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ON A SEMI-INVARIANT SUBMANIFOLD OF CODIMENSION 3 WITH CONSTANT MEAN CURVATURE IN A COMPLEX PROJECTIVE SPACE

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ABSTRACT. Let M be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space. Further, if the mean curvature of M is constant, then we prove that M is a real hypersurface of a complex projective space of codimension 2 in the ambient space.

0. Introduction

A submanifold M is called a *CR submanifold* of a Kaehlerian manifold with complex structure J if it is endowed with a pair of mutually orthogonal and complementary differentiable distribution $(\Delta, \Delta^{\perp})$ such that for any point $p \in M$ we have $J\Delta_p = M_p$, $J\Delta_p^{\perp} \subset M_p^{\perp}$, where $M_p^{\perp}$ denotes the normal space of M at p ([1]). In particular, M is said to be a *semi-invariant submanifold* of a Kaehlerian manifold if $\dim\Delta^{\perp} = 1$ ([2], [12]). In this case, M admits an almost contact metric structure. Furthermore new examples of nontrivial semi-invariant submanifolds in a complex projective space $\mathbb{C}P^n$ are constructed in [7] and [11]. Therefore we may expect to generalize some results which are valid in a real hypersurface to a semi-invariant submanifold. From this point of view, a semi-invariant submanifold of codimension 3 in a complex projective space are studied in [4], [5], [6], [7], [13] and so on by using properties of the third fundamental forms of the submanifold and those of the induced almost contact metric structure. One of them Ki, Li and Lee ([6]) assert that the following:

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THEOREM K-L ([6]). *Let M be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{n+1}$. If the scalar curvature of M is constant, then M is a real hypersurface in $\mathbb{C}P^n$.*

The main purpose of the present paper is to prove that a semi-invariant submanifold M of codimension 3 in a complex projective space is a real hypersurface provided that the mean curvature of M is constant.

All manifolds in this paper are assumed to be connected and of class C^∞ and the dimension of the submanifold is greater than 2.

1. Preliminaries

At first we review fundamental properties on a semi-invariant submanifold of codimension 3 in a Kaehlerian manifold.

Let $\tilde{M}$ be a real $2(n+1)$-dimensional Kaehlerian manifold equipped with parallel almost complex structure J and a Riemannian metric tensor G, and covered by a system of coordinate neighborhoods $\{\tilde{V}; y^A\}$.

Let M be a real (2n-1)-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V; x^h\}$ and immersed isometrically in $\tilde{M}$ by the immersion $i : M \to \tilde{M}$. We represent the immersion i locally by $y^A = y^A(x^h)$ and $B_j = (B_j{}^A)$ are $(2n-1)$-linearly independent local tangent vectors of M, where $B_j{}^A = \partial_j y^A$ and $\partial_j = \partial/\partial x^j$.

Let A, B, C, D and E run over $1, 2, \cdots, 2n+2$ and let h, i, j, k, r and s run from $1, 2, \cdots$, to $2n-1$. The summation convention will be used with respect to those system of indices. Three mutually orthogonal unit normals C, D and E may be chosen. Since the immersion i is isometric, the induced Riemannian metric tensor g with components on M is given by $g_{ji} = G(B_j, B_i)$.

Denoting by ∇_j the operator of van der Wareden-Bortolotti covariant differentiation with respect to g, equations of the Gauss for M of $\tilde{M}$ is obtained:

$$\nabla_j B_i = A_{ji} C + K_{ji} D + L_{ji} E, \tag{1.1}$$

where A_{ji}, K_{ji} and L_{ji} are components of the second fundamental forms in the direction C, D and E respectively. Equations of Weingarten are also given by

$$\begin{aligned} \nabla_j C &= -A_j{}^h B_h + l_j D + m_j E, \\ \nabla_j D &= -K_j{}^h B_h - l_j C + n_j E, \qquad (1.2) \\ \nabla_j E &= -L_j{}^h B_h - m_j C - n_j D, \end{aligned}$$

where $A = (A_j{}^h)$, $A_{(2)} = (K_j{}^h)$ and $A_{(3)} = (L_j{}^h)$, which are related by $A_{ji} = A_j{}^r g_{ir}$, $K_{ji} = K_j{}^r g_{ir}$ and $L_{ji} = L_j{}^r g_{ir}$ respectively, and l_j, m_j and n_j being components of the third fundamental forms.

As is well-known, a submanifold of a Kaehlerian manifold $\tilde{M}$ is said to a CR *submanifold* ([1], [14]) if it is endowed with a pair of mutually orthogonal complementary differentiable distribution $(\Delta, \Delta^{\perp})$ such that for any $p \in M$ we have $J\Delta_p = M_p$, $J\Delta_p{}^{\perp} \subset M_p{}^{\perp}$, where $M_p{}^{\perp}$ denotes the normal space of M at p. In particular, M is said to be a *semi-invariant submanifold* if $\dim\Delta^{\perp} = 1$, and the unit normal vector in $J\Delta^{\perp}$ is called a *distinguished normal* to the submanifold and denoted this by C ([2], [12]). Then we can write

$$JB_i = \phi_i{}^h B_h + \xi_i C, \quad JC = -\xi^h B_h, \quad JD = -E, \quad JE = D, \qquad (1.3)$$

where we have put $\phi_{ji} = G(JB_j, B_i)$, $\xi_j = G(JB_j, C)$, ξ^h being associate components of ξ_h ([7]). A tensor field of type (1,1) with components $\phi_j{}^h$ will be denoted by ϕ. By properties of the almost complex structure J, it is, using (1.3), seen that

$$\begin{aligned} &\phi_i{}^r \phi_r{}^h = -\delta_i^h + \xi_i \xi^h, \quad \xi_r \phi_i{}^r = 0, \quad \xi^r \phi_r{}^h = 0, \\ &\xi_r \xi^r = 1, \quad g_{rs} \phi_j{}^r \phi_i{}^s = g_{ji} - \xi_j \xi_i. \end{aligned}$$

In the sequel, we denote the normal components of $\nabla^{\perp} C$ by $\nabla_j C$. The distinguished normal is said to be *parallel in the normal bundle* if we have $\nabla^{\perp} C = 0$, that is, l_j and m_j vanish identically.

Since J is parallel, differentiating (1.3) covariantly along M and making use of (1.1), (1.2) and (1.3) itself, we find ([13])

$$\nabla_j \phi_i{}^h = -A_{ji} \xi^h + A_j{}^h \xi_i, \qquad (1.4)$$

$$\nabla_j \xi_i = -A_{jr} \phi_i{}^r, \qquad (1.5)$$

$$K_{ji} = -L_{jr} \phi_i{}^r - m_j \xi_i, \qquad (1.6)$$

$$L_{ji} = K_{jr} \phi_i{}^r + l_j \xi_i. \qquad (1.7)$$

REMARK 1. To write our formulas in a convention form, in what follows we denote by $\alpha = A_{rs}\xi^r\xi^s$, $\beta = A_{rs}^2\xi^r\xi^s$, $h = T_rA$, $k = T_rA_{(2)}$, $h_{(2)} = T_rA^2$, $K_{(2)} = T_rA_{(2)}^2$, $L_{(2)} = T_rA_{(3)}^2$, and for a function f we denote by ∇f the gradient vector field of f.

We notice here that we may assume $T_rA_{(3)} = 0$ (see [7]). Thus, it is, using (1.6) and (1.7), verified that

$$K_{jr}\xi^r = -m_j, \quad L_{jr}\xi^r = l_j, \tag{1.8}$$

$$m_r\xi^r = -k, \quad l_r\xi^r = 0. \tag{1.9}$$

Further, we obtain

$$\phi_{jr}m^r = -l_j, \quad \phi_{jr}l^r = m_j + k\xi_j, \tag{1.10}$$

$$K_{jr}L_i{}^r + K_{ir}L_j{}^r + l_jm_i + l_im_j = 0. \tag{1.11}$$

2. Auxiliary results

In order to prove our results we present in this section some notation, terminology and auxiliary results.

In the rest of this paper we shall suppose that $\tilde{M}$ is a Kaehlerian manifold of constant holomorphic sectional curvature c, which is called a *complex space form* and denoted by $M_{n+1}(c)$. Then equations of Gauss and Codazzi are given by

$$\begin{aligned} R_{kjih} =& \frac{c}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + \phi_{kh}\phi_{ji} - \phi_{jh}\phi_{ki} - 2\phi_{kj}\phi_{ih}) \\ &+ A_{kh}A_{ji} - A_{jh}A_{ki} + K_{kh}K_{ji} - K_{jh}K_{ki} \\ &+ L_{kh}L_{ji} - L_{jh}L_{ki}, \end{aligned} \tag{2.1}$$

$$\begin{aligned} &\nabla_kA_{ji} - \nabla_jA_{ki} - l_kK_{ji} + l_jK_{ki} - m_kL_{ji} + m_jL_{ki} \\ &= \frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj}), \end{aligned} \tag{2.2}$$

$$\nabla_k K_{ji} - \nabla_j K_{ki} + l_k A_{ji} - l_j A_{ki} - n_k L_{ji} + n_j L_{ki} = 0, \tag{2.3}$$

$$\nabla_k L_{ji} - \nabla_j L_{ki} + m_k A_{ji} - m_j A_{ki} + n_k K_{ji} - n_j K_{ki} = 0, \tag{2.4}$$

where R_{kjih} are covariant components of the Riemann-Christoffel curvature tensor of M, and those of the Ricci by

$$\nabla_k l_j - \nabla_j l_k + A_{kr} K_j{}^r - A_{jr} K_k{}^r + m_k n_j - m_j n_k = 0, \tag{2.5}$$

$$\nabla_k m_j - \nabla_j m_k + A_{kr} L_j{}^r - A_{jr} L_k{}^r + n_k l_j - n_j l_k = 0, \tag{2.6}$$

$$\nabla_k n_j - \nabla_j n_k + K_{kr} L_j{}^r - K_{jr} L_k{}^r + l_k m_j - l_j m_k = \frac{c}{2}\phi_{kj}. \tag{2.7}$$

Now, we put $U_j = \xi^r \nabla_r \xi_j$. Then U is orthogonal to the structure vector ξ. Because of (1.5) and properties of the almost contact metric structure, it follows that

$$\phi_{jr} U^r = A_{jr}\xi^r - \alpha\xi_j, \tag{2.8}$$

$$U^r \nabla_j \xi_r = A_{jr}^2 \xi^r - \alpha A_{jr}\xi^r. \tag{2.9}$$

From (2.8) we get $g(U, U) = \beta - \alpha^2$. Therefore we easily see that $A\xi = \alpha\xi$ if and only if $\beta - \alpha^2 = 0$. Differentiating (2.8) covariantly and taking account of (1.4) and (1.5), we find

$$\begin{aligned} &\xi_j (A_{kr} U^r + \alpha_k) + \phi_{jr} \nabla_k U^r \\ &= \xi^r \nabla_k A_{jr} - A_{jr} A_{ks} \phi^{rs} + \alpha A_{kr} \phi_j{}^r, \end{aligned} \tag{2.10}$$

where we put $\alpha_k = \nabla_k \alpha$, which shows that

$$(\nabla_k A_{rs})\xi^r \xi^s = 2A_{kr} U^r + \alpha_k,$$

which together with (1.8), (1.9) and (2.2) implies that

$$(\nabla_r A_{js})\xi^r \xi^s = 2A_{jr} U^r + \alpha_j + 2k l_j. \tag{2.11}$$

By means of (1.4), (1.5) and (2.11) it is verified that

$$\xi^r \nabla_r U_i = -3U^r A_{rs} \phi_i{}^s + \alpha A_{ir}\xi^r - \beta\xi_i - \phi_{ir}\alpha^r - 2k\phi_{ir} l^r. \tag{2.12}$$

The normal connection of a semi-invariant submanifold of codimension 3 in a complex space form is said to be lift-flat if it satisfies $dn = \frac{c}{2}\omega$, that is,

$$\nabla_j n_i - \nabla_i n_j = \frac{c}{2}\phi_{ji}, \tag{2.13}$$

where $\omega(X, Y) = g(\phi X, Y)$ for any vectors X and Y on M (see [9]).

LEMMA 2.1. *Let M be a semi-invariant submanifold of codimension 3 in a complex space form $M_{n+1}(c)$. Then the normal connection of M is lift-flat if and only if $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$.*

PROOF. Suppose that (2.13) is valid on M. Then we have by (2.7)

$$K_{jr}L_i{}^r - K_{ir}L_j{}^r + l_jm_i - l_im_j = 0 \tag{2.14}$$

or using (1.11)

$$K_{jr}L_i{}^r + l_jm_i = 0. \tag{2.15}$$

From this, (1.8) and (1.9), it is seen that $L_{ir}m^r = 0$ and hence $(m_rm^r)l_j = 0$. Thus it follows that $l_j = 0$ because of (1.10). Therefore (2.14) is reduced to $A_{(2)}A_{(3)} = A_{(2)}A_{(3)}$.

Conversely, if $A_{(2)}$ and $A_{(3)}$ mutually commutes, then (1.11) turns out to be

$$2K_{jr}L_i{}^r + l_jm_i + l_im_j = 0, \tag{2.16}$$

which together with (1.8), (1.9) and (1.10) gives

$$2K_{jr}l^r = kl_j, \quad 2L_{ir}m^r = -kl_i.$$

From the last three equations, we see that

$$l_j\{2m_rm^r - k^2\} = 0,$$

which connected with the first equation of (1.9) implies that

$$\{\|m_i\|^2 + \|m_i + k\xi_i\|^2\}l_j = 0.$$

If we take account of (1.10) and the last equation, then we verify that $l_j = 0$. Thus (2.16) becomes $K_{jr}L_i{}^r = 0$, which together with (1.6) yields $K_{ji}^2 - k\xi_j\xi_i = 0$ and hence $K_{(2)} = k^2$ and $K_{jr}\xi^r = k\xi_j$. From these relationships, it is clear that

$$K_{ji} = k\xi_j\xi_i.$$

Thus (2.7) is reduced to $\nabla_jn_i - \nabla_in_j = \frac{c}{2}\phi_{ji}$ since we have $l_j = 0$. This completes the proof. □

3. Semi-invariant submanifolds satisfying $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$

In the rest of this paper we shall suppose that M is a real $(2n-1)$-dimensional semi-invariant submanifold of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ and that $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$ is satisfied on M. Then we have $l_j = 0$ and

$$K_{ji} = k\xi_j\xi_i. \tag{3.1}$$

Further, we have

$$m_j = -k\xi_j, \tag{3.2}$$

$$L_{ji} = 0 \tag{3.3}$$

because of (1.7) and (1.10). Thus (2.4) and (2.6) are reduced respectively to

$$k\{\xi_j A_{ki} - \xi_k A_{ji} + (n_k\xi_j - n_j\xi_k)\xi_i\} = 0, \tag{3.4}$$

$$\nabla_j m_i - \nabla_i m_j = 0. \tag{3.5}$$

Multiplying $\xi^j\xi^i$ to (3.4) and summing for j and i, we find

$$k\{n_k - (n_t\xi^t)\xi_k + A_{kr}\xi^r - \alpha\xi_k\} = 0. \tag{3.6}$$

Now, let Ω be a set of points such that $k \neq 0$ on M and suppose that Ω be nonavoid. Then (3.4) and (3.6) imply

$$A_{ji} = \xi_j A_{ir}\xi^r + \xi_i A_{jr}\xi^r - \alpha\xi_j\xi_i \tag{3.7}$$

on Ω. From now on, we discuss our arguments on the open set Ω on M. Since the vector U is orthogonal to ξ, it is seen that

$$AU = 0. \tag{3.8}$$

Transforming (3.7) by ${\phi_k}^i$ and making use of (1.5), we get

$$\nabla_k\xi_j = \xi_k U_j. \tag{3.9}$$

If we transform this by ϕ^{kj} and use (1.5), then

$$h - \alpha = 0. \tag{3.10}$$

Multiplying (3.7) with A^{ji} and summing for j and i, we also find

$$h_{(2)} = 2\beta - \alpha^2. \tag{3.11}$$

REMARK 2. We notice here that $\beta-\alpha^2$ does not vanish on Ω. In fact, if not, then we have $A\xi = \alpha\xi$ and hence $A_{ji} = \alpha\xi_j\xi_i$ because of (3.7). From this fact and (3.9) we obtain $\nabla_k A_{ji} = \alpha_k\xi_j\xi_i$, which together with (2.2) and (3.3) gives

$$(\alpha_k\xi_j - \alpha_j\xi_k)\xi_i = \frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj}),$$

a contradiction.

Now, put $A\xi = \alpha\xi + \mu W$, where μ is a function on M which is not vanish on Ω and W is a unit vector field orthogonal to the structure vector field ξ. Then we have

$$\phi_{jr}U^r = \mu W_j \tag{3.12}$$

and $\mu^2 = \beta-\alpha^2$ because of (2.8). Thus W is also orthogonal to U. Thus (3.7) turns out to be

$$A_{ji} = \mu(\xi_j W_i + \xi_i W_j) + \alpha\xi_j\xi_i. \tag{3.13}$$

We notice here that it is, using (3.9), verified that

$$\xi^r\nabla_k W_r = 0 \tag{3.14}$$

because ξ is orthogonal to W.

Differentiating (3.13) covariantly along Ω and making use of (3.9), we find

$$\begin{aligned}\nabla_k A_{ji} =&\mu_k(\xi_j W_i + \xi_i W_j) + \mu\{(U_j W_i + U_i W_j)\xi_k + \xi_j\nabla_k W_i + \xi_i\nabla_k W_j\}\\ &+ \alpha_k\xi_j\xi_i + \alpha(U_j\xi_i + U_i\xi_j)\xi_k,\end{aligned}$$

from which, taking the skew-symmetric part with respect to indices k and j, and using (2.2) with $l_j = 0$ and (3.3),

$$\begin{aligned}&\frac{c}{4}(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj})\\ =&\mu_k(\xi_j W_i + \xi_i W_j) - \mu_j(\xi_k W_i + \xi_i W_k)\\ &+ \mu\{(U_j W_i + U_i W_j - \nabla_j W_i)\xi_k - (U_k W_i + U_i W_k - \nabla_k W_i)\xi_j\}\\ &+ \mu(\nabla_k W_j - \nabla_j W_k)\xi_i + (\alpha_k\xi_j - \alpha_j\xi_k)\xi_i + \alpha(U_j\xi_k - U_k\xi_j)\xi_i.\end{aligned}$$

Applying ξ^i to this and taking account of (3.14), we find

$$\begin{aligned}(3.15)\qquad &\mu_k W_j - \mu_j W_k + \mu(\nabla_k W_j - \nabla_j W_k) + \alpha_k \xi_j - \alpha_j \xi_k \\ &+\alpha(U_j \xi_k - U_k \xi_j) + \frac{c}{2}\phi_{kj} = 0.\end{aligned}$$

From the last two equations it follows that

$$\begin{aligned}(3.16)\qquad &\frac{c}{4}(\xi_k \phi_{ji} - \xi_j \phi_{ki}) + (\mu_j \xi_k - \mu_k \xi_j) W_i \\ &= \mu\{(U_j W_i + U_i W_j - \nabla_j W_i)\xi_k - (U_k W_i + U_i W_k - \nabla_k W_i)\xi_j\},\end{aligned}$$

which together with (3.14) implies that

$$\begin{aligned}(3.17)\qquad &\frac{c}{4}\phi_{ji} + \{\mu_j - (\mu_t \xi^t)\xi_j\} W_i \\ &= \mu\{U_j W_i + U_i W_j - \nabla_j W_i + (\xi^r \nabla_r W_i)\xi_j\}.\end{aligned}$$

If we apply this by W^i and use (3.12), then we obtain

$$(3.18)\qquad \mu\mu_j = \mu(\mu_t \xi^t)\xi_j + (\mu^2 + \frac{c}{4})U_j.$$

Multiplying (3.16) with $\xi^k W^j$ and summing for k and j, and making use of (3.14) and (3.18), we have

$$(3.19)\qquad \mu_t \xi^t = \alpha_t W^t.$$

LEMMA 3.1. *Let M be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{n+1}$. If the mean curvature of M is constant, then $\nabla_j U_i - \nabla_i U_j = 0$ on Ω.*

PROOF. Differentiating (3.2) covariantly along Ω, and using (3.9), we find

$$-\nabla_k m_j = \xi_j(\nabla_k k) + k\xi_k U_j,$$

which together with (3.5) yields

$$\xi_j(\nabla_k k) - \xi_k(\nabla_j k) + k(\xi_k U_j - \xi_j U_k) = 0.$$

Thus, it is seen that

$$(3.20)\qquad \nabla_j k = (\xi^t \nabla_t k)\xi_j + kU_j.$$

Since the mean curvature of M is assumed to be constant, it is, taking account of $T_rA_{(3)} = 0$ and (3.10), seen that $k^2 + \alpha^2 =$ const., which unable us to obtain

$$k\nabla_j k + \alpha\alpha_j = 0. \tag{3.21}$$

Because of the fact that W is orthogonal to U and ξ, we have from (3.20) and (3.21) $\alpha_t W^t = 0$. Thus (3.18) turns out to be

$$\frac{1}{2}\nabla_j\mu^2 = (\mu^2 + \frac{c}{4})U_j,$$

where we have used (3.19). From this, we obtain

$$\frac{1}{2}\nabla_k\nabla_j\mu^2 = 2(\mu^2 + \frac{c}{4})U_jU_k + (\mu^2 + \frac{c}{4})\nabla_k U_j,$$

which implies $(\mu^2 + \frac{c}{4})(\nabla_j U_i - \nabla_i U_j) = 0$. This completes the proof because we have $\mu^2 + \frac{c}{4} > 0$.

Finally, we are proved. □

THEOREM 3.2. *Let M be a real $(2n-1)$-dimensional semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{n+1}$. If the mean curvature of M is constant, then M is a real hypersurface in a complex projective space $\mathbb{C}P^n$.*

PROOF. Since the normal connection of M is lift-flat, we have $l_j = 0$, (3.8) and (3.13) are valid. Thus (2.12) is reduced to

$$\xi^r\nabla_r U_i = \mu(\alpha W_i - \mu\xi_i) - \phi_{ir}\alpha^r. \tag{3.22}$$

On the other hand, we have $U^r\nabla_j\xi_r + \xi^r\nabla_r U_j = 0$ by Lemma 3.1. Thus, it is, using (3.9) and (3.22), verified that$\phi_{jr}\alpha^r = \alpha\mu W_j$. So we have

$$\alpha_j = (\alpha_t\xi^t)\xi_j + \alpha U_j,$$

where we have used (3.12). From (3.20), (3.21) and the last equation, we obtain

$$\{k(\xi^t\nabla_t k) + \alpha(\alpha_t\xi^t)\}\xi_j + (k^2 + \alpha^2)U_j = 0,$$

which shows that $\mu(k^2 + \alpha^2) = 0$ and hence $k = 0$, a contradiction. Hence Ω is empty. Thus, by (3.1) $\sim$ (3.3) it follows that $A_{(2)} = A_{(3)} = 0$ and $\nabla^\perp C = 0$ on M.

Let $N_0(p) = \{\eta \in M_p^\perp | A_\eta = 0\}$ and $H_0(p)$ the maximal J-invariant subspace of $N_0(p)$. Then, the orthogonal complement of $H_0(p)$ is invariant under parallel translation with respect to the normal connection because we have $\nabla^\perp C = 0$. Therefore, by the reduction theorem in [3] or [10], we see that M is a real hypersurface of $\mathbb{C}P^n$ in $\mathbb{C}P^{n+1}$. Hence we arrive at conclusion. □

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