

## Output Feedback Stabilization using Integral Sliding Mode Control

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**Abstract** - We consider a single-input-single-output nonlinear system which can be represented in a normal form. The nonlinear system has a modeling uncertainties including the input coefficient uncertainty. A high-gain observer is used to estimate the states variables to reject a modeling uncertainty. A globally bounded output feedback integral sliding mode control is proposed to stabilize the closed loop system. The proposed integral sliding mode control can asymptotically stabilize the closed loop system in the presence of input coefficient uncertainty.

**Key Words** : integral sliding mode control, globally bounded control, high-gain observer, asymptotically stabilization, output feedback control.

## 1. Introduction

Since the separation principle does not hold in the nonlinear system which has a modeling uncertainty, a high-gain observer has been used to reject disturbances due to the imperfect feedback cancellation and modeling uncertainty for a nonlinear system with relative degree higher than one system in the output feedback control[1]. The use of high-gain observer to estimate state variable results in the peaking phenomenon of the state variable. A globally bounded control has been introduced to reduce the peaking phenomenon[2]. Since the globally bounded control was introduced, some works in the various control schemes used the globally bounded control with high gain observer. The works[3, 5] used the globally bounded control in the continuous control scheme. A state feedback controller was designed and analyzed first in the continuous control scheme, and then a Lipschitz property of the continuous controller was used to show that the output feedback controller can recover the state feedback properties. However an asymptotic stabilization was not achieved due to the presence of a nonvanishing perturbation caused by the estimation error and modeling uncertainty, but an ultimate boundness was achieved. The works[4,6] used the integral control to achieve the asymptotic stability in the continuous control. The

works[7,8] used a globally bounded control in the discontinuous control scheme such as a sliding mode control[7,8]. The works[7,8] also only achieved an ultimate boundness in the presence of a nonvanishing perturbation. In particular, an ultimate boundness was achieved in the presence of input coefficient uncertainty. Since the discontinuous controller does not have a Lipschitz property, the design and analysis are different with the continuous one. The work[9] used an integral control with sliding mode control to achieve an asymptotic stability in the presence of input coefficient uncertainty, but limited to the state feedback. We start with a nonlinear system which is a feedback linearizable system. In particular, the relative degree of nonlinear system is same as the dimension of state variables, since we are interested in output feedback control. We propose a new design scheme using an integral sliding mode control can asymptotically stabilize the closed-loop system with an high-gain observer in the presence of input coefficient uncertainty. We show that integral sliding mode control can reject disturbances due to the input coefficient uncertainty and estimation errors. The performance of the propose control is demonstrated in the example.

## 2. Problem statement

Consider the single-input single-output nonlinear system

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接受日字 : 2002年 9月 12日  
最終完了 : 2003年 2月 12日

$$\begin{aligned} \dot{w} &= F(w) + G(w)u \\ y &= h(w) \end{aligned} \quad (1)$$

where  $w \in R^n$  is the state,  $u$  is the control input,  $y$  is the measured output. Suppose that  $F$ ,  $G$ , and  $h$  are sufficiently smooth function on  $U$ , an open subset of  $R^n$ , and  $F(0)=0$ ,  $h(0)=0$ . Therefore the origin  $w=0$  is an equilibrium point of the open loop system. Since we are interested in input-output linearizable nonlinear system, we assume the following assumption on the nonlinear system (1).

**Assumption 1** For all  $w \in U$ ,

- The system (1) has an uniform relative degree, i.e.,  $L_G h(w) = \dots = L_G L_F^{n-2} h(w) = 0$  and  $L_G L_F^{n-1} h(w) \neq 0$
- The mapping  $x = T(w)$ , defined by

$x_i = L_F^{i-1} h(w)$ ,  $1 \leq i \leq n$  and  $x = [x_1 \ x_2 \ \dots \ x_n]^T$  is a proper map.

The uniform relative degree assumption is a necessary and sufficient condition for the mapping  $x = T(w)$  to be a local diffeomorphism in the neighborhood of every  $w \in U$  which is an open subset of  $R^n$ [10]. The properness of the mapping  $x = T(w)$  ensures that it is a diffeomorphism of  $U$  onto  $T(U)$ . The change of variables  $x = T(w)$  transforms the system (1) into the following normal form

$$\begin{aligned} \dot{x} &= Ax + B[f(x) + g(x)u] \\ y &= Cx \end{aligned} \quad (2)$$

$$A = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n \times 1},$$

$$C = [1 \ 0 \ \dots \ 0]_{1 \times n}.$$

where  $g(x) \neq 0, \forall x \in T(U)$ . Let  $f_o(x)$  and  $g_o(x)$  be a known nominal model of  $f(x)$  and  $g(x)$ , respectively. Suppose that  $f_o(x)$  and  $g_o(x)$  are sufficiently smooth,  $f_o(0)=0$ , and  $g_o(x) \neq 0$  for all  $x \in T(U)$ . Note that the mapping  $T$  could depend on unknown parameter, however the dependence of unknown parameter does not cause the problem, since we are interested in output feedback control. We also assume that the uncertainty of the equation (2) satisfies the following assumption which is a typical matching condition on the modeling uncertainty[11].

**Assumption 2** For all  $x \in T(U)$ , there is a scalar Lipschitz function  $\rho(x)$  such that

$$\begin{aligned} |f(x) - f_o(x)| &\leq \rho(x) \\ |g(x)/g_o(x) - 1| &< k_g < 1 \end{aligned} \quad (3)$$

where  $k_g$  is nonnegative constant.

Our goal is the design of output feedback controller to stabilize the nonlinear system given by the equation (2) over the domain  $T(U) = D$ .

### 3. Observer and sliding surface design

Since we are interested in an output feedback control, we use the following high-gain observer to estimate the state variable  $x$ ,

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + \frac{a_i}{\epsilon^i} (y - \hat{x}_1), \quad i=1, \dots, n-1 \\ \dot{\hat{x}}_n &= \frac{a_n}{\epsilon^n} (y - \hat{x}_1) + f_o(\hat{x}) + g_o(\hat{x})u \end{aligned} \quad (4)$$

where  $\hat{x}_i$  is the estimate of the state variables  $x_i$  and  $\epsilon$  is a positive constant to be specified. The positive constant  $a_i$  are chosen such that the roots of the following equation are in the open left half plane.

$$s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0$$

We rewrite the observer equation (4) into the compact form

$$\dot{\tilde{x}} = A\tilde{x} + B[f_o(\tilde{x}) + g_o(\tilde{x})u] + D(\epsilon)LC(x - \tilde{x}) \quad (5)$$

where  $L = [a_1, \dots, a_n]^T$ , and  $D(\epsilon) = \text{diag}[1/\epsilon \ 1/\epsilon^2 \ \dots \ 1/\epsilon^n]$ . We choose the following sliding surface

$$S(\tilde{x}, \sigma) = M\tilde{x} + \sigma \quad (6)$$

where  $M = [m_1, \dots, m_{n-1}, 1]$  and  $m_i$  is chosen such that

$$\overline{A} = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -m_1 & \dots & \dots & \dots & -m_{n-1} \end{bmatrix}_{(n-1) \times (n-1)}$$

is Hurwitz matrix,

$\dot{\sigma} = -M(A\tilde{x} + BK\tilde{x}) - MD(\epsilon)LC(x - \tilde{x}) - r(\tilde{x})$  where  $K$  is chosen such that  $A+BK$  is a Hurwitz matrix, and

$$r(\tilde{x}) = -\frac{1}{1-k_g} (\rho(\tilde{x}) + k_g |f_o(\tilde{x})| + k_g |K\tilde{x}|) \text{SGN}(\tilde{x}^T P B)$$

where

$\text{SGN}(\cdot)$  denotes the signum function and  $P$  is a positive definite matrix such that  $P(A+BK) + (A+BK)^T P = -I$ . It is described that an integral part is useful to reject disturbance due to the input coefficient uncertainty for state feedback, since  $\sigma$  term is appeared in the

equivalence control input during the sliding mode[9]. We use the idea suggested from [9] in the design of sliding surface. However the sliding surface is different with [9], since we are interested in output feedback. In fact, the first term of sliding surface is used for stabilization of system in the sliding mode, the second term of sliding surface is used to reject estimation error term and the third term is used to reject the uncertainty of the input coefficient. The reason for the choice of the sliding surface will be more clear as the stability analysis of the closed-loop system is progressed later on. Let  $e_i = x_i - \hat{x}_i$  be the estimation error, and define the scaled variables  $\zeta_i = (1/\varepsilon^{n-i})e_i$ . The closed-loop equations (2) and (5) can be rewritten as

$$\begin{aligned} \dot{x} &= Ax + B[f(x) + g(x)u] \\ \varepsilon \dot{\zeta} &= (A - LC)\zeta + \varepsilon B[f(x) - f_0(\hat{x}) + \{g(x) - g_0(\hat{x})\}u] \end{aligned} \quad (7)$$

where  $\varepsilon$  is the same one used in the observer equation (4). Note that  $(A - LC)$  is a Hurwitz matrix. Let  $V(x) = x^T P x$ . Define

$$\begin{aligned} \Omega_v &= \{x \in R^n \mid V(x) \leq v_r\} \subset D \\ \Omega_{\zeta} &= \{\zeta \in R^n \mid \|\zeta\| < c_{\zeta} / \varepsilon^{n-1}\} \\ \Omega &= \Omega_v \times \Omega_{\zeta} \end{aligned}$$

where  $v_r$  is a positive constant such that  $v_r > \frac{1}{\lambda_{\min}(P)} (2\lambda_{\max}(P)(1+k_g)\delta_1 \|PB\|)^2$ ,  $\delta_1$  and  $c_{\zeta}$  are arbitrary positive numbers, and  $\|\cdot\|$  denotes the Euclidean norm. Note that  $\lambda_{\min}(\cdot)$  and  $\lambda_{\max}(\cdot)$  denote the minimum eigenvalue and maximum eigenvalue of the arguments, respectively. The set  $\Omega$  is taken as the region of interest. We use a globally bounded control function as a control input. We will specify the control input  $u$  to make a globally bounded control later on. The following lemma states that the fast variables  $\zeta$  decays very rapidly during a short time period with a globally bounded control. The proof of the lemma is the same as the proof of Lemma 1 in [7], hence it is omitted.

**Lemma 1** Consider the closed-loop system (7) and suppose that the control input  $u$  is globally bounded. Then, for all  $(x(0), \zeta(0)) \in \Omega_0 \subset \Omega$ , there exist  $\varepsilon_1$  and  $T_1 = T_1(\varepsilon) \leq T_3$  such that for all  $0 < \varepsilon < \varepsilon_1$ ,  $\|\zeta\| < k_{\zeta}\varepsilon$  for all  $t \in [T_1, T_4)$  where  $k_{\zeta}$  is some positive constant,  $T_3$  is a

finite time, and  $T_4 > T_3$  is the first time  $x(t)$  exits from the compact set  $\Omega$ .

**Proof:** see [7]

#### 4. Globally bounded controller design

We will design globally bounded control as we opposed it in the previous section. Consider the function

$$\bar{u}(\hat{x}) = \frac{1}{g_0(\hat{x})} [-f_0(\hat{x}) + K\hat{x} + r(\hat{x}) - \delta_1 \text{SGN}(S)] \quad (8)$$

We take a control input  $u$  as  $\bar{u}$ , saturated outside set  $\Omega_v$ . In particular, let  $\bar{u}_1 = \frac{1}{g_0(\hat{x})} [-f_0(\hat{x}) + K\hat{x} + r(\hat{x})]$ ,  $\bar{u}_2 = -\frac{\delta_1}{g_0(\hat{x})}$ ,  $s_i = \max_{\hat{x} \in \Omega} |\bar{u}_i(\hat{x})|$ , and take the control input

$$u = s_1 \text{sat}(\bar{u}_1/s_1) + s_2 \text{sat}(\bar{u}_2/s_2) \text{SGN}(S) \quad (9)$$

where  $\text{sat}(\cdot)$  is defined by  $\text{sat}(x/s_i) = \begin{cases} 1, & |x| \geq s_i \\ x, & |x| < s_i \end{cases}$ . One can verify that  $u(\hat{x})$  is a globally bounded control input.

**Lemma 2** Consider the closed-loop system (7) with control input  $u$  defined by (9). Then

- the sliding mode condition  $SS \leq -\delta_2 |S|$

is satisfied as long as  $\|\zeta\| < k_{\zeta}\varepsilon$  where  $\delta_2$  is some positive constant.

- $\|\zeta\| < k_{\zeta}\varepsilon$  for all  $t \geq T_1$ .

**Proof:** The proof of this lemma has two parts. One part is to prove that the sliding mode condition is satisfied with the control input (9). The second part is to prove that  $\zeta$  is  $O(\varepsilon)$  for all  $t \geq T_1$ . The first part can be proved using the equation (9) and the fact that  $u = \bar{u}$  for  $\hat{x} \in \Omega$ , which is provided by  $\|\zeta\| < k_{\zeta}\varepsilon$ . Using the derivative of  $S$  along the trajectories of the equation (7), the control input  $u$  (9), and the fact that  $MB=1$ , it can be seen that

$$\begin{aligned} \dot{S}S &= S(M\dot{\hat{x}} + \dot{\delta}) \\ &= S(MA\hat{x} + MB(f_0(\hat{x}) + g_0(\hat{x})u) + MD\dot{\epsilon})LC(x - \hat{x}) \\ &\quad - M(A\hat{x} + BK\hat{x}) - MD\dot{\epsilon})LC(x - \hat{x}) - r(\hat{x}) \end{aligned}$$

$$\begin{aligned}
 &= S[f_0(\hat{x}) + g_0(\hat{x})u - K\hat{x} - r(\hat{x})] \\
 &= S[f_0(\hat{x}) - f_0(\hat{x}) + K\hat{x} + r(\hat{x}) - \delta_1 \text{SGN}(S) - K\hat{x} - r(\hat{x})] \\
 &= S[-\delta_1 \text{SGN}(S)] \\
 &\leq -\delta_2 \text{SGN}(S)
 \end{aligned}$$

where  $\delta_2 < \delta_1$  is some positive constant. Lemma 1 implies that  $\|\zeta\| < k_1 \varepsilon$  as long as the state variable  $x \in \Omega_r$  for all time. Therefore we will show that  $x \in \Omega_r$  for all time. Using  $\hat{x} = x - \bar{D}(\varepsilon)\zeta$ , the derivative of  $V(x) = x^T P x$  along the trajectories of the equation (7) is given by

$$\begin{aligned}
 \dot{V}(x) &= x^T P(Ax + BKx - BK\bar{D}(\varepsilon)\zeta) + (Ax + BKx - BK\bar{D}(\varepsilon)\zeta)^T \\
 &\quad Px + 2x^T PB[f(x) - f(\hat{x}) + f(\hat{x}) - f_0(\hat{x}) \\
 &\quad + \frac{g(x) - g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x}) + K\hat{x}) \\
 &\quad + \frac{g(x)}{g_0(\hat{x})}r(\hat{x}) - \frac{g(x)}{g_0(\hat{x})}\delta_1 \text{SGN}(S)]
 \end{aligned} \tag{10}$$

where  $\bar{D}(\varepsilon) = \text{diag}[\varepsilon^{-n-1}, \varepsilon^{-n-2}, \dots, 1]$ . Using

$$r(\hat{x}) = -\frac{1}{1-k_g}(\rho(\hat{x}) + k_g f_0(\hat{x})) + k_g |K\hat{x}| \text{SGN}(\hat{x}^T PB)$$

$\hat{x} = x - \bar{D}(\varepsilon)\zeta$ , it can be verified that

$$\begin{aligned}
 \dot{V}(x) &= -\|x\|^2 - 2x^T PBK\bar{D}(\varepsilon)\zeta + 2x^T PB(f(x) - f(\hat{x})) \\
 &\quad - \frac{g(x)}{g_0(\hat{x})}\delta_1 \text{SGN}(S) - 2\frac{g(x)}{g_0(\hat{x})}\frac{1}{(1-k_g)} \\
 &\quad | \hat{x}^T PB(\rho(\hat{x}) + k_g f_0(\hat{x})) + k_g |K\hat{x}| \\
 &\quad + 2\hat{x}^T PB[f(\hat{x}) - f_0(\hat{x}) + \frac{g(x) - g_0(\hat{x})}{g_0(\hat{x})} \\
 &\quad (-f_0(\hat{x}) + K\hat{x}) \\
 &\quad + 2\zeta^T \bar{D}(\varepsilon)PB[f(\hat{x}) - f_0(\hat{x}) \\
 &\quad + \frac{g(x) - g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x}) + K\hat{x}) + \frac{g(x)}{g_0(\hat{x})}r(\hat{x})] \\
 \dot{V}(x) &\leq -\|x\|^2 - 2x^T PBK\bar{D}(\varepsilon)\zeta + 2x^T PB(f(x) - f(\hat{x})) \\
 &\quad - \frac{g(x)}{g_0(\hat{x})}\delta_1 \text{SGN}(S) + 2\zeta^T \bar{D}(\varepsilon)PB[f(\hat{x}) - f_0(\hat{x}) \\
 &\quad + \frac{g(x) - g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x}) + K\hat{x}) + \frac{g(x)}{g_0(\hat{x})}r(\hat{x})] \\
 &\leq -\|x\|^2 + 2k_1 \|x\| \|\zeta\| + 2\|x\| \|PB\| (1 + k_g) \delta_1 + 2k_2 \|\zeta\|^2 \\
 &\leq -\frac{V(x)}{\lambda_{\max}(P)} + 2(1 + k_g) \delta_1 \|PB\| \sqrt{\frac{V(x)}{\lambda_{\min}(P)}} + O(\varepsilon)
 \end{aligned} \tag{11}$$

for sufficiently small  $\varepsilon$ . Note that  $k_1$  and  $k_2$  are independent with  $\varepsilon$ . Therefore  $V(x) \leq 0$  for  $V(x) > c_1$  where  $c_1 = \frac{1}{\lambda_{\min}(P)} (2\lambda_{\max}(P)(1 + k_g)\delta_1 \|PB\|)^2$ . Since  $v_r > c_1$ ,  $x$  can not leave the set  $\Omega_r$ .

Lemma 2 implies that there is a finite time to reach the sliding manifold  $S=0$  and  $S=0$  holds thereafter. We can reach the following conclusion after performing the

Lyapunov analysis for the closed-loop system in the sliding manifold.

**Theorem 1** Consider the closed-loop system(7) with the control input (9). Suppose that Assumption 1 and 2 are satisfied. Then for all  $(x(0), \zeta(0)) \in \Omega_0$ , there is  $\varepsilon_2 > 0$  such that for all  $0 < \varepsilon < \varepsilon_2$  such that  $\lim_{t \rightarrow \infty} (x, \zeta) = 0$  and  $\sigma$  is bounded.

**Proof:** Since the sliding mode condition is satisfied, the control input  $u$  can be replaced by[11]

$$u_{eq}(\hat{x}, \sigma) = \frac{1}{g_0(\hat{x})}[-f_0(\hat{x}) + K\hat{x} + r(\hat{x})]$$

in the sliding manifold which is the same as the control input  $u$  defined in the equation (9) with  $\delta_1=0$ . Let  $W(x, \zeta) = x^T P x + \zeta^T P_1 \zeta$  where  $P_1$  is a positive definite matrix such that  $P(A-LC) + (A-LC)^T P = -I$ . The derivative of  $W(x, \zeta)$  along the trajectories of the equation (7) is given by

$$\begin{aligned}
 \dot{W}(x, \zeta) &= V(x)|_{\delta_1=0} - \frac{1}{\varepsilon} \|\zeta\|^2 + 2\zeta^T PB[f(x) - f_0(\hat{x}) \\
 &\quad + (g(x) - g_0(\hat{x}))u_{eq}] \\
 &= V(x)|_{\delta_1=0} - \frac{1}{\varepsilon} \|\zeta\|^2 + 2\zeta^T PB[f(x) - f(\hat{x}) \\
 &\quad + f(\hat{x}) - f_0(\hat{x}) \\
 &\quad + \frac{g(x) - g_0(\hat{x})}{g_0(\hat{x})}(-f_0(\hat{x}) + K\hat{x} + r(\hat{x}))]
 \end{aligned}$$

where  $V(x)|_{\delta_1=0}$  denotes  $V(x)$  with  $\delta_1=0$  in the equation (10). Using the inequality (11) with  $\delta_1=0$ , it can be verified that

$$\begin{aligned}
 \dot{W}(x, \zeta) &\leq -\|x\|^2 + 2k_1 \|x\| \|\zeta\| + 2k_2 \|\zeta\|^2 - \frac{1}{\varepsilon} \|\zeta\|^2 + 2k_3 \|\zeta\|^2 \\
 &\quad + 2k_4 \|\zeta\| \|x\| \\
 &= -\|x\|^2 + 2k_5 \|x\| \|\zeta\| + 2k_6 \|\zeta\|^2 - \frac{1}{\varepsilon} \|\zeta\|^2 \\
 &= -[\|x\| \|\zeta\|] \begin{bmatrix} 1 & k_5 \\ k_5 & \frac{1}{\varepsilon} - 2k_6 \end{bmatrix} \begin{bmatrix} \|x\| \\ \|\zeta\| \end{bmatrix}
 \end{aligned}$$

for some positive constant  $k_3, k_4, k_5$ , and  $k_6$ . Note that  $k_3 \sim k_6$  are independent with  $\varepsilon$ . Let  $P_2 = \begin{bmatrix} 1 & k_5 \\ k_5 & \frac{1}{\varepsilon} - 2k_6 \end{bmatrix}$ .  $P_2$  is a positive definite matrix for sufficiently small  $\varepsilon$ . Thus implies that  $\lim_{t \rightarrow \infty} (x, \zeta) = 0$ . Since  $\hat{x}$  is bounded and  $SS \leq -\delta_2 |S|$ ,  $\sigma$  is bounded for all time.

### 5. Example

Consider the system

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= \theta_1 \sin x_1 + (0.8 + \theta_2)u \\
 y &= x_1
 \end{aligned}$$

where  $\theta_1$  and  $\theta_2$  are unknown constant that satisfy  $|\theta_1| < 0.4$  and  $|\theta_2| < 0.2$  and domain  $D = \{x \in R^2 \mid \|x\| \leq 5\}$ . Suppose that initial states belong to the set  $\Omega_0 = \{x \in R^2 \mid \|x\| \leq 0.5\}$ . A high gain observer is constructed as

$$\begin{aligned} \hat{x}_1 &= \hat{x}_2 + (1/\epsilon)(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= (1/\epsilon^2)(y - \hat{x}_1) + 0.8u \end{aligned}$$

Choose the sliding surface  $S(\hat{x}, \sigma) = M\hat{x} + \sigma$  with  $M = [1 \ 1]$ ,  $K = [-2 \ -3]$ ,  $k_k = 0.25$ ,  $f_0(\hat{x}) = 0$ , and  $\rho(\hat{x}) = 0.5|\hat{x}_1|$  in the equation (6). Let  $\bar{u}(\hat{x}) = \frac{1}{0.8} [K\hat{x} + r(\hat{x}) - \delta_1 \text{SGN}(S)]$  where  $\delta_1 = 0.1$ . Define the set  $\Omega = \{\hat{x} \in R^2 \mid V(\hat{x}) = \hat{x}^T P \hat{x} \leq 1\}$  where  $P = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$ . One can verify that  $\Omega_0 \subset \Omega \subset D$ .

We take saturation level as  $s_1 = 11.5$ ,  $s_2 = 0.12$  in the equation (9). We simulate the closed loop system with  $x(0) = [0.5 \ 1]^T$ ,  $\hat{x}(0) = [0 \ -0.5]^T$ ,  $\sigma(0) = 0$ , and  $\epsilon = 0.02$ . Note that it is desirable that a small value of  $\delta_1$  is used to reduce a chattering in practice. Fig. 1 shows that state variables,  $x_1(t)$  and  $x_2(t)$ , does not exhibit peaking phenomenon, since a globally bounded controllers are used. One can also observe that the estimate of state  $\hat{x}_2$  exhibits a peaking phenomenon in the Fig. 2. Fig. 1 and Fig. 2 show that asymptotic stability is achieved in the closed-loop system. Fig. 3 shows that the output of integrator is bounded. Fig. 4 shows that the reaching condition is satisfied as we expected. It is possible that the use of a small value of  $\delta_1$  takes a long reaching time to the sliding surface. In this case, one can use the sliding surface  $S = M\hat{x} + \sigma - M\hat{x}(t_1) - \sigma(t_1)$  to reduce a reaching time where  $t_1$  is some time instant after  $\hat{x}$  returning the set  $\Omega$  [9]. Since the derivative of constant equals to 0, the use of the term,  $-M\hat{x}(t_1) - \sigma(t_1)$ , does not have an effect in our analysis.

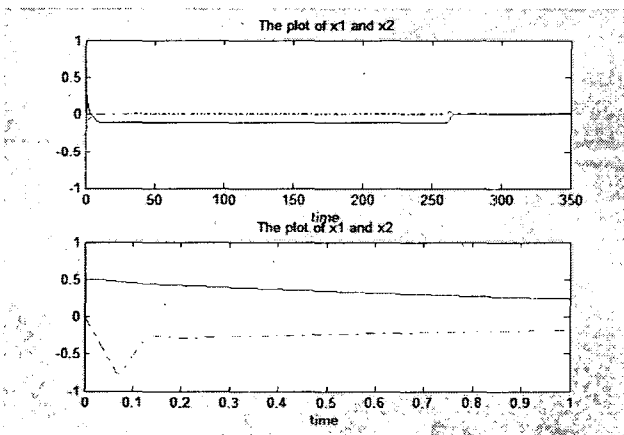


그림 1 상태변수값  
Fig. 1 The plot of states variables  
(solid line is  $x_1$  and dashed line is  $x_2$ )

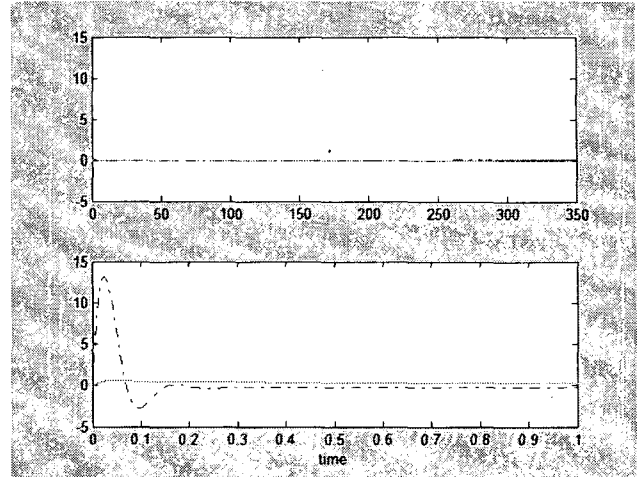


그림 2 상태변수 예측치 값  
Fig. 2 The plot of estimates of states variables  
(solid line is  $\hat{x}_1$  and dashed line is  $\hat{x}_2$ )

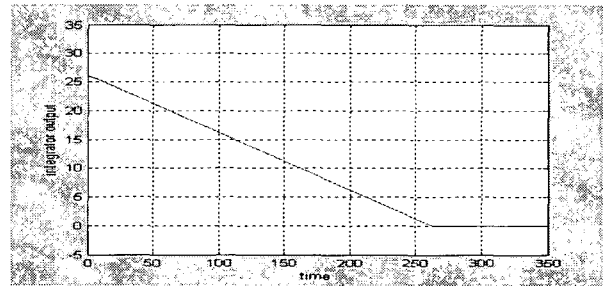


그림 3 적분기 출력값  
Fig. 3 The plot of the integrator output

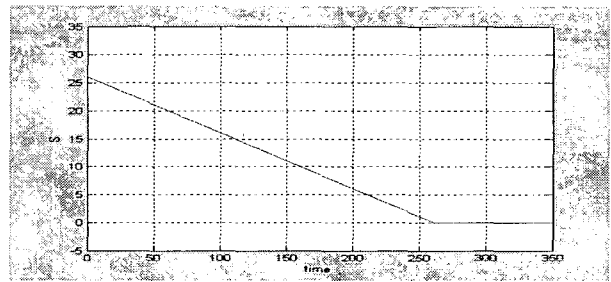


그림 4 S(슬라이딩 면)값의 변화  
Fig. 4 The plot of value of S(sliding surface)

## 6. Conclusion

We have designed a globally bounded output feedback integral sliding mode control. The integral sliding mode control can asymptotically stabilize the closed loop system in the presence of input coefficient uncertainty. The peaking does not exhibit in the states variables, even though a high-gain observer is used to estimate the state variable. We demonstrate the performance of the integral sliding mode controller via an example.

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