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TRIPLE DIFFERENCE APPROACH TO LOW EARTH ORBITER PRECISION ORBIT DETERMINATION

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ABSTRACT

A precise kinematic orbit determination (P-KOD) procedure for Low Earth Orbiter(LEO) using the GPS ion-free triple differenced carrier phases is presented. Because the triple differenced observables provide only relative information, the first epoch's positions of the orbit should be held fixed. Then, both forward and backward filtering was executed to mitigate the effect of biases of the first epoch's position. P-KOD utilizes the precise GPS orbits and ground stations data from International GPS Service (IGS) so that the only unknown parameters to be solved are positions of the satellite at each epoch. Currently, the 3-D accuracy of P-KOD applied to CHAMP (CHAllenging Minisatellite Payload) shows better than 35 cm compared to the published rapid scientific orbit (RSO) solution from GFZ (GeoForschungsZentrum Potsdam). The data screening for cycle slips is a particularly challenging procedure for LEO, which moves very fast in the middle of the ionospheric layer. It was found that data screening using SNR (signal to noise ratio) generates best results based on the residual analysis using RSO. It is expected that much better accuracy are achievable with refined prescreening procedure and optimized geometry of the satellites and ground stations.

Keywords: orbit determination, triple differencing, algorithms and implementation

1. INTRODUCTION

With current fully operational configuration and technology, the orbit of a Low Earth Orbiter (LEO) can be determined with accuracy better than 5 cm of RMS as demonstrated in the satellite altimetry mission TOPEX/POSEIDON (Bertiger et al. 1994, Tapley et al. 1994).

Conventionally, there are three strategies to determine precise orbit of a LEO with GPS: dynamic, kinematic and reduced-dynamic or hybrid strategies depending on the degree of incorporated dynamic modeling of the physical forces on the satellite such as gravity, solar radiation, atmospheric drag, etc. as well as the physical properties of the satellites like shape and dimension. Among those methods, under the assumption of a complex dynamic behavior of LEO satellite caused by low altitude (below 700 km), the kinematic strategy potentially generates more accurate orbit than the dynamic approach.

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Although the reduced dynamics was proven to be the best OD method in terms of accuracy as in TOPEX/POSIDON mission (Yunck et al. 1994), the procedures for the reduced dynamics are much complex than that of the kinematic cases because of the all dynamic models and optimal weighting assignment. Therefore, with its much simpler and efficient procedures, the kinematic strategy could generate much faster orbit compared to the other strategies, which is very important in some of the applications like near real time weather forecasting (Grejner-Brzezinska et al. 2001).

As a matter of fact, the kinematic approach has already applied to the TOPEX/POSEIDON mission as well. Using double differenced carrier phase observables, the accuracy of better than 5 cm in radial direction and 16 cm in 3D has been achieved (Byun & Schutz 2001). Considering the receiver on TOPEX/POSEIDON has only 6 channels, better results are expected with a receiver having more channels.

In this paper, a kinematic LEO positioning algorithm and results using the triple differenced GPS carrier phase data from CHAMP satellite is presented. The triple difference technique used in this study is a modified and extended version of the previous algorithm GODIVA applied to GPS POD for the LEO OD (Grejner-Brzezinska 1995, Goad et al. 1996). The triple differenced phases have some advantages such as reduced number of unknowns caused by no ambiguity resolution, easy detection of cycle slips, and reduced computation memory and processing time. On the other hand, the complicated structure of the covariance matrix is the major disadvantage of the triple differencing.

The main goal of this study includes the fast generation of orbit with full analysis of CHAMP data on the aspect of data quality and geometry. The fast orbit also contributes the near-real time weather forecasting through GPS occultation technique

2. DATA PROCESSING

The P-KOD primarily consists of three main procedures, namely preprocessing of GPS data, main estimation of LEO POD, and post processing of the estimated orbit as shown in Figure 1. In the preprocessing, the construction of the database using IGS reference stations' and LEO observation data, the detection of the cycle slips and outliers, the correction of the stations' clock error, and the triple differencing using data from LEO and ground stations are performed. The calculation of a priori orbit from double differenced pseudoranges, construction and reduction of the normal matrix, and estimation of the orbits are performed in the main step.

One of the disadvantages in the kinematic POD is that the estimated orbits still contain some spikes caused by bad geometry and even no solution is calculated when the number of the tracked satellites are not sufficient. Therefore, after the converged positions are obtained, the spikes have to be removed and the epochs showing singularity should be filled with interpolation.

Once the continuous orbit is obtained, the velocity of the LEO is computed through the numerical differentiation since velocity is not the state vector in kinematic approach. Currently, P-KOD calculates 24 hrs orbits within 2 hrs on the platform of 1 GHz processor of PC with 60-70 ground station data.

2.1 Preprocessing

The quality of the GPS data and geometry are the most important factors for the kinematic orbit determination. Two traditional cycle slip detection algorithms are implemented in this study using the test quantities of (1) ionosphere-only linear phase combination and phase/code combination (Hofmann-Wellenhof et al. 1997), (2) widelane ambiguity based on the double difference phase-code combination (Blewitt 1990). In addition, the Signal-to-Noise Ratio (SNR) from the receiver on

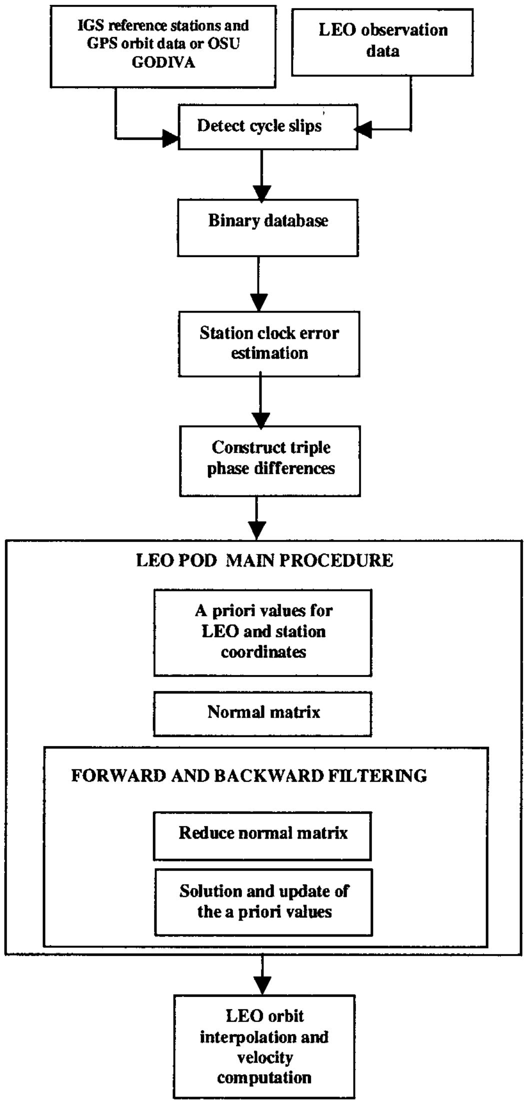

Figure 1. Schematic diagram of the kinematic LEO POD.

board CHAMP is implemented to detect the cycle slip.

To analyze and verify the results of cycle slip detection, the rapid science orbit (RSO) from GFZ were used to generate a reference marks for the cycle slips. The residuals using the RSO as approximated orbit clearly show the quality of the data for each epoch. Unfortunately, it has turned out that none of the traditional methods for the cycle slip is working properly for CHAMP data because of the low SNR caused by variable ionosphere and high speed of the vehicle. According to the cycle slip detection using RSO, the data from CHAMP contains 5.5 % of cycle slip while ground stations have only 0.2 %. This bad quality of CHAMP basically makes those conventional

Table 1. Standard deviation of the estimated orbits compared to RSO of the CHAMP after forward & backward adjustment (unit: m).

	Std(x)	Std(y)	Std(z)	Std(3d)
Forward	0.54	1.01	1.22	1.68
Backward	0.12	0.25	0.19	0.33

Table 2. Standard deviation of estimated velocity with respect to RSO.

	Std(x)	Std(y)	Std(z)	Std(3d)
Velocity	0.6mm/s	0.7mm/s	1.1mm/s	1.4mm/s

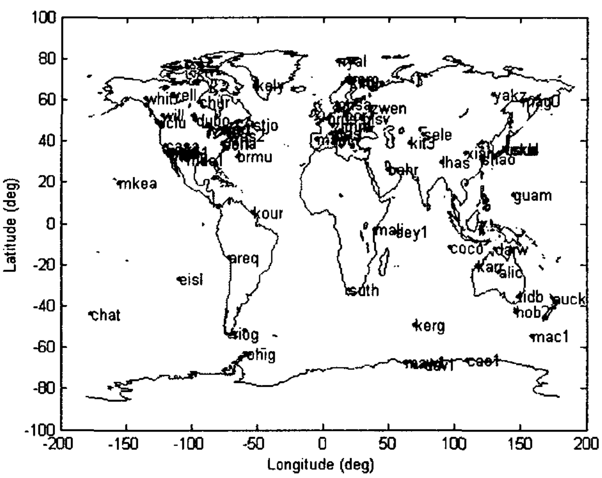

Figure 2. Distribution of selected ground stations for kinematic LEO POD.

cycle slip methods be failed. Using SNR after setting appropriate threshold for declaring the cycle slip, however, about 80-86 % of the cycle slips are successfully detected. For detailed and complete discussion on cycle slip detection, see Grejner-Brzezinska et al. (2001).

Figure 2 shows the distribution of the 65 ground stations selected from the International GPS Service (IGS) centers for this study. At this point, the station configuration is not optimally designed yet. While many stations are densely located in the area of North America and Europe, not many stations are located in Africa, South America and polar region. After selecting the ground stations, the data for all station is screened for cycle slips and the station contains too much cycle slips (> 2 %) is removed and replaced by other neighbor station with good quality data. Overall, the selected ground stations contain the cycle slip less than 1 % in 24 hr span.

2.2 Orbit estimation

2.2.1 Ion-free triple differenced phases

In this study, the ion-free triple difference phases are used as observations:

$$\Phi^{kl}_{ij,iono-free,dt} = \rho^{kl}_{ij,dt} + T^{kl}_{ij,dt} + \alpha_1 \varepsilon^{kl}_{ij,1,dt} + \alpha_2 \varepsilon^{kl}_{ij,2,dt}, \tag{1}$$

where $\Phi^{kl}_{ij,iono-free,dt}$ is the ion-free triple-differenced phases between satellites k, l and stations i, j; $\rho^{kl}_{ij,dt}$ is the triple-differenced geometric ranges; $T^{kl}_{ij,dt}$ is the triple-differenced tropospheric effect; $\alpha_1 = f_1^2/f_1^2 - f_2^2$, $\alpha_2 = -f_2^2/f_1^2 - f_2^2$ are the coefficients used to eliminate the ionospheric effect; $\varepsilon^{kl}_{ij,1,dt}$ and $\varepsilon^{kl}_{ij,2,dt}$ are triple-differenced phase errors in L1 and L2 frequencies, respectively. Note that only remaining unknowns in equation (1) would be the positions of the receiver, namely the positions of the LEO.

2.3.2 Cholesky decomposition

In kinematic batch procedure, the number of observations could be tremendously large depending on the number of epochs of the batch. In this case, a conventional inverse routine might not be sufficient to handle the complex large covariance matrix of the triple differenced observables in terms of efficiency as well as the accuracy. Therefore, a convenient and efficient decorrelation scheme using Cholesky factorization is applied to the covariance matrix. In this scheme only the memory for two consecutive epochs are assigned and decorrelated. After that, the first epoch's data is eliminated and second epoch's data is moved to the location of the first block in the memory and the second block is filled with the third epoch's data. After this decorrelation, the normal matrix is stored in a physical memory and then solved for the unknowns using an adjustment technique. For details on this scheme, see Grejner-Brzezinska (1995) and Yang (1995).

2.3.3 Estimation procedure

As shown before, the observations for the estimation of LEO positions are the ion-free triple differenced phases (1) given more explicitly:

$$\begin{aligned} \Phi^{kl}_{ij,iono-free,dt} &= \rho^{kl}_{ij,dt} + T^{kl}_{ij,dt} + \alpha_1 \varepsilon^{kl}_{ij,1,dt} + \alpha_2 \varepsilon^{kl}_{ij,2,dt} \\ &= [\rho^{kl}_{ij} + T^{kl}_{ij}]_{t_2} - [\rho^{kl}_{ij} + T^{kl}_{ij}]_{t_1} \\ &\quad +[\alpha_1 \varepsilon^{kl}_{ij,1} + \alpha_2 \varepsilon^{kl}_{ij,2}]_{t_2} - [\alpha_1 \varepsilon^{kl}_{ij,1} + \alpha_2 \varepsilon^{kl}_{ij,2}]_{t_1} \end{aligned} \tag{2}$$

Since the observations are non-linear with respect to the unknowns, the equation (2) should be linearlized to construct the observation model in the usual form of

$$Y = A\xi + e, \quad e \sim N(0, \Sigma), \tag{3}$$

where Y is the vector of the measurement and e is the measurement error with assumed normal distribution with zero mean and variance of Σ.

Assuming the station i is the LEO, the observation and design matrix are derived as follows for the triple differenced phase between epoch t_1 and t_2:

$$\begin{aligned} Y &:= \Phi^{kl}_{ij,iono-free,dt} - \rho^{kl\ O}_{ij,dt} \\ A\xi &:= [A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6] \begin{bmatrix} dx_{t_1} \\ dy_{t_1} \\ dz_{t_1} \\ dx_{t_2} \\ dy_{t_2} \\ dz_{t_3} \end{bmatrix}, \end{aligned} \tag{4}$$

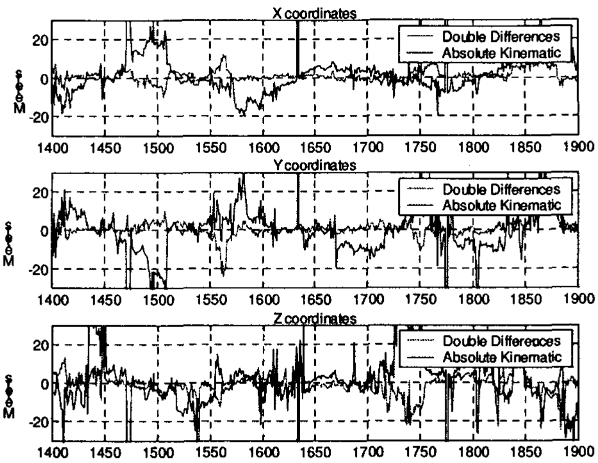

Figure 3. Difference between RSO and a priori orbits from absolute kinematic and DD pseudorange algorithm.

where ${\rho^{kl}_{ij,dt}}^{O}$ is the triple differenced geometric ranges using a priori values of LEO orbit, and $A_1 \sim A_6$ are the partial derivatives of the observations with respect to the unknowns. Using the measurements and design matrix in equation (4), a least square estimation was conducted to estimate the unknowns.

Because the triple differencing provides only relative information like the conventional leveling survey, the position of the first epoch is fixed in every batch. Fixing the first epoch position with a priori information means the first epoch position is biased and the estimates from the first portion of the batch are considerably affected by the bias. Therefore, using the results from the forward batch filter, backward filtering should be conducted to eliminate the bias effect from the wrong initial values.

2.3 Velocity and interpolation

Usually, the estimated positions are suffered from the singularity and bad estimates from poor geometry appearing as gaps or spikes. Therefore, a spike removal and interpolation routines should be applied to the estimated positions to obtain smoothed and continuous ones. Then, the continuous positions are numerically differentiated to get velocities. Because of the high frequency noise caused by the numerical differentiation, a smoothing has to be applied with numerical differentiation.

3. RESULTS AND ANALYSIS

3.1 A priori positions

For a priori positions of LEO, absolute kinematic positioning using pseudorange and double differenced pseudorange data were conducted (Figure 3). The data used is obtained on June 15, 2001 and the data interval is 30 seconds. Although the accuracy of the positions is not sufficient for any scientific applications, it is sufficient to be used as initial values for further estimation procedure, namely triple differenced phase procedure. Furthermore, this initial result provides the information about the geometry of satellites and quality of the LEO clock. For example, one can instantly notice that some peaks such as epochs around 1500 and 1630 have disappeared at the DD estimates

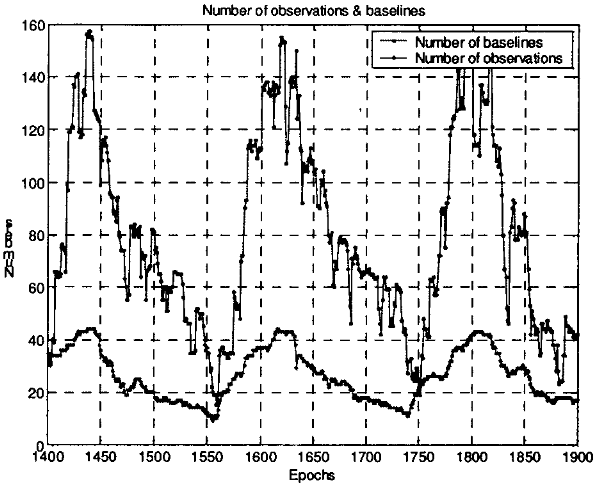

Figure 4. Number of baseline and observations for DD estimates.

indicating that the clock of LEO was particularly unstable for the epochs. In addition, the big peak around epoch 1750 appearing in both absolute and DD estimates indicates that the geometry for those epochs is relatively poor.

It should be noted that overall accuracies of the estimated orbits from the absolute and DD pseudorange are much poorer than expected. This is considered to be the effect of low SNR from low altitude satellites. In general, the less number of observation and baseline results in poorer estimates as expected (see Figure 3 and Figure 4).

3.2 Orbit from the triple differencing

As mentioned before, the most critical procedure to get a good estimated orbit is to clean the data at the pre-processing stage. If we have a priori values with sufficient accuracy like in dynamic or reduced dynamic approach, then the cycle slip as well as outliers can be easily detected by investigating the residual, namely the differences between the observations and calculated ranges (Colombo et al. 2002, personal communication with Da Kuang at JPL). Since the accuracy of the a priori orbits in this case is no better than 5 meters in RMS, those orbits cannot be used for the detection of the cycle slip and outliers at the stage of normal matrix construction. Naturally, the only step for the detection of the bad observations or cycle slip is at the stage of data prescreening and current cycle slip detection using SNR was applied for the data cleaning. In Figure 5, the difference between RSO and estimated orbits from the forward filtering is presented. These 500 epochs are one continuous segment between the singularities and will be used for the detailed analysis of the result from orbit to velocity. Note the effect of the biases at the first epoch lasts for certain period, up to 120 epochs in this case, of the estimated orbits as shown in Figure 5.

After the backward filtering, significant reduction of the biases and 3 dimensional RMS of 33 cm was achieved as seen in Figure 6 and Table 1.

According to Zhao (1998), the orbit error of 30 cm is required to estimate the temperature profile in GPS sounding better than 1 degree up to altitude of 40 km. Therefore, the kinematic orbit could be successfully applied on the fast atmospheric profiling and weather forecasting when the cycle slip detection algorithm is more refined.

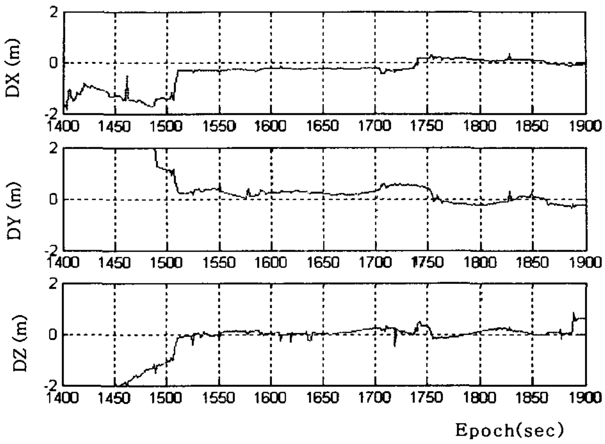

Figure 5. Difference between RSO and the estimated orbits from the forward filtering.

Table 3. Number of singularities and duration in 24 hr data set.

# of Segment	Duration of epochs	# of epochs
1	253-255	3
2	678-682	5
3	752-753	2
4	1080-1080	1
5	1314-1314	1
6	1392-1392	1
7	1904-1904	1
8	2394-2395	2
9	2792-2792	1
SUM		17

3.3 Derived velocity

To provide proper data for atmospheric sounding, the velocity of LEO should be calculated as a final step. As shown in Table 2, the 3D accuracy of 1.2 mm/s is achieved relative to the published RSO.

The velocity of calculation in the kinematic OD has a couple of disadvantages. Firstly, there is no estimated variance for the velocity since it is not estimated as states. Secondly, the result of the numerical differentiation depends on many factors like order of polynomial and length of smoothing window. Therefore, to obtain consistent results on the velocity through numerical differentiation, many tests have to be carried out to find optimal values for those factors by comparing it to another good estimated velocity, for example the velocity from dynamic approach.

3.4 Discussions

In kinematic POD, the number of observations at each epoch is the most critical factor for its dependence on the observations. If the number of the observation at an epoch is less than four, the orbit could not be calculated because of the singularity. On the date for the test data set, thanks to the state-of-art 12-channel receiver on CHAMP, there is no epoch having less than four observations.

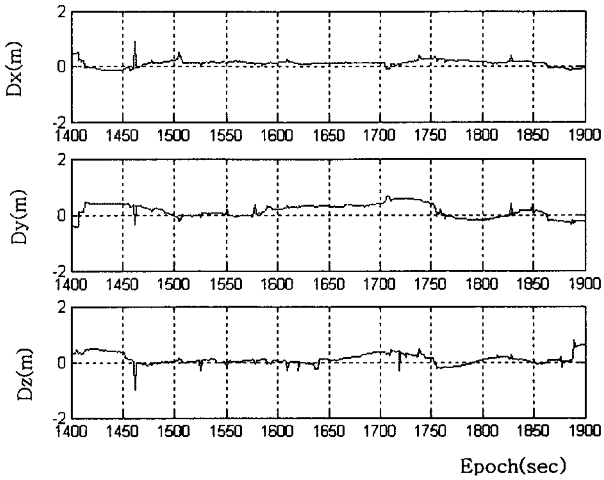

Figure 6. Difference between RSO and the estimated orbits from the forward & backward filtering.

There are, however, about 10 minutes of gaps in data for unknown reason. In most of the time, CHAMP tracked six or more satellites for the test date. After excluding cycle slips, bad observations and taking triple differencing, 17 out of total 2880 epochs in 24 hr span appeared as singular, which leaves 10 segments in 24 hr data set (Table 3).

Whenever the singularity occurs, the batch should be rebuilt excluding the epoch and restart the estimation procedure. The ten segments spanning 24 hrs are processed to estimate orbit in separate batch. As seen in Table 4, it was found that each batch produces consistent results with accuracy ranged 20-50 cm in RMS except last segment. The reason for the poor result on last segment is the small size of the batch as well as bad geometry.

The magnitude of bias at the starting epoch and overall geometry also affect the convergence as one can see in segment 3 and 6. In segment 3, the geometry was better but the initial bias was larger than the segment 10. Therefore, with good geometry, it converges to the solution but very slowly, after 13 iterations, because of the short span. The segment 6 had a better geometry and initial value compared to the segment 10 and converged in the first iteration to the solution.

4. CONCLUSIONS

An algorithm of the precise kinematic orbit determination for LEO using the triple differenced phases has been developed and applied to CHAMP. The three-dimensional accuracy of the estimated positions and velocities from P-KOD are better than 35 cm and 1.5mm/sec, respectively with respect to the published RSO. An analysis on the quality of the CHAMP data was performed utilizing CHAMP RSO. Because of the low SNR caused by the fast movement of CHAMP through a part of ionosphere, the detection of outliers and cycle slips was not sufficient using conventional methods. Using SNR, 80-86 % of the cycle slip was detected and the algorithm is under refinement.

We expect that many estimated orbits from different approach are available within this year. A comparison and analysis of orbits from different methods like no differenced and double differenced approach should be conducted to identify the best methods in terms of accuracy and efficiency,

Table 4. Standard deviation of the estimated orbits compared to RSO of the CHAMP after forward & backward adjustment (unit: m).

# of segment and epochs	X (m)	Y (m)	Z (m)	3D (m)	# of iteration
1 (251)	4.3	15.0	14.7	21.4	1
2 (421)	9.1	25.7	18.4	32.9	2
3 (68)	11.1	7.5	26.6	29.8	13
4 (325)	11.3	20.8	16.1	28.7	1
5 (232)	20.4	6.0	19.0	28.6	3
6 (76)	20.3	7.1	17.9	27.9	1
7 (510)	18.4	18.7	16.3	30.9	1
8 (488)	22.9	12.0	29.4	39.1	1
9 (395)	26.5	17.9	35.1	47.5	2
10 (87)	6.6 2	7.0	88.4	92.9	1

especially for LEO. Eventually, the advantages and disadvantages of the approach using the triple differences will be clearly defined and analyzed.

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