (u^2 + u - 1)x_4 + (u+1)x_2\theta, \\
\dot{x}_4 &= -x_4, \\
y &= (x_1 \quad x_2)^T,
\end{aligned} \tag{24}$$

where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \in \mathbb{R}^4$, $\theta \in \mathbb{R}$ is unknown. In the notation of Section 3,

$$\begin{aligned}
f_1(x,u) &= (-x_1 + ux_3 + x_4 \quad -x_2 + u^2 x_4)^T, \\
f_2(x,u) &= ((u^2 - 1)x_3 + (u^2 + u - 1)x_4 \quad -x_4)^T, \\
g_1(x,u) &= (x_2 \quad x_2)^T, \\
g_2(x,u) &= ((u^2 + u - 1)x_4 + (u+1)x_2 \quad 0)^T.
\end{aligned}$$

Let $K(x_1, u) = \begin{pmatrix} u & 1 \\ 0 & 0 \end{pmatrix}$, and then $L = (ux_1 + x_2 \quad 0)^T$ will satisfy the relation in Assumption 1,

$$\frac{\partial L}{\partial x_1}(x_1, u) = K(x_1, u).$$

Moreover, the following system

$$\begin{aligned}
\begin{pmatrix} \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} &= F_2(0, e_2; x_1, x_2, u) - KF_1(0, e_2; x_1, x_2, u) \\
&= \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e_3 \\ e_4 \end{pmatrix}
\end{aligned}$$

is exponentially stable at $(e_3, e_4) = 0$. Then, the change of coordinates as in Proposition 1,

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \begin{pmatrix} \xi_3 \\ \xi_4 \end{pmatrix} = \begin{pmatrix} x_3 - ux_1 - x_2 \\ x_4 \end{pmatrix}$$

is conducted so that ξ-dynamics is obtained as

$$\begin{aligned}
\dot{\xi}_1 &= \dot{x}_1 = ux_3 + x_4 + x_2\theta \\
&= u(\xi_3 + u\xi_1 + \xi_2) + \xi_4 + \xi_2\theta \\
&= u^2\xi_1 + u\xi_2 + u\xi_3 + \xi_4 + \xi_2\theta, \\
\dot{\xi}_2 &= \dot{x}_2 = u^2 x_4 + x_2\theta \\
&= u^2\xi_4 + \xi_2\theta, \\
\dot{\xi}_3 &= \dot{x}_3 - \dot{u}x_1 - u\dot{x}_1 - \dot{x}_2 \\
&= -x_3 - x_4 - \dot{u}x_1 \\
&= -\xi_3 - u\xi_1 - \xi_2 - \xi_4 - \dot{u}\xi_1 \\
&= -(u + \dot{u})\xi_1 - \xi_2 - \xi_3 - \xi_4, \\
\dot{\xi}_4 &= \dot{x}_4 = -x_4 \\
&= -\xi_4.
\end{aligned}$$

The adaptive observer for (24) with the form proposed in Proposition 1 is

$$\begin{aligned}
\dot{z}_1 &= u^2\xi_1 + u\xi_2 + uz_3 + z_4 + \xi_2\hat{\theta} + \gamma_1(*), \\
\dot{z}_2 &= u^2 z_4 + \xi_2\hat{\theta} + \gamma_2(*), \\
\dot{z}_3 &= -(u + \dot{u})\xi_1 - \xi_2 - z_3 - z_4, \\
\dot{z}_4 &= -z_4,
\end{aligned}$$

where $\gamma(*) = [\gamma_1(*) \quad \gamma_2(*)]^T$ is a function of known signals such as e_1, e_2, and z. Then the error dynamics is

$$\begin{aligned}
\dot{e}_1 &= ue_3 + e_4 - \xi_2\tilde{\theta} + \gamma_1(*), \\
\dot{e}_2 &= u^2 e_4 - \xi_2\tilde{\theta} + \gamma_2(*), \\
\dot{e}_3 &= -e_3 - e_4, \\
\dot{e}_4 &= -e_4.
\end{aligned} \tag{25}$$

Next, consider a positive-semidefinite function

$$V(e, \tilde{\theta}) = \frac{1}{2}e^T e + \frac{1}{2}\tilde{\theta}^2.$$

Then, the time-derivative of V along the system in (24) and (25) is

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \dot{\tilde{\theta}}\tilde{\theta} \\ &= e_1(ue_3 + e_4 - \xi_2\tilde{\theta}) + e_1\gamma_1 + e_2(u^2 z_4 + \xi_2\hat{\theta}) + e_2\gamma_2 \\ &\quad + e_3(-e_3 - e_4) + e_4^2 + \dot{\tilde{\theta}}\tilde{\theta} \\ &= -e_3^2 - e_3e_4 - e_4^2 + ue_1e_3 + e_1e_4 + u^2 e_2 e_4 \\ &\quad + e_1\gamma_1(*) + e_2\gamma_2(*) - (e_1 + e_2)\xi_2\tilde{\theta} + \dot{\tilde{\theta}}\tilde{\theta}.\end{aligned}$$

Choosing the adaptive law as

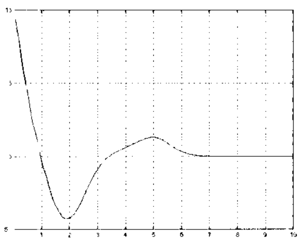
$$\dot{\tilde{\theta}} = (e_1 + e_2)\xi_2$$

and using Young's inequality,

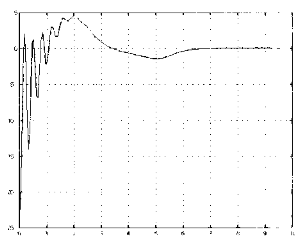
$$|ab| \le \frac{1}{2k}|a|^2 + \frac{k}{2}|b|^2, k > 0,$$

the upper bound of $\dot{V}$ is found:

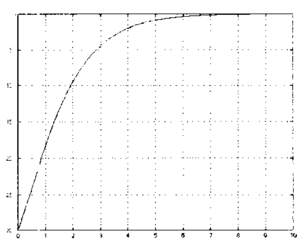
$$\dot{V} \le -\frac{3}{8}e_3^2 - \frac{1}{4}e_4^2 + (2u^2 + 2)e_1^2 + 2u^4 e_2^2 + e_1\gamma_1 + e_2\gamma_2.$$



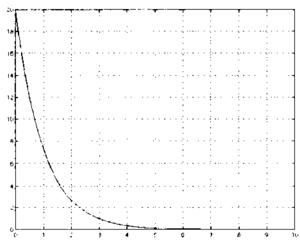
(a) $e_1(0) = 10$



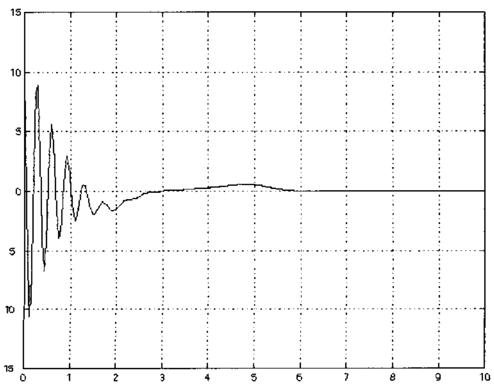
(b) $e_2(0) = -22$



(c) $e_3(0) = -30$



(d) $e_4(0) = 20$



(e) $\tilde{\theta}$

Fig. 1. State and parameter estimation errors.

To make $\dot{V}$ nonpositive, we select $\gamma_1(*) = (-2u^2 - 3)e_1$ and $\gamma_2(*) = (-2u^4 - 1)e_2$ so that

$$\dot{V} \le -e_1^2 - e_2^2 - \frac{3}{8}e_3^2 - \frac{1}{4}e_4^2.$$

Fig. 1 shows the result of simulation for 10 seconds. The simulation is performed under the condition that the input $u = \sin t$ and the unknown parameter $\theta = 1$ with the initial state estimation error $e(0) = [10 \quad -22 \quad -30 \quad 20]^T$ and the parameter estimation error $\tilde{\theta} = 11$. The result of the simulation validates that the state estimation error goes to zero, and so does the parameter estimation error, since the plant (24) has bounded states with the input and the parameter assigned above for simulation.

5. CONCLUSIONS

In this paper, we have studied the adaptive observer design for a class of nonlinear systems. The design is based on the observer design for the system which has the property of a detectability of subsystem. With additional technical assumption on the function multiplied by uncertain parameters, asymptotic observers have been constructed and all the parameter estimation errors are guaranteed to be bounded.

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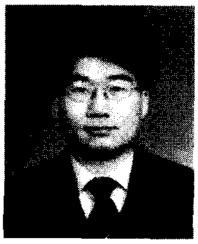

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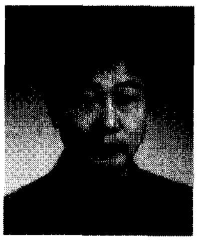

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