Journal of the Korean Association for Shell and Spatial Structures
Vol. 3, No. 2 (통권 8호), pp. 77~83, June. 2003

A Study on Multi-Objective Fuzzy Optimum Design of Truss Structures

牟在根* Mu, Zai-gen　葛 心** Ge, Xin　颜 谋** Yan, Mou　陈云周** Chen, Yun-zhou

Abstract

This paper presents decision making method of structural multi-objective fuzzy optimum problem. The data and behavior of many engineering systems are not know precisely and the designer is required to design the system in the presence of fuzziness in the multi-goals, constraints and consequences of possible actions. In this paper, in order to find a satisfactory solution, the membership functions are constructed for the fuzzy objectives subject to the fuzzy constraints, and two approaches are presented by using the different types of fuzzy decision making. Thus, multi-objective fuzzy optimum problem can be converted into single objective non-fuzzy optimum problem and satisfactory solution of the multi-objective fuzzy optimum problem can be found with general optimum programming. Illustrative numerical example of the ten bar truss for minimum weight and minimum deflection is provided to demonstrate the process of finding the solution and the results are discussed.

keywords : multi-objective fuzzy optimization; fuzzy decision making; truss structures

1. Introduction

In fact, for an optimum design of structures there exists a vast amount of uncertainties in resistances as well as in applied loads. It has been recognized that some uncertainties which are not random in nature play important roles in the establishment of rational design methods. It seems that these uncertainties should be treated by using fuzzy sets.[1,2] Consequently, various efforts are being made to apply fuzzy set theory to solve structural optimization problems since there exists a vast amount of fuzzy information on both the objective and constraint functions for the optimum design of structures.

* University of Science and Technology Beijing Associate Professor
** University of Science and Technology Beijing Graduate Student

Most practical structural design problems involve optimization of several, often conflicting, objectives subject to the constraints containing fuzzy information. Thus, the multi-objective fuzzy optimum design of structures aims at finding a satisfactory solution which is taken to be the best compromise between several conflicting objectives subject to the fuzzy constraints.[3,4] In order to find a satisfactory solution of the multi-objective fuzzy optimum problem of truss structures, the membership functions are constructed for the subsets of the fuzzy objectives subject to the fuzzy constraints, and then two approaches are presented by using the different types of fuzzy decision making, namely, the intersection decision and the non-symmetry decision. Finally, the application of the fuzzy optimization techniques proposed herein is illustrated by numerical examples of ten bar truss structure.

2. Multi-objective fuzzy optimization problem

2.1 Mathematical model

A multi-objective fuzzy optimum problem of structures can be stated as, find the design vector X which minimize the multi-objective function $F(X)$ subject to the fuzzy constraints on performance characteristics and dimensions. So the mathematical programming can be expressed in a standard form as follows:[5]

$$\left.\begin{array}{l}\min F(X)\\ s.t.\ g_j(X) \tilde{\leq} b_j^u,\ j=1,2,\ldots,m\\ \quad\ \ g_j(X) \tilde{\geq} b_j^l,\ j=m+1,m+2,\ldots,p\end{array}\right\} \tag{1}$$

where $X=(x_1,x_2,\ldots,x_n)^T$ is the design vector, $F(X)=(f_1(X),f_2(X),\ldots,f_q(X))^T$ is the vector of the objective function, $f_i(X)$ is the ith objective function, $g_j(X)$ is the jth constraints function, the wave symbols indicate that the constraints contain fuzzy information, and b_j^u, b_j^l are the allowable upper and lower limits of the jth constraints, respectively.

2.2 Membership function of fuzzy constraint subset

In the fuzzy constraint set $\tilde{C}_j$ for the allowable limits b_j of each fuzzy constraint $\tilde{C}_j$ and the maximums of tolerances d_j are given, so the membership functions of inclined straight lines[3] can be adopted.

Decreasing membership function:

$$\mu_{\tilde{C}_j}(X)=\begin{cases}1, & \text{if } g_j(X)<b_j^u\\ \dfrac{(b_j^u+d_j^u)-g_j(X)}{d_j^u}, & \text{if } b_j^u\leq g_j(X)\leq b_j^u+d_j^u\\ 0, & \text{if } b_j^u+d_j^u<g_j(X)\end{cases}$$
$$(j=1,2,\cdots,m) \tag{2}$$

Increasing membership function:

$$\mu_{\tilde{C}_j}(X)=\begin{cases}0, & \text{if } g_j(X)<b_j^l-d_j^l\\ \dfrac{g_j(X)-(b_j^l-d_j^l)}{d_j^l}, & \text{if } b_j^l-d_j^l\leq g_j(X)\leq b_j^l\\ 1, & \text{if } b_j^l<g_j(X)\end{cases}$$
$$(j=m+1,m+2,\cdots,p) \tag{3}$$

2.3 Membership function of fuzzy objective subset

For each of the objective functions $f_k(X)$, the possible minimum solution m_k is governed by the fuzziness of the constraints while the possible maximum solution M_k is affected by the minimum points of other objective functions. So the individual single-objective function $f_k(X)$ is changed in a specific range and the membership functions of the fuzzy objective $\tilde{G}_k$ can be written as follows:[1]

$$\mu_{\tilde{G}_k}(X)=\begin{cases}1, & \text{if } f_k(X)<m_k\\ \dfrac{M_k-f_k(X)}{M_k-m_k}, & \text{if } m_k\leq f_k(X)\leq M_k\\ 0, & \text{if } M_k<f_k(X)\end{cases}$$
$$(k=1,2,\cdots,q) \tag{4}$$

where $\mu_{\tilde{G}_k}(X)\in[0,1]$.

3. Fuzzy decision-making of multi-objective fuzzy optimum design

The fuzzy decision-making is to construct a fuzzy feasible domain $\tilde{D}$ characterized by its membership function $\mu_{\tilde{D}}(X)$ with fuzzy objectives $\tilde{F}$ and fuzzy constraints $\tilde{C}$, then, find the optimum point X^* which maximizes $\mu_{\tilde{D}}(X)$.

$$\mu_{\tilde{D}}(X^*)=\max_{x\in R^n}\mu_{\tilde{D}}(X) \tag{5}$$

For engineering design, the different types of fuzzy decision-making can be utilized. In this paper, we mainly introduced intersection decision making and non-symmetry decision making.

3.1 Intersection decision making

Intersection decision making can be viewed as the intersection of the fuzzy objectives and the fuzzy constraints, in which objectives and constraints are in the same degree and symmetrical. Thus, intersection decision also can be called symmetry decision.

$$\widetilde{D} = \widetilde{F} \cap \widetilde{C} \tag{6}$$

For the multi-objective fuzzy optimum problem, the fuzzy objectives and the fuzzy constraints are,

$$\widetilde{F} = \bigcap_{i=1}^{q} \widetilde{f}_i \quad \widetilde{C} = \bigcap_{j=1}^{p} \widetilde{C}_j \tag{7}$$

so the membership function of the intersection decision making is given as follows,

$$\begin{aligned} \mu_{\widetilde{D}}(X) &= \mu_{\widetilde{F}}(X) \wedge \mu_{\widetilde{C}}(X) \\ &= \left\{ \bigwedge_{i=1}^{q} \mu_{\widetilde{f}_i}(X) \right\} \wedge \left\{ \bigwedge_{j=1}^{p} \mu_{\widetilde{C}_j}(X) \right\} \\ &= \min_{i,j} \{ \mu_{\widetilde{f}_i}(X),\ \mu_{\widetilde{C}_j}(X) \} \end{aligned} \tag{8}$$

and substitution equation (8) for equation (5), namely,

$$\mu_{\widetilde{D}}(X^*) = \max_{x \in R^n} \min_{ij} \{ \mu_{\widetilde{f}_i}(X),\ \mu_{\widetilde{C}_j}(X) \} \tag{9}$$

Thus, the multi-objective fuzzy optimum problem can be transformed into the following single-objective non-fuzzy optimum problem[6]:

$$\begin{aligned} &max \quad \mu_{\widetilde{D}}(X) = \lambda, \\ &s.t. \quad \mu_{\widetilde{f}_i}(X) \geq \lambda, \quad i = 1, 2, \ldots, q \\ &\qquad \mu_{\widetilde{C}_j}(X) \geq \lambda, \quad j = 1, 2, \ldots, p \\ &\qquad 0 \leq \lambda \leq 1 \end{aligned} \tag{10}$$

The intersection decision making reflects the most conservative idea of the decision-maker, because for all of the objectives and constraints only the worst component is optimized and the rest are neglected so that unfortunately much information is lost.

3.2 non-symmetry decision making

In non-symmetry decision making fuzzy objectives and fuzzy constraints are not in the same degree and symmetrical. It aims at finding the optimal solution in condition of satisfied constraints. So the non-symmetry decision making can be viewed as the intersection of the fuzzy objectives. It is expressed as follows:

$$\mu_{\widetilde{D}}(X) = \bigwedge_{i=1}^{q} \mu_{\widetilde{f}_i}(X) \tag{11}$$

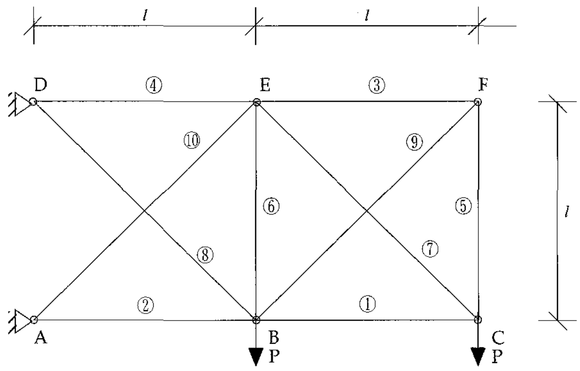
Thus, the multi-objective fuzzy optimum problem can be transformed into the following single objective non-fuzzy optimum problem:

$$\begin{aligned} &max \quad \lambda \\ &s.t. \quad g_j(X) \leq b_j^u + d_j^u \quad j = 1, 2, \cdots, m \\ &\qquad g_j(X) \geq b_j^l - d_j^l \quad j = m+1, m+2, \cdots, p \\ &\qquad \mu_{fi}(X) \geq \lambda \quad i = 1, 2, \cdots, q \\ &\qquad 0 \leq \lambda \leq 1 \end{aligned} \tag{12}$$

Based on the above analysis, a computer program has been written for the multi-objective fuzzy optimum problem of structures.

4. Numerical example

The numerical example of the ten-bar truss shown in Fig.1 and multi-objectives of minimizing weight and vertical deflection of node C are considered.



〈Fig. 1〉 Ten-bar truss

Initial design condition is as follows:

$p=444920N$, $l=9144mm$, $\rho=7850\times10^{-9}kg/mm^3$, $E=206\times10^3N/mm^2$;

and the allowable limits, tolerances of various physical variables are given by

$D^u=630mm$, $D^l=32mm$, $d_D{}^u=63mm$, $d_D{}^l=3.2mm$; $t^u=16mm$, $t^l=2.5mm$, $d_t{}^u=1.6mm$ $d_t{}^l=0.25mm$,;

$f^u=215N/mm^2$, $315N/mm^2$, $350N/mm^2$, $d_f{}^u=10.75N/mm^2$, $15.75N/mm^2$, $17.50N/mm^2$;

$\lambda^u=150$, $\lambda^{u\prime}=250$, $d_\lambda{}^u=15$, 25; $k^u=100$, $d_k{}^u=10$; $(f_2/L)^u=1/400$, $d_f{}^u=0.000125$.

where D, t are the diameter and the thickness of the steel tube, f is the tensile or compression strength, is slenderness of tube bar member, k is the ratio of the diameter to the thickness.

Let x_1 is the cross sectional area of top chord bars and bottom chord bars, x_2 is the cross sectional area of the erect bars, x_3 is the cross sectional area of string web bars. And N is the internal force under loads, $\overline{N}$ is the internal force under unit load.

Thus, the multi-objective fuzzy optimum problem of ten bar truss can be formulated as follows (take Q235 steel as an example):

$$min\quad f_1(X)=\rho l(4x_1+2x_2+4\sqrt{2}x_3)$$

$$f_2(X)=\frac{l}{Ex_1}(\sum_{i=1}^{4}\overline{N_i}\cdot N_i)+\frac{l}{Ex_2}(\sum_{i=5}^{6}\overline{N_i}\cdot N_i)+\frac{l}{Ex_3}(\sum_{i=7}^{10}\overline{N_i}\cdot N_i)$$

$$s.t.\quad C_1=D_i\gtrsim 32\ \ i=1,2,3;\ \ C_2=t_i\gtrsim 2.5\ \ i=1,2,3;$$

$$C_3=D_i\lesssim 630\ \ i=1,2,3;\ \ C_4=t_i\lesssim 16\ \ i=1,2,3;$$

$$C_5=\frac{f_2}{2l}\lesssim\frac{1}{400};$$

$$C_6=\frac{N_i}{\phi_j x_j}\lesssim 215\quad i=2,10\quad j=1,3;$$

$$C_7=\frac{N_i}{x_j}\lesssim 215\quad i=4,6,8\quad j=1,2,3;$$

$$C_8=\frac{D_i}{t_i}\lesssim 100\quad i=1,2,3;$$

$$C_9=\frac{L_i}{r_i}\lesssim 150\quad i=1,3;$$

$$C_{10}=\frac{L_i}{r_i}\lesssim 250\quad i=2;$$

where,

$$N_1=-(\frac{x_2}{x_1^2}+\frac{\sqrt{2}x_2}{x_1x_3}+\frac{5}{2x_1}+\frac{1}{2x_2}+\frac{3\sqrt{2}}{x_3})\cdot\frac{p}{\triangle},$$

$$N_2=-(\frac{4x_2}{x_1^2}+\frac{4x_2}{x_3^2}+\frac{6\sqrt{2}x_2}{x_1x_3}+\frac{13}{2x_1}+\frac{1}{x_2}+\frac{5\sqrt{2}}{x_3})\cdot\frac{p}{\triangle};$$

$$N_3=(\frac{x_2}{x_1^2}+\frac{4x_2}{x_3^2}+\frac{3\sqrt{2}x_2}{x_1x_3}+\frac{1}{2x_1})\cdot\frac{p}{\triangle},$$

$$N_4=(\frac{4x_2}{x_1^2}+\frac{12x_2}{x_3^2}+\frac{10\sqrt{2}x_2}{x_1x_3}+\frac{11}{2x_1}+\frac{1}{x_2}+\frac{7\sqrt{2}}{x_3})\cdot\frac{p}{\triangle};$$

$$N_5=N_3=(\frac{x_2}{x_1^2}+\frac{4x_2}{x_3^2}+\frac{3\sqrt{2}x_2}{x_1x_3}+\frac{1}{2x_1})\cdot\frac{p}{\triangle},$$

$$N_6=(\frac{x_2}{x_1^2}+\frac{8x_2}{x_3^2}+\frac{5\sqrt{2}x_2}{x_1x_3}+\frac{\sqrt{2}}{x_3})\cdot\frac{p}{\triangle};$$

$$N_7=(\frac{\sqrt{2}x_2}{x_1^2}+\frac{2x_2}{x_1x_3}+\frac{5\sqrt{2}}{2x_1}+\frac{\sqrt{2}}{2x_2}+\frac{6}{x_3})\cdot\frac{p}{\triangle},$$

$$N_8=(\frac{2\sqrt{2}x_2}{x_1^2}+\frac{4x_2}{x_1x_3}+\frac{7\sqrt{2}}{2x_1}+\frac{\sqrt{2}}{2x_2}+\frac{4}{x_3})\cdot\frac{p}{\triangle};$$

$$N_9=-(\frac{\sqrt{2}x_2}{x_1^2}+\frac{4\sqrt{2}x_2}{x_3^2}+\frac{6x_2}{x_1x_3}+\frac{\sqrt{2}}{2x_1})\cdot\frac{p}{\triangle},$$

$$N_{10}=-(\frac{2\sqrt{2}x_2}{x_1^2}+\frac{8\sqrt{2}x_2}{x_3^2}+\frac{12x_2}{x_1x_3}+\frac{5\sqrt{2}}{2x_1}+\frac{\sqrt{2}}{2x_2}+\frac{8}{x_3})\cdot\frac{p}{\triangle};$$

$$\triangle=(\frac{2x_2}{x_1^2}+\frac{4x_2}{x_3^2}+\frac{4\sqrt{2}x_2}{x_1x_3}+\frac{3}{x_1}+\frac{1}{2x_2}+\frac{3\sqrt{2}}{x_3}).$$

Based on the analysis of Section 1 and Section 2, the multi-objective fuzzy optimum numerical example can be transformed into single objective non-fuzzy optimum problem and can be solved, the results of single-objective optimization are given in <Table 1>.

〈Table 1〉 Results of single-objective optimization

Items	Q235	16Mn	15MnV
f_{1min}	3480.76	3465.68	3466.78
f_{2max}	48.01	48.00	48.01
f_{2min}	6.27	6.29	6.27
f_{1max}	27648.53	27563.85	26218.36

Thus, the membership functions of the fuzzy objectives are,

$$\mu_{f_1}(x)=\begin{cases}1 & f_1(x)<3480.76\\ \dfrac{27648.53-f_1(x)}{24167.77} & 3480.76\le f_1(x)\le 27648.53\\ 0 & f_1(x)>27648.53\end{cases}$$

$$\mu_{f_2}(x)=\begin{cases}1 & f_2(x)<6.27\\ \dfrac{48.01-f_2(x)}{41.74} & 6.27\le f_2(x)\le 48.01\\ 0 & f_2(x)>48.01\end{cases}$$

and the membership functions of the fuzzy constraints are,

$$\mu_{C_1}(x)=\begin{cases}1 & C_1>32\\ \dfrac{C_1-28.8}{3.2} & 28.8\le C_1\le 32\\ 0 & C_1<28.8\end{cases}$$

$$\mu_{C_2}(x)=\begin{cases}1 & C_2>2.5\\ \dfrac{C_2-2.25}{0.25} & 2.25\le C_2\le 2.5\\ 0 & C_2<2.25\end{cases}$$

$$\mu_{C_3}(x)=\begin{cases}1 & C_3<630\\ \dfrac{693-C_3}{63} & 630\le C_3\le 693\\ 0 & C_3>693\end{cases}$$

$$\mu_{C_4}(x)=\begin{cases}1 & C_4<16\\ \dfrac{17.6-C_4}{1.6} & 16\le C_4\le 17.6\\ 0 & C_4>17.6\end{cases}$$

$$\mu_{C_5}(x)=\begin{cases}1 & C_5<0.0025\\ \dfrac{0.002625-C_5}{0.000125} & 0.0025\le C_5\le 0.002625\\ 0 & C_5>0.002625\end{cases}$$

$$\mu_{C_6}(x)=\begin{cases}1 & C_6>-215\\ \dfrac{225.75+C_6}{10.75} & -225.75\le C_6\le -215\\ 0 & C_6>-225.75\end{cases}$$

$$\mu_{C_7}(x)=\begin{cases}1 & C_7<215\\ \dfrac{225.75-C_7}{10.75} & 215\le C_7\le 225.75\\ 0 & C_7>225.75\end{cases}$$

$$\mu_{C_8}(x)=\begin{cases}1 & C_8<100\\ \dfrac{110-C_8}{10} & 100\le C_8\le 110\\ 0 & C_8>110\end{cases}$$

$$\mu_{C_9}(x)=\begin{cases}1 & C_9<150\\ \dfrac{165-C_9}{15} & 150\le C_9\le 165\\ 0 & C_9>165\end{cases}$$

$$\mu_{C_{10}}(x)=\begin{cases}1 & C_{10}<250\\ \dfrac{275-C_{10}}{25} & 250\le C_{10}\le 275\\ 0 & C_{10}>275\end{cases}$$

In accordance with equations (10) and (12), two different types of fuzzy decision-making are used for the optimum design of the ten bar truss. The mathematical programming can be expressed as follows:

The intersection decision-making,

$$\begin{aligned}&\max && \mu_{\tilde{D}}(x)=\lambda\\ &s.t. && \mu_{\tilde{F}}(x)\ge\lambda \quad \mu_{\tilde{C}}(x)\ge\lambda\\ &&& \mu_X^u(x)\ge\lambda \quad \mu_X^l(x)\ge\lambda\\ &&& 0\le\lambda\le 1\end{aligned}$$

The non-symmetry decision-making,

$$\begin{aligned}&\max && \lambda\\ &s.t. && \mu_{\tilde{F}}(x)\ge\lambda\\ &&& g_j(x)\le b_j^u+d_j^u\\ &&& g_j(x)\ge b_j^l-d_j^l\\ &&& 0\le\lambda\le 1\end{aligned}$$

Finally, the results of the multi-objective fuzzy optimization of the ten bar truss are given in <Table 2>.

<Table 2> shows that with the increasing of the yield strength, objective function f_1 (weight) is decreasing, objective function f_2 (deflection) is increasing and membership function $\mu_{\tilde{D}}(X^*)$ are not changed much. Holding yield strength, the objective functions f_1 and f_2 got from intersection design are smaller than those $\mu_{\tilde{D}}(X^*)$ got from non-symmetry design and got from intersection design are bigger than those got from non-symmetry design. That is the results from intersection design are better.

5. Conclusion

In this paper, the fuzzy decision making method of structural multi-objective fuzzy optimum problem has been presented. Illustrative numerical example of the ten bar truss for minimum weight and minimum deflection shows that multi-objective fuzzy optimum design can be easily converted into single objective non-fuzzy optimum design using the fuzzy decision making and can be further solved by using conventional mathematical programming methods. Besides, the fuzzy range of goals and constraints are subjectively decided by designer, which is necessary for the optimum design of structure.

Acknowledgement

The authors gratefully acknowledge the support from National Natural Science Foundation of China No. 50078004.

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〈Table 2〉 Results of the multi-objective fuzzy optimization

Item	Q235		16Mn		15MnV	
	Intersection decision	non-sym. decision	Intersection decision	non-sym. decision	Intersection decision	non-sym. decision
D1(mm)	350.917	296.811	420.880	554.304	395.554	510.695
D2(mm)	188.943	142.087	140.678	148.235	154.046	177.840
D3(mm)	329.689	517.451	388.626	467.562	378.349	416.698
T1(mm)	14.546	16.994	12.225	9.375	12.447	10.179
T2(mm)	2.579	4.559	3.716	2.838	3.259	4.129
T3(mm)	12.566	7.940	10.267	8.364	10.662	8.90
F1(kg)	9708.98	9727.47	9686.34	9688.42	9519.27	9547.78
F2(mm)	17.03	17.06	17.06	17.05	17.38	17.41
$\mu_{\tilde{D}}(X^*)$	0.742	0.741	0.742	0.741	0.734	0.733

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