# Generation of RMS Hazard-Compatible Artificial Earthquake Ground Motions

### RMS 가속도에 의한 인공 지진파 생성기법

김 진 만<sup>\*</sup> Kim, Jin Man

#### 국문요약

지진응답 해석 시 불확실한 지진헌상을 추정하여 설계지진파를 선정하는 것은 어려운 일 중의 하나이다. 게다가 제한된 숫자의 설계인자에 상응하는 지진파가 결코 유일하지 않다는 문제도 있다. 따라서 동일한 설계진도에 상응하는 여러 지진파들로부터 구한 응답치들이 서로 크게 차이가 날 수 있다. 본 논문은 이 같은 지진하중의 불확실성을 체계적으로 고려하는 실용적인 지진파 생성 기법을 제시한다. 이 기법은 에너지 개념의 RMS 지진가속도에 기반하며 주요 지진파 설계인자의 불확실성을 고려한다. 시뮬레이션을 통해, 이 새로운 RMS 기법이 지진재해에 상응하는 지진파를 대량 생성하는 경우에 적합하며 따라서 소량의 지진파 생성에 적합한 기존의 방법들과 비교할 때 특히 확률론적 지진응답 해석 시 유용하다는 점을 확인하였다.

주요어 : 지진, 인공지진파, 추계과정, 생성, RMS 재해, 불확실성

#### **ABSTRACT**

Due to the random nature of earthquake, the definition of the input excitation is one of the major uncertainties in the seismic response analysis. Furthermore, ground motions that correspond to a limited number of design parameters are not unique. Consequently, a broad range of response values can be obtained even with a set of motions, which match the same target parameters. The paper presents a practical probabilistic approach that can be used to systematically model the stochastic nature of seismic loading. The new approach is based on energy-based RMS hazard and takes account for the uncertainties of key ground motion parameters. The simulations indicate that the new RMS procedure is particularly useful for the rigorous probabilistic seismic response analysis, since the procedure is suitable for generation of large number of hazard-compatible motions, unlike the conventional procedures that aim to generate a small number of motions.

Key words: earthquake, artificial motion, stochastic process, generation, RMS hazard, uncertainty

#### 1. Introduction

Due to the random nature of earthquake motions, the definition of the input excitation is one of the major uncertainties in the seismic response analysis. Hence, the estimation of key design parameters of seismic ground motion and subsequent quantitative assessment of the associated uncertainties has been an important issue in geotechnical earthquake engineering. The most common approach has been to use a deterministic procedure in which several time histories of acceleration are selected as free field or bedrock design motions. The design motions are often defined as motions of which response spectra match a given design response spectrum. However, a single time history of acceleration matching the design response spectrum represents only one possible realization among infinite numbers of possible scenarios of earthquake motions. By the same token, ground motions that correspond to a limited number of design parameters are not

There are, in general, three main methods for generation of design ground motion: (1) modification of recorded ground motions(e.g., Lilhanand and Tseng<sup>(1)</sup>), which either rescale recorded motions to the target amplitude or adjust the time scale to get desirable frequency content, or splice parts of recorded motions together, (2) generation of genuine artificial motion in terms of stochastic processes, (e.g., Der Kiureghian and Crempien<sup>(2)</sup>), and (3) generation of artificial motion using Green's function techniques(e.g., Hartzell<sup>(3)</sup>). Among them, the stochastic representation of the earthquake motion hazard provides, at least theoretically, a systematic way of representing the infinite number of possible scenarios, which correspond to a certain level of hazard.

The paper presents a practical probabilistic approach that can be used to systematically model the stochastic

unique. Consequently, a broad range of response values can be obtained even with a set of motions, which match the same target parameters. The problem may be effectively approached by generating a large series of hazard-compatible artificial motions, and by using them in subsequent response analyses.

<sup>\*</sup> 정회원·현대건설 기술연구소 선임연구원(대표저자: jmkim@hdec.co.kr) 본 논문에 대한 토의를 2003년 4월 30일까지 학회로 보내 주시면 그 결과를 계재 하겠습니다. (논문접수일: 2002. 11. 29 / 심사종료일: 2003. 1. 21)

nature of seismic loading. The new approach, which is based on energy-based RMS hazard, is suitable for generating a large series of hazard-compatible artificial motions. Uncertainties in modeling seismic source, path attenuation, and local soil conditions are taken into account in the process of artificial earthquake generation. Finally, the applicability of the proposed approach is illustrated with example seismic slope stability analyses.

#### 2. Stochastic characterization of ground motion

#### 2.1 Characterization of stationary ground motions

Earthquakes are essentially non-stationary processes, and in many circumstances the use of stationary models may prove inadequate for earthquake engineering purposes. However, due to simplicity, many investigators have characterized earthquake ground motions as a stationary random process(e.g., Housner and Jennings<sup>(4)</sup>, Shinozuka<sup>(5)</sup>). A zero mean stationary Gaussian random process can be completely defined by its(one-sided) power spectral density (PSD) function,  $G(\omega)$ . The PSD describes how the power (energy per unit time) is distributed among the frequencies of vibration. Based on Kanai's study<sup>(6)</sup> of the frequency content of a limited number of recorded strong ground motion, Tajimi<sup>(7)</sup> proposed the following widely quoted form for the power spectral density(PSD) function of ground motion.

$$G(\omega) = G_0 \frac{1 + 4\zeta_g^2 (\omega/\omega_g)^2}{\left[1 - (\omega/\omega_g)^2\right]^2 + 4\zeta_g^2 (\omega/\omega_g)^2} \tag{1}$$

Where  $G(\omega)$  is the energy content at frequency  $\omega$ ,  $G_0$  is some measure of shaking intensity,  $\zeta_g$  and  $\omega_g$  are the parameters termed the K-T damping coefficient and K-T frequency. Eq. (1) is the form exactly same as the acceleration response PSD of SDOF linear system of natural frequency  $\omega_g$  and damping ratio  $\zeta_g$  upon the input of white noise basal acceleration  $G_0$ .

It is sometimes convenient to define normalized K-T PSD with unit variance. The normalized PSD can be obtained by dividing the PSD(Eq. (1)) with its variance as:

$$G^*(\omega) = \frac{1}{\sigma^2} G(\omega) \tag{2}$$

For the Kanai-Tajimi PSD, the variance is related to its parameters as(Crandall and Mark<sup>(8)</sup>):

$$\sigma^2 = \int_0^\infty G(\omega) d\omega = \frac{\pi G_0 \omega_g}{4\zeta_g} (1 + 4\zeta_g^2)$$
 (3)

The intensity parameter  $G_0$  can then be computed from the variance as:

$$G_0 = \frac{4\zeta_g \sigma^2}{\pi \omega_\sigma (1 + 4\zeta_\sigma^2)} \tag{4}$$

#### 2.2 RMS, Energy-based intensity parameter

Traditionally, most seismic hazard analyses use either maximum values of ground motion or a response spectrum to characterize the intensity of ground shaking(e.g., Housner<sup>(9)</sup>, Abrahamson and Silva<sup>(10)</sup>). Response spectrum is the maximum response of a single-degree-of-freedom(SDOF) system to a particular input motion and can be defined as a function of the natural frequency and damping ratio of the SDOF system. However, using a maximum value to characterize the intensity of shaking is often inadequate, especially when dealing with structures whose response depends on total seismic energy rather than peak values of ground shaking. One of the energy-based parameters that include the effects of the intensity and frequency content of ground shaking is the RMS(root mean square) acceleration, which is defined as(Housner<sup>(11)</sup>):

$$RMS_{a} = \frac{E(T_{d})}{T_{d}} = \left[ \frac{1}{T_{d}} \int_{t_{0}}^{t_{0}+T_{d}} a^{2}(\tau) d\tau \right]^{1/2}$$
 (5)

where  $a(\tau)$  is a acceleration time history,  $t_0$  is a initial time of interest,  $T_d$  is duration of the strong ground shaking, and  $E(T_d)$  is a total energy for the duration  $T_d$ . Similarly, temporal RMS(Fig. 1) can be defined by replacing  $T_d$  with a small time interval  $\Delta t$  as:

$$RMS_a(t) = \left[ \frac{1}{\Delta t} \int_t^{t+\Delta t} a^2(\tau) d\tau \right]^{1/2} \text{ for } \Delta t \to 0$$
 (6)

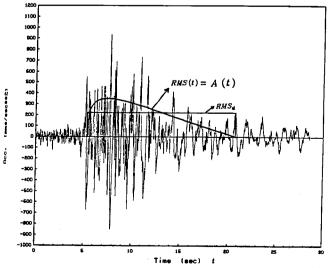


Fig. 1 Time-invariant and temporal RMS(Wang and Kavazanjian (12)

The RMS is a time-averaged value of shaking intensity and the RMS acceleration is not strongly sensitive to large, high frequency parts of accelerations, which is a desirable characteristic for engineering purposes. Its value, however, is sensitive to the definition of duration(e.g., McCann<sup>(13)</sup>). The RMS definition(Eq. (5)) is closely related to another energy-based parameter, Arias intensity(Arias<sup>(14)</sup>), which is written as:

$$I_a = \frac{\pi}{2g} \int_0^\infty a^2(t) dt \tag{7}$$

The Arias intensity in the above original form is independent of the method used to define the duration of strong shaking because its evaluation is defined over the whole time domain of earthquake ground motion. As a result, the Arias intensity has been used in a number of recent geotechnical applications(e.g., Kayen and Mitchell<sup>(15)</sup>).

#### 3. Generation of Stationary Ground Motions

A number of procedures have been developed for generating earthquake ground motions as random(or stochastic) processes. Majority of them are either based on ARMA models(e.g., Chang et al.<sup>(16)</sup>, Shamaras et al.<sup>(17)</sup>) or spectral representation(e.g., Housner and Jennings<sup>(4)</sup>, Shinozuka<sup>(5)</sup>). Spectral representation of ground motion with the PSD function, in general, provides both clear interpretation and computational efficiency. It can also easily incorporate the non-stationarity of the intensity and frequency content, as will be shown later.

A periodic function can be expressed by series of sinusoidal motions and, especially, the zero-mean process can be represented as:

$$X(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \theta_i)$$
 (8)

Where  $A_i$  is the Fourier amplitude,  $\omega_i$  is the circular frequency, and  $\theta_i$  is the phase angle of the  $i^{th}$  contributing sinusoid. By fixing an array of amplitudes and randomly generating different arrays of phase angles, we can produce different motions with the same general appearance but different details(Gasparini and Vanmarcke<sup>(18)</sup>). The total power of the steady state motion X(t) is  $\sum_{i=1}^{n} \frac{1}{2} A_i^2$ . For stationary random process this total power may be expressed in terms of the power spectral density function as:

$$\frac{1}{2}\sum A_i^2 = \sum G(\omega_i) \Delta \omega \approx \int_0^\infty G(\omega) d\omega \tag{9}$$

Therefore the Fourier amplitude  $A_i$  is related to the (one-sided) power spectral density function  $G(\omega)$  as:

$$\frac{1}{2}A_i^2 = G(\omega_i)\Delta\omega \tag{10}$$

Thus,  $G(\omega_i)\Delta\omega$  can be interpreted as the contribution from the sinusoid with frequency  $\omega_i$  to the total power of the motion. Each different array of phase angles can be modeled by statistically independent random phase angles  $\theta_i$ , which are uniformly distributed between 0 and  $2\pi$ . Eq. (8) then becomes(Rice<sup>(19)</sup>):

$$X(t) = \sum_{i=1}^{n} \sqrt{2G(\omega_i)\Delta\omega} \sin(\omega_i t + \theta_i)$$
 (11)

As the number n of sinusoidal motions becomes large, the distribution of the process X(t) approaches Gaussian distribution by virtue of the central limit theorem, as long as  $A_i$  are of similar magnitude (Yang<sup>(20)</sup>). The above formula defines an infinite ensemble of time histories with the same frequency content but with randomly distributed phase angles between the individual components. Sample earthquake motion can then be obtained either by filtering white noise through a SDOF linear filter with natural frequency  $\omega_{\scriptscriptstyle g}$  and viscous damping  $\zeta_{\scriptscriptstyle g}$  or by directly transforming them into the time domain by FFT(fast fourier transform, Cooley and Tukey<sup>(21)</sup>). Fig. 2 shows a stationary time history and its corresponding key parameters that are self-explanatory. The theoretical and actual Fourier amplitude and PSD are also shown. This study defines an upper cutoff frequency  $\omega_n$ , beyond which the contribution is insignificant, to be 100rad/sec(about 16cycles/sec).

#### 4. Generation of non-stationary ground motions

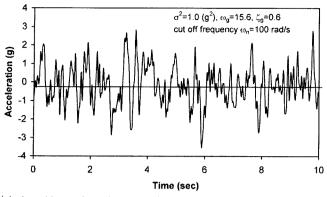
#### 4.1 Representation of non-stationary ground motions

Earthquakes are essentially non-stationary processes, and in many circumstances the use of stationary models may prove inadequate for earthquake engineering purposes. Transient character of the intensity content can be added by multiplying the stationary motion by a deterministic modulating(envelope) function m(t). The non-stationary motion Y(t) then becomes

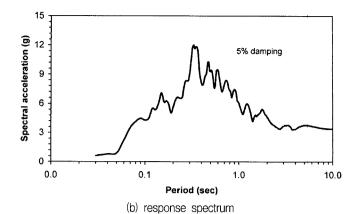
$$Y(t) = m(t)X(t)$$

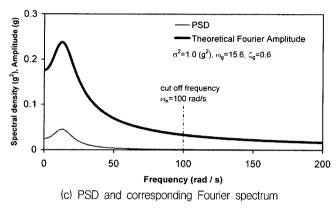
$$= m(t) \sum_{j=1}^{n} \sqrt{2G_{j}(\omega_{j})\Delta\omega} \sin(\omega_{j}t + \theta_{j})$$
(12)

It should be noted that the resulting motion is still



(a) time history based on the frequency parameters for firm soil proposed by Kanai(1957)





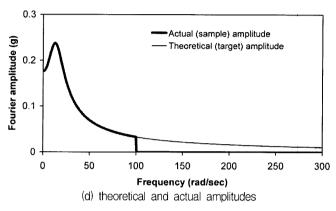


Fig. 2 Sample stationary artificial earthquake ground motion

stationary in terms of frequency content. Various modulating functions have been proposed to incorporate a non-stationary aspect of the earthquake ground motions. Almost all recorded ground motions show initial build-up followed by a strong-motion part and then a die-down segment. The most common forms of the modulating function are in types of the "Exponential", "Trapezoidal", and "Compound". Even though these types of the modulating function have been used successfully they may not be adequate in a situation of generating a large number of input ground motions, which correspond to the certain level of seismic hazard. That is primarily because of difficulties in developing the consistent statistics of model parameters, which define the size and shape of the modulating function since those modulating functions are duration-dependent. Lack of statistics of the model parameters makes it difficult to consistently assess the model uncertainty.

Assuming that the shape of ground motions is, in general, independent of the duration, the non-stationary of ground motion can be modeled by a duration-independent modulating function. A set of model parameters can thus be used to describe any ground motions regardless of the duration. In their liquefaction-related analytical approaches, Wang and Kavazanjian<sup>(12)</sup> proposed a trigonometric modulating function that has two model parameters to define

the shape of the modulating function, defined as:

$$m(t) = \sin^{\alpha}(\pi(t/t_d)^{\beta}) \tag{13}$$

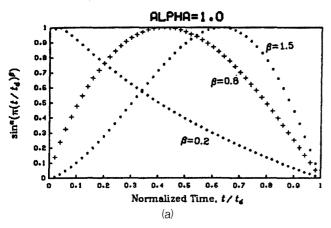
Where  $\alpha$  and  $\beta$  are two parameters to determine the shape of the modulating function and  $t_d$  is the duration of motion(Fig. 3). The function is in a normalized form for both intensity and duration. This model provides a convenient way in developing the statistics of shape parameters since it is in a normalized form and can thus be used independent of the size and the duration of ground motion. For this reason, the model is adopted in this study.

## 4.2 Relationship between the modulating function and RMS hazard

As discussed earlier, the normalized PSD is obtained by dividing the PSD by its corresponding variance. When the stationary process  $X^*(t)$  is a normalized process with a unit variance and PSD function  $G^*(\omega)$ , Eq. (12) (non-stationary ground motion) can be rewritten in terms of the normalized PSD function and modulating function m(t) as:

$$Y(t) = m(t)X^{*}(t)$$

$$= m(t) \sum_{i=1}^{n} \sqrt{2G^{*}(\omega_{i})\Delta\omega} \sin(\omega_{i}t + \theta_{i})$$
(14)



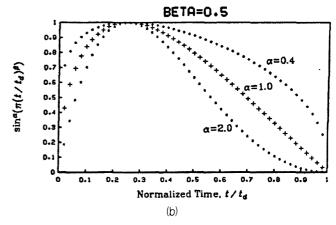


Fig. 3 Trigonometric modulating function  $\sin^{\alpha}(\pi (t/t_{d})^{\beta})$  and effects of parameters  $\alpha$  and  $\beta$  on the modulation function (Wang and Kavazanjian)

It is straightforward to relate the time-variant RMS to the deterministic modulating function m(t) as follows:

$$RMS_{Y}^{2}(t) = \sigma^{2}(t) = E[Y^{2}(t)] = E \mid m^{2}(t)X^{*2}(t) \mid$$
$$= m^{2}(t)E \mid X^{*2}(t) \mid = m^{2}(t)$$
(15)

The time-variant RMS of the motion Y(t) can thus be identical to the modulating function m(t). In other words, if the normalized power spectral functions are given, nonstationary ground motion can be obtained by a product of the temporal-variant RMS(or time dependent standard deviation) and normalized stationary process of PSD  $G^*(\omega)$ . The temporal RMS and/or modulating function can be obtained by fitting the suggested analytical forms to smoothened accelerograms(Iyengar and Iyengar (22)). It is, however, a common practice in seismic hazard analysis that the RMS hazard is estimated in terms of a single constant value, not in terms of temporal RMS, and thus a determination of the model parameters is required to relate the modulating function m(t) to constant RMS. Wang and Kavazanjian<sup>(12)</sup> derived the relationship between the constant RMS hazard and their trigonometric modulating function  $\sin^{\alpha}(\pi(t/t_d)^{\beta})$ based on the concept of the normalized expected energy (or normalized expected Arias intensity) and 5-95% Arias duration(often called significant duration) such as:

$$m(t) \approx C \cdot \sin^{\alpha}(\pi (t/t_d)^{\beta})$$
 (16)

where the intensity coefficient *C* of modulating function is given as:

$$C = \frac{RMS_Y}{\left[\int_0^1 \sin^{2\alpha}(\pi \tau^{\beta}) d\tau\right]^{0.5}}$$
(17)

The above procedure is, in general, applicable to other types of modulating functions as long as the modulating functions are in a normalized form both in time and intensity such that:

$$0 \le m(\tau) \le 1$$
 for  $0 \le \tau \le 1$  (18)

Wang and Kavazanjian<sup>(12)</sup> also developed the statistics of the model parameters  $\alpha$  and  $\beta$  based on their study of Northern Californian 122 strong ground motions, of which 74 motions are recorded on soil sites and 48 others are rock-site recorded. The data is limited to events with magnitudes of 5.3 to 7.7, and to epicentral distances to the recording site from 8km and 135km. The statistics is based on the 5-95% Arias duration(often called significant duration) of the motion.

#### 4.3 Representation of non-stationary frequency content

Not only the intensity content, but also the frequency content of earthquake ground motion changes with time. Due to simplicity, previous researchers have not taken account of the temporal variation of spectral content of earthquake ground motion(e.g., Housner and Jennings<sup>(4)</sup>, Gasparani and Vanmarcke<sup>(18)</sup>, Lai<sup>(23)</sup>). However, there is a number of methods for characterizing the temporal variation of the spectral content of earthquake motion. Simple but yet efficient approach is to divide the ground motion into several sections small enough so that stationarity of the frequency content within each section can be assumed without much error(Saragoni and Hart<sup>(24)</sup>). The proposal by Saragoni and Hart<sup>(24)</sup> is based on dividing the motion into three sections of equal time interval as:

$$Y(t) = m(t) \sum_{j=1}^{3} (X_j(t_j))$$

$$= m(t) \sum_{j=1}^{3} \left[ \sum_{i=1}^{n} \sqrt{2G_j(\omega_i) \Delta \omega} \sin(\omega_i t_j + \theta_i) \right],$$

$$t = \sum_{j=1}^{3} t_j$$
(19)

In order to avoid the selection of arbitrary number of divided sections, Der Kiureghian and Crempien<sup>(2)</sup> proposed a procedure, with which the number of sections can be determined on a theoretical basis. They found that less than 10 sections are enough in most cases.

$$Y(t) = m(t) \sum_{j=1}^{NS} (X_j(t_j))$$

$$= m(t) \sum_{j=1}^{NS} \left[ \sum_{i=1}^{n} \sqrt{2G_j(\omega_i) \Delta \omega} \sin(\omega_i t_j + \theta_i) \right],$$

$$t = \sum_{i=1}^{NS} t_j$$
(20)

where NS is the number of the sections divided into. One of the problems of this approach(i.e., discontinuous evolutionary process) is that frequency content changes abruptly between each sections of the motion. This study adopts the discontinuous approach, since alternative continuous approaches (e.g., STOCAL-II, Wung and Der Kiureghian<sup>(25)</sup>) appear to be equally problematic in other aspects including their modeling complexity.

Once the number of sections is determined, the frequency and damping parameters of Kanai-Tajimi PSD need to be estimated for each section. Based on the approach by Saragoni and Hart<sup>(24)</sup>, Wang and Kavazanjian<sup>(12)</sup> analyzed 80 out of the same set of 122 earthquake records, which were used to estimate the modulating function parameters and proposed the statistics of the parameter. The PSD functions were obtained from the squared Fourier amplitude  $F^2(\omega)$  of motion records through their well-known relationship and fitted to smoothened T-K PSD functions to estimate PSD function parameters  $\omega_g$  and  $\zeta_g$  using the spectral moments method(Lai<sup>(23)</sup>). Tung et al.<sup>(26)</sup> updated the statistics of modulating and PSD function parameters by including an additional set of 36 earthquake records from the 1989 Loma Prieta earthquake. These values will be used in this study.

#### 4.4 Duration of strong ground motion

The duration of strong motion has received relatively less attention than the intensity and frequency content. The reason may be partly because the duration has a relatively small, if any, influence on the response of the linear system. The duration, however, can have a strong contribution to earthquake-related damage for the system of non-linear characteristics, especially for the system consisting of hysteretic materials.

There have been a number of proposed methods to define the duration of strong ground motion for engineering

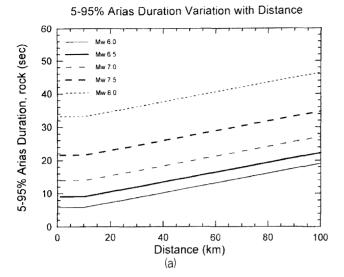
purposes(Bolt<sup>(27)</sup>, Trifunac and Brady<sup>(28)</sup>, Vanmarcke and Lai<sup>(29)</sup>, McCann and Shah<sup>(30)</sup>). The bracketed duration(Bolt<sup>(27)</sup>) is defined as the time between the first and last exceedances of the threshold acceleration(usually 0.05g). Trifunac and Brady<sup>(28)</sup> proposed the time interval between the points at which 5% and 95% of the Arias intensity has been recorded. Vanmarcke and Lai<sup>(29)</sup> suggested the strong motion duration based on the relationships of the Arias intensity and power spectral density on a basis of random vibration theory. The rate of change of cumulative RMS acceleration has also been used to define the strong motion part of an accelerogram, which exhibits a consistent RMS level(McCann and Shah<sup>(30)</sup>)It has been known that earthquake magnitude and distance to site are two most significant factors to affect the duration of strong motion. Since the time required for the release of strain energy increases with magnitude, the duration of strong motion in general increases with increasing magnitude. Local geology can have influence on the duration(Chang and Krinitszky(31)), as the duration at soil sites is likely to be longer than rock sites. Several different relationships(e.g., Trifunac and Brady<sup>(28)</sup>, Dobry et al. (32) have been proposed to estimate the duration of strong motion as a function of magnitude m, distance r, and other factors including local geology as:

$$Duration = f(m, r, etc)$$
 (21)

The 5-95% Arias duration(often called significant duration) is adopted in this study since it is simple to estimate and provides a clear relationship to RMS hazard. The duration for 5-95% Arias intensity is nothing but the 5-95% RMS acceleration's duration. Fig. 4 shows the variation of 5-95% Arias duration with respect to the distance and moment magnitude(Abrahamson and Silva<sup>(33)</sup>) that is used in this study. It should be noted that the 5-95% Arias duration increases linearly with increasing distance at large distances. Since acceleration amplitude decease with distance, duration based on absolute acceleration levels, such as the bracketed duration, decreases with increasing distance. On the other hand the duration based on relative acceleration levels(i.e., significant duration) increases with increasing distance.

#### 5. Generation of response spectra matching motions

The aforementioned RMS-hazard compatible procedure can be extended to generate earthquake ground motions with the response spectra that match the target response spectra. Generation of response spectra-matching motions



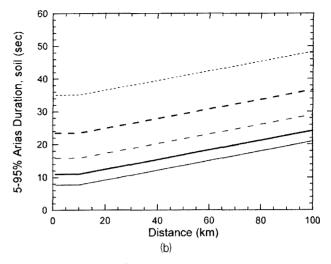


Fig. 4 Variation of 5-95% Arias duration with respect to the distance and moment magnitude(based on the relationships proposed by Abrahamson and Silva<sup>(33)</sup>)

may be appealing since the seismic design criteria are usually given in terms of a set of smooth response spectra (e.g., Newmark et al.<sup>(34)</sup>). In order to produce response spectra-matching motions from the previously described procedure, it is necessary to estimate PSD functions from the target response spectra. Gasparini and Vanmarcke<sup>(18)</sup> proposed a relationship between PSD and response spectra, which is derived from a random vibration analysis of linear SDOF systems. The derivation involves the so-called first-passage problem(e.g., Crandell<sup>(35)</sup>, Corotis et al.<sup>(36)</sup>). The relationship between the response spectrum and the power spectral density(PSD) function of the ground motion is given as:

$$G(\omega_i) \approx \frac{1}{\omega_i \left(\frac{\pi}{4\zeta_s} - 1\right)} \left[ \frac{(S_a)_{s,p}^2}{r_{s,p}^2} - \int_0^{\omega_i} G(\omega_i) d\omega \right]$$
 (22)

Where  $r_{s,p}$  is the peak factor and  $(S_a)_{s,p}$  is the peak acceleration response at  $\omega_i$  with probability p and duration s. Eq. (22) yields the median response when p=0.5. By beginning at the lowest natural frequency  $\omega_1$ , the  $G(\omega_i)$  can be found iteratively from  $\omega_1$  to  $\omega_n$ . It should be noted that the relationship between response spectrum  $S_a$  and PSD function  $G(\omega)$  is not unique but depends on the chosen values of the probability p, duration s, and damping ratio s. The motions, which were generated with this relationship, may need to be further modified to have their response spectra be matched with the target response spectrum. The procedure based on this relationship may therefore be useful for generation of small number of response spectra-matching motions, not for large number of motion generations.

#### 6. Example simulation of artificial motion

The purpose of the example analyses is to illustrate the procedure for the simulation of the RMS-compatible artificial earthquake motions, and its use in seismic analyses. Suppose that we are interested in evaluating the risk of failure of a cohesive slope(Fig. 5) that is located in Berkeley, California, in the seismically active San Francisco Bay Area. For the purpose of seismic analyses, the Hayward fault that is the closest major fault to the site of interest is considered. The first step in the seismic risk analysis is to identify sources of earthquake and to estimate probable hazard levels(e.g., RMS acceleration in this study) at the site of interest. Seismic hazard analyses, using the empirical RMS attenuation relationship proposed by Kavazanjian et al. (37), are performed with an aid of HAZ20A, a probabilistic seismic hazard analysis program developed by Abrahamson. (38) The computed RMS hazards are shown in Fig. 6. The computed RMS hazards are de-aggregated into several intervals of intensity, magnitude, and distance. In order to generate RMS-compatible ground motion, we need to specify the frequency content and duration in addition to the RMS acceleration. We adopt the stochastic ground motion

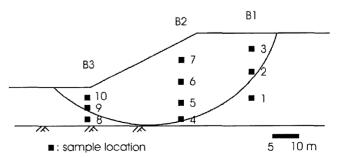


Fig. 5 Geometry and sample location of a slope with a circular slip surface

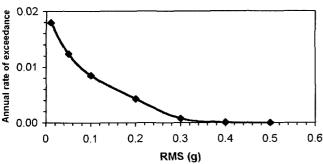


Fig. 6 Annual hazard curve(annual rate of exceeding each hazard

parameters, which were suggested by Wang and Kavazanjian (12) and updated by Tung et al. (26) Tables 1 and 2 summarize stochastic ground motion parameters selected in this study. Another strong ground motion parameter, which is important in the nonlinear deformation analyses, is duration. The hazard compatible duration can be assigned to each generated ground motion by mean of de-aggregation of total hazard into appropriate intervals of earthquake magnitude and distance to site. Table 3 shows the significant duration (5-95% Arias duration or 5-95% RMS duration), which is estimated using the empirical relationship proposed by Abrahamson and Silva. (33) Once the intensity, duration, and frequency content of the ground motion at the site are determined, RMS hazard-compatible ground motion can be produced by directly transforming the spectral representation (Eq. (19)) into the time history by FFT.

Fig. 7 shows a sample motion generated with an aid of GenMotion, a computer code for generation of artificial motion written by the author(Kim<sup>(39)</sup>), based on the aforementioned statistics of ground motion parameters. The amplitude of the time history closely follows the trend of the modulating function, as it should be. Through their simulation-based analyses of seismic slope stability with hazard-compatible artificial motions, Kim<sup>(39)</sup> and Kim and Sitar<sup>(40)</sup> have shown that the stochastic nature of seismic loading can be systematically accessed by generating a

Table 1 Power spectral density parameters for rock sites(from Tung et al.(26)

Parameter	Distribution	Segment 1		Segment 2		Segment 3	
		μ	σ	μ	σ	μ	σ
$\omega_g$	Gamma	23.57	3.46	21.12	3.60	18.38	3.50
$\zeta_g$	Gamma	0.352	0.360	0.394	0.380	0.417	0.162

Table 2 Parameters for modulating function(from Wang and Kavazanjian<sup>(12)</sup>)

Parameter	Distribution	μ	σ
α	Rayleigh	0.73	0.45
β	Exponential	0.22	0.18

Table 3 Significant Duration(5-95% Arias duration), estimated based on the empirical relationship reported by Abrahamson and Silva (33)

	<del></del>			
Magnitude	Distance(kilometers)	Significant Duration(seconds)		
5.0 - 6.0	0 - 10	3.80		
	10 – 20	4.53		
	20 - 30	6.00		
	30 - 50	8.20		
	50 - 100	13.33		
6.0 - 7.0	0 – 10	9.05		
	10 – 20	9.78		
	20 - 30	11.25		
	30 - 50	13.45		
	50 - 100	18.58		
7.0 - 7.5	0 - 10	17.34		
	10 - 20	18.08		
	20 - 30	19.54		
	30 - 50	21.74		
	50 - 100	26.87		
7.5 - 8.0	0 - 10	26.76		
	10 – 20	27.49		
	20 - 30	28.96		
	30 - 50	31.16		
	50 - 100	36.29		

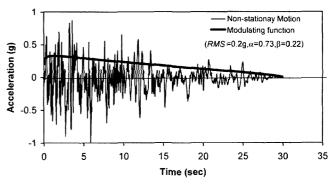


Fig. 7 Sample time history of non-stationary ground motion (RMS=0.2g,  $\alpha$ =0.73,  $\beta$ =0.22, t<sub>d</sub>=30sec)

large series of hazard-compatible ground motions, and by using them in subsequent response analyses(Fig. 8). It should be noted that unlike in deterministic analyses, the results(i.e., Fig. 8) allow us to quantify the risk level, and can be used as a systematic aid in making decisions. Detailed description to their analyses is beyond the scope

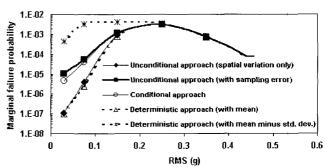


Fig. 8 Marginal annual risk curve(annual probability of failure with respect to each hazard level)

of this paper but it can be found in Kim<sup>(39)</sup> or Kim and Sitar.<sup>(40)</sup>

#### 7. Conclusions

RMS acceleration may be one of the best ways to characterize the intensity of ground shaking, since the time-variant RMS of the motion is nothing but the modulation function that defines the shape of ground motion.

The study shows that the RMS procedure is useful for generation of large number of hazard-compatible motions, unlike the conventional procedures that aim to generate a small number of motions that match deterministic targets such as design response spectra.

Therefore, the procedure is particularly useful for the rigorous probabilistic seismic response analysis where hundreds or thousands of ground motions are often required. The RMS procedure can also be easily extended to take account of coherencies among motions at different locations in case where earthquake ground motion is not spatially invariant.

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