A Classification and Selection of Reliability Growth Models

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Abstract

In the development of a complex systems, the early prototypes generally have reliability problems, and, consequently these systems are subjected to a reliability growth program to find problems and take corrective action. A variety of models have been proposed to account for the reliability growth phenomena. Clear guidelines need to be established to assist the reliability engineers for model selection. In this paper, some of more well-known growth models are surveyed and classified. These models are classified based upon distinguishing model features. A procedure for model selection is introduced which is based on this classification.

1. Introduction

Development programs for complex systems require considerable resources such as time, equipment and manpower, to achieve a level of system reliability acceptable to the user. The reliability requirements for many systems are high, and to attain these high goals it is common practice to subject the system to a TAAF(test-analyze-and-fix) process. During tested to failure, system failure modes are determined, and design and/or engineering changes are made as attempts to eliminate these modes or, at least, to decrease their rate of occurrence. If this process is continued, and design and engineering modifications are made in a competent manner, then the system reliability will increase. <Figure1> shows typical TAAF process.

Duane[Duane, 1964] postulated a reliability growth model for planning the reliability of electromechanical systems.

Duane observed that a plot of cumulative failure rate versus cumulative operational hours follows a straight line on log-log paper. Although Duane's model is

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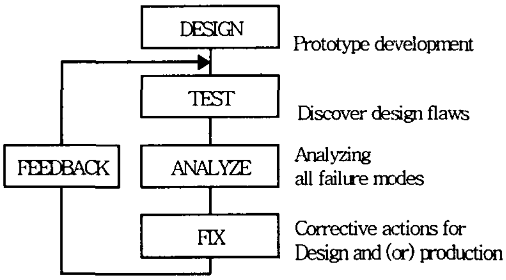
widely accepted in its use for modeling reliability growth phenomena, a plethora of alternative models have been reported in the literature. Each of these models is unique with respect to data requirements, model assumptions, etc. The intent of this article is generate a taxonomy of reliability growth models which can then be made available to the reliability engineer.

This article surveys well-known reliability growth models, including both continuous and discrete reliability growth models. These models are seen to differ significantly with respect to data requirements, model features and assumptions. Due to these differences, the reliability engineer should be able to the application at hand.



<Figure1> Typical TAAF process for a reliability growth program

2. Characteristics of models

Based upon a survey of the reported models for reliability growth, several model characteristics which are useful for model differentiation are differentiated according to the data requirements, the nature of the failure process, failure mode and failure source. For example, the allowance of a prior distribution on any of the growth parameters is a distinguishing characteristic. In general, lifetime data will either be discrete (counted) or continuous. If the reliability study is being conducted Istages, then the data will be counted with respect to how many failures were observed at each stage. As shown in Figure 2, only certain growth models are designed for the case where failure data is grouped[Barlow 1966, Gross 1968, Lloyd 1962, Pollock 1968, Smith 1977, Wolman 1963].

The growth models are primarily distinguished in the way in which the details of the underlying TAAF process are captured. For example, if the incremental growth pattern associated with corrective action is approximated with a continuous function, then the reliability engineers may use time-continuous, parametric models for reliability growth. The effectiveness of such a modeling technique resides in the capability to derive accurate estimates of the parameters as function of accumulated test time. Duane's reliability growth model is based upon the relationship on log-log paper of cumulative failure rate versus cumulative test time.

By contrast, reliability growth can be

modeled as a stepwise series of growth spurts during the test phase when the effort of program activities such as redesign or failure sources analysis are expended. These models may assume that there are a known number of faults which have occurrence rates that vary according to some distribution[Jelinsky 1972, Littlewood 1981, Rosner 1961]. By removal of successive faults, the overall reliability improves. Such a modeling approach is believed that the computer software contains only a finite number of "bugs."

All models are designed to show the sequence of the successive times between failure as monotonically non-decreasing. This means that the corrective actions made between stages of testing are assumed effective or at least not detrimental to the system. The rate of growth is represented by the slopes of continuous growth lines or by the magnitude of the jumps for the discontinuous growth models. This growth rate is correlated with the amount of the effort on reliability improvement and indicates the effectiveness of the program. The most common assumption of reliability growth models is immediate perfect fix when the system fails. At the

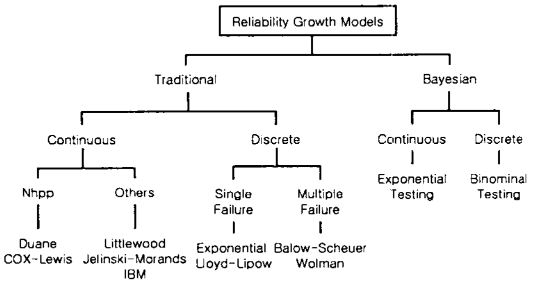
<Table1> Characteristics of the reliability growth models

	Model	**Characteristics**
Modeling Approach	Traditional Model	Model are in the form of math formulas. Parameters are estimated by maximum likelihood method or least squires method
	Bayesuan Model	Growth parameter are drawn form a prior distribution and observed data
Data Requirement	Continuous Model	In the form of a smooth continuous curve as a function of accumulated test time.
	Discrete Model	For each test stage, the system is considered either a success of failure.
Failure Process	NHPP Model	Point process satisfies the poisson property with time varying rates.
	Other process Model	Overall reliability is proportional to the remaining faults.
Failure Mode	Single Failure Mode	On any given trial, the system considers only one failure mode.
	Multiple Failure Model	Multiple and independent failure modes. The distinguishability can be characterized by definition.
Failure Source	Infinite Failure Source	System contains infinite number of failure.
	Finite Failure Source	System terminates after a known number of failures occurred.
Correctability	Perfect Fix	The corrective action is perfect and never causes the same problem.
	Imperfect Fix	The corrective action is successful with some finite probability. p.

time of each repair, it is assumed that the system is repaired without delay. Of course, not all corrective action is "perfect" or immediate. The corrective action, itself, may be imperfect with respect to the timeliness of the modification to the product. Crow propose a model whereby with some probability, p, the repairs are made immediately, but with probability, 1-p, the repair is delayed[Crow, 1983]. Pollock considers Bayesian models with imperfect fix[Pollock, 1968]. In his models, if the system fails, the designers attempt a modification which has a given probability of being successful.

3. Classification of models

A useful taxonomy for classification of reliability growth models is described by <figure2>. The individual in the following section. A primary distinction is made between traditional and Bayesian modeling approaches.



<Figure2> Classification of reliability growth models

3.1 Traditional Models

The traditional models are classified into two types - continuous and discrete models. The discrete models are classified according to the level of detail used in modeling the failure process (i.e., single failure versus multiple failure models). The continuous models are classified according to the nature of the failure process. These models are subdivided according to whether the failure process follows a no homogenous failure process.

3.1.1 Continuous Models

a) NHPP Models

For continuous models, the no homogeneous Poisson process (NHPP) is widely applied in the modeling of the underlying failure process. Duanel demonstrated that plots of the cumulative failure rate against cumulative test time on loglog paper follow a straight line. Crow added a NHPP assumption to the Duane postulate since the rate of occurrence of failure (ROCOF) is changing with time[Crow 1974, 1975]. The Cox-lewis model14 is also in this category. The Cox-lewis model starts the test process with an unknown number of failures[Cox, 1966]. For each stage, observations satisfy all the conditions for an NHPP. This model can be applied to a nonlinear model as well as a linear model on log paper.

b) Other Process Models

The Jelinski-Moranda model[Jelinski, 1972] is an example of a continuous model in which a primary assumption is made that the number of failure sources is finite. The failure processes of this type of growth models do not have the independent increment property. Instead, the failure rate of the system after the ith fault has been removed, is proportional to the remaining faults contained in the system. The Jelinski-Moranda model assumes that the system starts life with N faults. Faults are corrected through stages and each fault contributes the same amount to the overall unreliability. Littlewood suggested that the faults contained in the system would cause failures with different rates[Littlewood, 1981A, 1981B]. After the ith fault is corrected, the failure rate is the sum of (N-i) different failure rates. The IBM model assumes that the failure rate of the system with respect to operational time is proportional to the amount of correctable failures remaining in the system at that time.[Rosner, 1961]

3.1.2 Discrete Models

a) Single Failure Models

The single failure models consider either success or failure of the trials. Lloyd-Lipow propose a model in which a system has a single failure modeLloyd, 1962]. In their model, a test program is conducted in N stages; each stage consists of a certain number of trials under the test, and the number of successes and failures in each test is recorded. When the Nth stage of the curve to the N groups of success-failure data. Gross and Kamin3 considered the same testing situation in their Exponential model.

b) Multiple Failure Models

As the reliability test program progresses, the correctable cause failures are eliminated by means of equipment or operational modifications which are taken after each such system failure. However, not every system fault can be accompanied by a corrective action which results in system improvement and hence, reliability growth. For example, in the extreme case, it may be that nothing short of a costly, complete system redesign will reduce the underlying failure rate associated with the cause. As such, it may be desirable to model the underlying failure process as that of an underlying, inherent or residual failure process which cannot be fixed, along with a finite number of failure sources due to design or manufacturing deficiencies which can be corrected. During testing, the observed outcomes are successes, inherent failures or assignable cause failures. For the latter case, it is up to management to prioritize design modification activities, and to allocate resources for the design fix.

Wolman proposed a model with two types of failure modes[Wolman, 1963]. The failures are either inherent failures or assignable cause failures. Barlow-Scheuer proposed a model for discrete failure data which incorporated independent probabilities of inherent and correctable failure modes [Barlow, 1966]. It is assumed that the probability of an inherent failure remains constant throughout the test program while the probability of a correctable failure mode does not increase with consecutive stages.

3.2 Bayesian Models

The traditional (non-Bayesian) approach to parameter estimation in reliability growth models does not account for past experience. This means that parameter estimates are not updated as new information on the process becomes available. Bayesian models allow for the existence of a prior distribution on the parameter apace. That is, a parameter may be regarded as a random variable, and hence a posterior distribution is used to model likelihood of any realized parameter estimates. Generally, the prior distribution is chosen such that the posterior distribution will be of a similar functional form, and hence, termed a conjugate distribution. In this case, one would expect that the mean time between failures would increase each time that the posterior distribution is updated. The Bayesian reliability growth models are classified according to the methods utilized to conduct the life test.

a) Exponential Test Model

A Bayesian model is classified as an "exponential testing model" when the exponential distribution is used to model time to failure. Littlewood and Verrall developed a Bayesian growth model for computer software[Littlewood, 1973]. The exponential failure rate parameter is described by a gamma prior distribution, which is the natural conjugate of the Poisson. Pollock presented Bayesian models in which a probability of successful repair is considered[Pollock, 1968]. In his exponential testing model, the system remains either in an unrepaired or a repaired state after n tests have been performed.

b) Binomial Testing model

A Bayesian model is classified as the binomial testing model if the model is applicable to a success-failure testing situation. For example, if a number of systems are placed on test and x is the random variable representing the number of failures, then the probability density function is binomial with parameter being the fraction failing. Smith presented a Bayesian model by considering the prior belief that the binomial parameters are uniformly distributed[Smith, 1977]. Pollock presented models for both the exponential testing and the binomial testing systems.

Figure 2 shows a classification tree for the growth models.

The surveyed models are classified according to the prominent model features including the modeling approach used, any special data requirements, the nature of the failure process, and the allowed failure mode. As discussed, the growth models can be subdivided into two groups according to the nature of the failure source. The first division is the infinite failure source models which consider infinite sequences of inter-arrival times. This means that in an infinite time interval, the expected number of failures is infinite. The second division is the finite failure source models which consider the situation where the failure process of the system terminates after a known number of failures. In this case, the maximum number of failures for an infinite time interval is finite. Table 2 shows examples of models according to failure source and data requirements.

<Table2> Examples of Models According to Failure Sources

	Continuous Model	Discrete Model
Infinite Failure Source Model	Duane, Cox-Lewis, Littlewood-Verrall Pollock(1)	Barlow-Scheuer, Exponential, Smith, Lloyd-Lipow, Pollock(2)
Finite Failure Source Model	Jelinski-Moranda Gamma, IBM	Wolman

4. Literature Survey Comment

In the preceding section, we presented reliability growth models for both hardware and software systems. Although we think of a TAAF process in hardware terms, the reliability growth models that have been developed for analyzing and predicting failure occurrences can just as readily be applied for software systems. Mathematically there is no reason to distinguish between hardware and software systems in the application of growth models. Through the fault removal process, software systems improve by debugging and correction. This process is the same as the TAAF process for hardware, with the added difference that when software faults are removed, they do not occur again. In both cases, the removal of design flaws may be imperfect, causing new problems which will be detected later on.

Most of the reliability growth models presented here show that the reliability tends to be perfect for long-term operation. Ascher and Feingold pointed out that long-term forecasting is questionable because usual TAAF programs operate for finite periods[Ascher, 1984]. These extremes provide unusable results for fintie testing times. Models such as the IBM model classified failures into residual/removable sets. As $t \rightarrow \infty$, the limiting ROCOF is the residual failure rate. The multiple failure mode models, such as the Wolman model and the Barlow-Scheuer model, considered inherent failures which will lead nonzero ultimate failure rate. This modeling approach has advantages when failures have been established and that those future occurrences cannot be effectively prevented.

In addition, the existing growth models are not applicable for the case of no-growth or deterioration situations. Under such circumstances, the results would be meaningless and misleading. For example, if the observed mean time between failures decreases, the maximum likelihood estimates for the number of initial fault contents in finite failure source models are infinite. It is shown that such a test result can have occurred with a finite probability[Littlewood, 1981B]. The Duane model gives implausible results when the growth rate $\alpha<Q$. In reality, the possibility of no-growth always exists; for instance, deliberate delaying of fixes, growth not occurring after installation of design fixes, or in some cases corrections that are harmful to the system. To remedy these problems, the models should provide necessary and sufficient conditions for ML estimates of the model parameters. When these conditions are met, models must perform well with the current set of data to projected the reliability of the system. For delayed fixes, Crow presented a model which yields, under reasonable assumptions an unbiased estimate of the true system reliability after delayed fixes have been incorporated[Crow, 1983].

5. Selecting the Appropriate Model

Most of the reliability growth models presented in this article are used for special cases. Thus, the current problem is how should the reliability engineers select the best fit model for a particular application. Generally, reliability growth tests are involved in complexities, there is no straightforward analytical way to prove or disprove the validity of any of the reliability growth models. It would be more useful, from a practical point of view, to have tests available to determine the appropriateness of the models for representing the reliability growth of the system. Some literature suggest goodness-of-fit tests to demonstrate the validity of the models against real data. See the Duane model[Duane, 1964], the Cox-Lewis model[Cox, 1966], the Littlewood model[Littlewood, 1981B] and the Littlewood-Verrall model[Littlewood 1973, 1981B]. If a test statistics is greater than the critical value, the undergoing model is rejected at the designated significance level, otherwise the model is accepted and may be used to track and project system reliability. Although these tests are applicable in some situations, a global goodness-of-fit criteria for selecting the best for particular application is not available. Therefore, engineers must consider carefully the assumptions made about theunderlying failure process. These assumptions can be incorporated with prior engineering experience obtained from related development testing programs. From the assumptions made for the testing, one can choose different models with different growth forms and testing formats. The different characteristics which were provided in the section "Characteristics of Models" will give valuable guidelines for model selection. In addition, it appears that the form of the existing models can be modified without a great deal cf effort. Such extended models may acccommodate variations in several important factors and could be appropriate for a particular growth testing situation.

6. Conclusion

The classification scheme presented in this article provided a coherent framework for the literature based upon the salient features of the existing models. The characteristics used to classify the literature are the modeling approach, the data requirement, the failure process, and the failure mode.

Through the results of the literature survey and classification, one can recognize that a best fit model for a certain environment could be useless for other applications. This survey would be useful now for reliability engineers to evaluate and select reliability growth models for specific applications.

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