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A STUDY ON FAITHFUL AND MONOGENIC R-GROUPS

YONG UK CHO

ABSTRACT. Throughout this paper, we will consider that R is a near-ring and G is an R-group. We initiate the study of monogenic and strongly monogenic R-groups and their basic properties Also, we investigate some properties of D G. R-groups, faithful R-groups and monogenic R-groups and we determine that when near-rings are rings.

1. Introduction

In this paper, R is a near-ring, that is, R is an algebraic system $(R, +, \cdot)$ with two binary operations $+$ and $\cdot$ such that $(R, +)$ is a group (not necessarily abelian), $(R, \cdot)$ is a semigroup and the left distributive law holds: $a(b+c) = ab + ac$ for all a, b, c in R. If R has a unity 1, then R is called *unitary*. If 0 is the neutral element of the group $(R, +)$ then the left distributive law implies the identity $a0 = 0$ for all $a \in R$. However, $0a$ is not equal to 0, in general. An element d in R is called *distributive* if $(a+b)d = ad + bd$ for all a and b in R. A near-ring R with $(R, +)$ is abelian is called an *abelian* near-ring.

We consider the following notations: Given a near-ring R,

$$R_0 = \{a \in R \mid 0a = 0\}$$

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which is called the *zero symetric part* of R,

$$R_c = \{a \in R \mid 0a = a\} = \{a \in R \mid ra = a, \ for\ all\ r \in R\}$$

which is called the *constant part* of R, and

$$R_d = \{a \in R \mid a\ is\ distributive\}$$

which is called the *distributive part* of R.

We note that R_0 and R_c are subnear-rings of R, but R_d is not a subnear-ring of R. A near-ring R with the extra axiom $0a = 0$ for all $a \in R$, that is, $R = R_0$ is said to be *zero symmetric,* also, in case $R = R_c$, R is called a *constant* near-ring, and in case $R = R_d$, R is called a *distributive* near-ring. From the Pierce decomposition theorem, we get

$$R = R_0 \oplus R_c$$

as additive groups. So every element $a \in R$ has a unique representation of the form $a = b + c$, where $b \in R_0$ and $c \in R_c$.

An *ideal* of R is a subset I of R such that (i) $(I, +)$ is a normal subgroup of $(R, +)$, (ii) $a(I + b) - ab \subseteq I$ for all $a, b \in R$, (iii) $(I + a)b - ab \subseteq I$ for all $a, b \in R$. If I satisfies (i) and (ii) then it is called a *left ideal* of R. If I satisfies (i) and (iii) then it is called a *right ideal* of R.

On the other hand, a *(two-sided) R-subgroup* of R is a subset H of R such that (i) $(H, +)$ is a subgroup of $(R, +)$, (ii) $RH \subseteq H$ and (iii) $HR \subseteq H$. If H satisfies (i) and (ii) then it is called a *left R-subgroup* of R. If H satisfies (i) and (iii) then it is called a *right R-subgroup* of R.

Note that normal R-subgroups of R may not be ideals of R.

Similarly, a subset H of R such that $RH \subseteq H$ is called a *left R-subset* of R, a subset H of R such that $HR \subseteq H$ is called a *right R-subset* of R, and a left and right R-subset H is said to be a *(two-sided) R-subset* of R.

Also, a subset H of R is called a *base* (of equality) if for all $a, b \in R$ and all $x \in H$ $xa = xb$ implies $a = b$.

Let $(G, +)$ be a group (not necessarily abelian). In the set

$$M(G) := \{f \mid f : G \longrightarrow G\}$$

of all the self maps of G, if we define the sum $f + g$ of any two mappings $f,\ g$ in $M(G)$ by the rule $x(f+g) = xf + xg$ for all $x \in G$ (called the *pointwise addition of maps*) and the product $f \cdot g$ by the rule $x(f \cdot g) = (xf)g$ for all $x \in G$, then $(M(G),\ +,\ \cdot)$ becomes a near-ring. It is called the *self map near-ring* of the group G or *near-ring of self maps* on G.

Also, if we define the set

$$M_0(G) := \{f \in M(G) \mid of = o\}$$

for additive group G with identity o, then $(M_0(G),\ +,\ \cdot)$ is a zero symmetric near-ring.

Let R and S be two near-rings. Then a mapping θ from R to S is called a *near-ring homomorphism* if for all $a,\ b \in R$, (i) $(a+b)\theta = a\theta + b\theta$ and (ii) $(ab)\theta = a\theta b\theta$.

We can replace homomorphism by momomorphism, epimorphism, isomorphism, endomorphism and automorphism, if these terms have their usual meanings as for rings ([1]).

Let R be any near-ring and G an additive group. Then G is called an *R-group* if there exists a near-ring homomorphism

$$\theta . (R,\ +,\ \cdot) \longrightarrow (M(G),\ +,\ \cdot).$$

Such a homomorphism θ is called a *representation* of R on G, we write that xr (right scalar multiplication in R) for $x(\theta_r)$ for all $x \in G$ and $r \in R$. If R is unitary, then R-group G is called *unitary*. Thus an R-group is an additive group G satisfying (i) $x(a+b) = xa + xb$, (ii) $x(ab) = (xa)b$ and (iii) $x1 = x$ (if R has a unity 1), for all $x \in G$ and $a,\ b \in R$. Sometimes, we denote an R-group G simply by G_R. We note that R itself is an R-group called the *regular group*.

Moreover, naturally, every group G has an $M(G)$-group structure, from the representation of $M(G)$ on G given by applying $f \in M(G)$ to the $x \in G$ as a scalar multiplication xf.

An R-group G with the property that for each $x,\ y \in G$ and $a \in R$, $(x+y)a = xa + ya$ is called a *distributive R-group*, and also an R-group G with $(G,\ +)$ is abelian is called an *abelian R-group*. For example, if $(G,\ +)$ is abelian, then $M(G)$ is an abelian near-ring and moreover, G is an abelian $M(G)$-group, on the other hand, every distributive near-ring R is a distributive R-group. We can seek a distributive abelian R-groups at lemma 2.22, in section 2.

We denote that the neutral element of G by o, this is different from the neutral element 0 of the near-ring R, also we write the trivial groups (or ideals) of G and R as $\{o\} =: o$ and $\{0\} =: 0$ respectively.

A representation θ of R on G is called *faithful* if $Ker\theta = \{0\}$ In this case, we say that G is a *faithful R-group*, or that R *acts faithfully* on G.

For an R-group G, a subgroup T of G such that $TR \subseteq T$ is called an *R-subgroup* of G, and an *R-ideal* of G is a normal subgroup N of G such that $(N+x)a - xa \subseteq N$ for all $x \in G,\ a \in R$. The R-ideals of the regular group R are precisely the right ideals of R. Also, a subset V of G such that $VR \subseteq V$ is called an *R-subset* of G.

Let $G,\ T$ be two additive groups (not necessarily abelian). Then the set

$$M(G,\ T) := \{f \mid f : G \longrightarrow T\}$$

of all maps from G to T becomes an additive group under pointwise addition of maps. Since $M(T)$ is a near-ring of self maps on T, we obtain that $M(G,\ T)$ is an $M(T)$-group with a scalar multiplition.

$$M(G,\ T) \times M(T) \longrightarrow M(G,\ T)$$

defined by $(f,\ g) \longmapsto f \cdot g$, where $x(f \cdot g) = (xf)g$ for all $x \in G$.

Let G and T be two R-groups. Then a mapping $f : G \longrightarrow T$ is called a *R-group homomorphism* if for all $x,\ y \in G$ and $a \in R$, (i) $(x+y)f = xf + yf$ and (ii) $(xa)f = (xf)a$.

Also, we can replace R-group homomorphism by R-group momomorphism, R-group epimorphism, R-group isomorphism, R-group endomorphism and R-group automorphism, if these terms have their usual meanings as for modules ([1]).

A near-ring R is called *distributively generated* (briefly, $D.G.$) by S if

$$(R,+) = gp < S >= gp < R_d >$$

where S is a semigroup of distributive elements in R.

In particular, $S = R_d$ (this is motivated by the fact that the set of all distributive elements of R is multiplicatively closed and contains the unity of R if it exists), where $gp < S >$ is a group generated by S, we denote this D G. near-ring R which is generated by S is (R, S).

We also note that the set of all distributive elements of $M(G)$ are obviously the set $End(G)$ of all endomorphisns of the group G, that is,

$$(M(G))_d = End(G)$$

which is a semigroup under composition, but not yet a near-ring. Here we denote $E(G)$ is the D.G. near-ring generated by $End(G)$, that is,

$$E(G) = (M(G)),\ End(G).$$

Obviously, $E(G)$ is a subnear-ring of $(M_0(G), +, \cdot)$. Thus we say that $E(G)$ is the *endomorphism near-ring* of the group G.

Let (R, S) and (T, U) be D.G. near-rings. Then a near-ring homomorphism

$$\theta : (R, S) \longrightarrow (T, U)$$

is called a *D.G. near-ring homomorphism* if $S\theta \subseteq U$. Clearly, any near-ring epimomorphism $\theta : (R, S) \longrightarrow (T, U)$ is a D.G. near-ring homomorphism.

Note that a semigroup homomorphism $\theta : S \longrightarrow U$ is a D.G. near-ring homomorphism if it is a group homomorphism from $(R, +)$ to $(T, +)$ ([5], [6]).

For any group G, $M(G)$-group G and $M_0(G)$-group G are strongly monogenic which are appeared in Pilz [9].

For the remainder concepts and results on near-rings and R-groups, we refer to Meldrum [8], and Pilz [9].

2. Some results of faithful and related R-groups

There is a module like concept as follows: Let (R, S) be a D.G. near-ring. Then an additive group G is called a *D.G. (R, S)-group* if there exists a D.G. near-ring homomorphism

$$\theta : (R, S) \longrightarrow (M(G), End(G)) = E(G)$$

such that $S\theta \subseteq End(G)$. If we write xr instead of $x(\theta_r)$ for all $x \in G$ and $r \in R$, then an D.G (R, S)-group is an additive group G satisfying the following conditions:

$$x(rs) = (xr)s$$

and

$$x(r + s) = xr + xs,$$

for all $x \in G$ and all $r, s \in R$,

$$(x + y)s = xs + ys,$$

for all $x, y \in G$ and all $s \in S$.

Such a homomorphism θ is called a *D.G. representation* of (R, S) on G. This D.G. representation is said to be *faithful* if $Ker\theta = \{0\}$. In this case, we also say that G is a *faithful D.G (R, S)-group.*

Let R be a near-ring and let G be an R-group. If there exists an x in G such that $G = xR$, that is, $G = \{xr \mid r \in R\}$, then G is called a *monogenic R-group* and the element x is called a *generator* of G, more specially, if G is monogenic and for each $x \in G$, $xR = o$ or $xR = G$, then G is called a *strongly monogenic R-group.* It is clear that $G \neq 0$ if and only if $GR \neq 0$ for any monogenic or strongly monogenic R-group G.

LEMMA 2.1. *Let R be a near-ring and G an R-group. Then we have the basic concepts:*

(1) *If I is a right ideal of R, then $IR_0 \subseteq I$.*
(2) *If A is an R-ideal of G, then A is an R_0-subgroup of G.*

From this useful lemma, we obtain the following several properties.

PROPOSITION 2.2. *For a near-ring R, the following are equivalent:*

(1) *R is a zero symmetric near-ring;*
(2) *Every right ideal of R is an R-subgroup of R.*

Proof. (1) $\Longrightarrow$ (2) is obtained from Lemma 2.1 (1).

(2) $\Longrightarrow$ (1) Suppose that every right ideal of R is an R-subgroup of R. Since 0 is a right ideal of R, 0 is an R-subgroup of R. Thus $0R = 0$. This implies that $R = R_0$. □

LEMMA 2.3 ([9]). *For an R-group G, we have the following.*

(1) *For any x in G, xR is an R-subgroup of G.*
(2) *For any R-subgroup A of G, we have that $oR = oR_c \subseteq A$.*

In Lemma 2.3 (2), oR is the smallest R-subgroup of G, so throughout this paper, we will write that

$$oR = oR_c =: \Omega.$$

We note that if R is zero symmetric, then $\Omega = \{o\} =: o$, and $\Omega = xR_c$ for all $x \in G$.

From Lemma 2.3 (2), we define the following concepts: An R-group G is called *simple* if G has no non-trivial ideal, that is, G has no ideals except o and G. Similiarly, we can define simple near-ring as ring case Also, R-group G is called *R-simple* if G has no R-subgroups except Ω and G.

We can explain the previous concepts elementwise: for example, a subgroup A of G such that $ar \in A$ for all $a \in A, r \in R$, is an *R-subgroup* of G, and an *R-ideal* of G is a normal subgroup N of G such that

$$(x + g)a - ga \in N$$

for all $x \in N$, $g \in G$ and $a \in R$ (Meldrum [8]).

LEMMA 2.4. *For an R-group G and a subgroup A of G, we have the following:*

(1) *A is an R-ideal of G if and only if A is an R_0-ideal of G.*

(2) *A is an R-subgroup of G if and only if A is an R_0-subgroup of G and $\Omega \subseteq A$.*

Proof. (1) Necessity is obvious. Suppose A is an R_0-ideal of G. Let $a \in A$, $x \in G$ and $r \in R$. Then since $R = R_0 \oplus R_c$, we rewrite that $r = s + t$, where $s \in R_0$ and $t \in R_c$. Thus we have

$$(a+x)r - xr = (a+x)(s+t) - x(s+t) = (a+x)s + (a+x)t - xt - xs.$$

Here, since $t \in R_c$, $(a+x)t - xt = t - t = 0$ so that $(a+x)r - xr = (a+x)s - xs$. Also since $s \in R_0$ and A is an R_0-ideal of G, $(a+x)s - xs \in A$, that is $(a+x)r - xr \in A$. Consequently, A is an R-ideal of G.
(2) This statement can be proved as a similar method of the proof of (1). □

Lemma 2.1 (2) and Lemma 2.4 imply the following proposition.

PROPOSITION 2.5. *For an R-group G with $\Omega \neq o$, we have the following:*

(1) *$G = \Omega$ if and only if G is strongly monogenic.*
(2) *R_0-simplicity implies simplicity for G.*

LEMMA 2.6 ([7]). *Let (R, S) be a D.G. near-ring. Then all R-subgroups and all R-homomorphic images of a (R, S)-group are also (R, S)-groups.*

Let G be an R-group and K, K_1 and K_2 be subsets of G. Define

$$(K_1 : K_2) := \{a \in R; K_2 a \subseteq K_1\}.$$

We abbreviate that for $x \in G$

$$(\{x\} : K_2) =: (x : K_2).$$

Similarly for $(K_1 : x)$. $(0 : K)$ is called the *annihilator* of K, sometimes denoted it by $A(K)$. Easily, we can drive that G is a faithful R-group, that is, R acts faithfully on G if $A(G) = \{0\}$, that is, $(0 : G) = \{0\}$.

LEMMA 2.7 ([3]). *Let G be an R-group and K_1 and K_2 subsets of G. Then we have the following conditions:*

(1) *If K_1 is a normal subgroup of G, then $(K_1 : K_2)$ is a normal subgroup of a near-ring R.*
(2) *If K_1 is an R-subgroup of G, then $(K_1 : K_2)$ is an R-subgroup of R as an R-group.*
(3) *If K_1 is an R-ideal of G and K_2 is an R-subset of G, then $(K_1 : K_2)$ is a two-sided ideal of R.*

Proof. (1) and (2) are proved by Pilz [9] and Meldrum [8]. Now, we prove only (3) : Using the condition (1), $(K_1 : K_2)$ is a normal subgroup of R. Let $a \in (K_1 : K_2)$ and $r \in R$. Then

$$K_2(ra) = (K_2r)a \subseteq K_2a \subseteq K_1,$$

also, since K_2 is an R-subset of G, $K_2r \subseteq K_2$ we have $ra \in (K_1 : K_2)$. Whence $(K_1 : K_2)$ is a left ideal of R.

Next, let $r_1, r_2 \in R$ and $a \in (K_1 : K_2)$. Then

$$k\{(a + r_1)r_2 - r_1r_2\} = (ka + kr_1)r_2 - kr_1r_2 \in K_1$$

for all $k \in K_2$, since $K_2a \subseteq K_1$ and K_1 is an ideal of G. Thus $(K_1 : K_2)$ is a right ideal of R. Consequently, $(K_1 : K_2)$ is a two-sided ideal of R.

□

COROLLARY 2.8 ([8], [9]). *Let R be a near-ring and G an R-group.*

(1) *For any $x \in G$, $(0 : x)$ is a right ideal of R.*
(2) *For any R-subset K of G, $(0 : K)$ is a two-sided ideal of R.*
(3) *For any subset K of G, $(0 : K) = \bigcap_{x \in K}(0 : x)$.*

REMARK 2.9. *For any R-group homomorphism $f : G \longrightarrow T$, we have $(0 : G) \subseteq (0 : f(G))$. So every momomorphic image of a faithful R-group is also faithful. Moreover, for any R-group isomorphism $f : G \longrightarrow T$, we have $(0 : G) = (0 : T))$. In this case, G is faithful iff T is faithful.*

The following statement can be proved easily, but it is important later.

LEMMA 2.10 [9]. *Let G be a faithful R-group. Then we have the following conditions:*

(1) *If $(G, +)$ is abelian, then $(R, +)$ is abelian.*
(2) *If G is distributive, then R is distributive.*

From this Lemma, we get the following proposition:

PROPOSITION 2.11. *If G is a distributive abelian faithful R-group, then R is a ring.*

PROPOSITION 2.12. *Let R be a near-ring and G an R-group. Then we have the following conditions:*

(1) *$A(G)$ is a two-sided ideal of R. Moreover G is a faithful $R/A(G)$-group.*
(2) *For any $x \in G$, we get $xR \cong R/(0 : x)$ as R-groups.*

Proof. (1) By Corollary 2.8 and Lemma 2.7, $A(G)$ is a two-sided ideal of R. We now make G an $R/A(G)$-group by defining, for $x \in R$, $A(G)+r \in R/A(G)$, by $x(A(G)+r) = xr$. If $A(G)+r = A(G)+s$, then $r - s \in A(G)$ hence $x(r-s) = 0$ for all x in G, that is, $xr = xs$. This tells us that

$$x(A(G) + r) = xr = xs = x(A(G) + s)$$

Thus the action of $R/A(G)$ on G has been shown to be well defined. The verification of the structure of an $R/A(G)$-group is a routine triviality Finally, to see that G is a faithful $R/A(G)$-group, we note that if $x(A(G) + r) = 0$ for all $x \in G$, then by the definition of $R/A(G)$-group structure, we have $xr = 0$. Hence $r \in A(G)$, that is,

$$A(G) + r = A(G)$$

This says that only the zero element of $R/A(G)$ annihilates all of G. Thus G is a faithful $R/A(G)$-group.
(2) For any $x \in G$, clearly xR is an R-subgroup of G. The map $\phi : R \longrightarrow xR$ defined by $\phi(r) = xr$ is an R-group ephimorphism, so

that from the isomorphism theorem for R-groups, since the kernel of ϕ is $(0 : x)$, we deduce that

$$xR \cong R/(0 : x)$$

as R-groups. □

PROPOSITION 2.13. *If R is a near-ring and G an R-group, then $R/A(G)$ is isomorphic to a subnear-ring of $M(G)$.*

Proof. Let $a \in R$. We define $\tau_a : G \longrightarrow G$ by $x\tau_a = xa$ for each $x \in G$. Then τ_a is in $M(G)$. Consider the mapping $\phi : R \longrightarrow M(G)$ defined by $\phi(a) = \tau_a$. Then obviously, we see that

$$\phi(a + b) = \phi(a) + \phi(b) \text{ and } \phi(ab) = \phi(a)\phi(b),$$

that is, ϕ is a near-ring homomorphism from R to $M(G)$.

Next, we must show that $Ker\phi = A(G)$: Indeed, if $a \in Ker\phi$, then $\tau_a = 0$, which implies that $Ga = G\tau_a = 0$, that is, $a \in A(G)$. On the other hand, if $a \in A(G)$, then by the definition of $A(G)$, $Ga = 0$ hence $0 = \tau_a = \phi(a)$, this implies that $a \in Ker\phi$. Therefore from the first isomorphism theorem for R-groups, the image of R is a near-ring isomorphic to $R/A(G)$. Consequently, $R/A(G)$ is isomorphic to a subnear-ring of $M(G)$. □

COROLLARY 2.14. *If G is a faithful R-group, then R is embedded in $M(G)$. Furthermore, G is a faithful R-group iff G is both faithful R_0-group and faithful R_c-group.*

PROPOSITION 2.15. *If (R, S) is a D.G. near-ring, then every monogenic R-group is an (R, S)-group.*

Proof. Let G be a monogenic R-group with x as a generator. Then the map $\phi : r \longmapsto xr$ is an R-group epimorphism from R to G. We see that by Proposition 2.12 (2),

$$G \cong R/A(x),$$

where $A(x) = (0 : x) = Ker\phi$. From Lemma 2.6, we obtain that G is an (R, S)-group. □

LEMMA 2.16. *Let G be an R-group. Then G is faithful iff for each $x \in G$, $R \cong xR$.*

Proof. Suppose G is a faithful R-group. Then we can define the map $f : a \longmapsto xa$ which is an R-group epimorphism from R to xR as R-groups for each $x \in G$.

To show that f is one-one, if $f(a) = f(b)$ for $a,\ b \in R$, then $xa = xb$, that is, $x(a - b) = 0$ for all $x \in G$. This implies that $a - b \in \bigcap_{x \in G}(0 : x)$, which is equal to $(0 : G) = A(G)$ from Corollary 2.8 (3). Since G is faithful, $a - b = 0$. Hence for all $x \in G$, $R \cong xR$.

Conversely, assume the condition that $R \cong xR$ for all $x \in G$. Consider the map $f : R \longrightarrow xR$ given by $a \longmapsto xa$ is an R-group isomorphism. To show that G is faithful, take any element $a \in A(G)$, that is, $Ga = 0$. This implies that for all $x \in G$, $xa = 0$, that is, $f(a) = 0$. Since f is an R-group isomorphism, $a = 0$. Consequently, G is faithful. □

The following statement can be easily proved from Lemma 2.16 and Corollary 2.14.

PROPOSITION 2.17. *Let A be a right R-subgroup of a near-ring R. Then the following statements are equivalent:*

(1) *A is faithful;*
(2) *A is a base (of equality);*
(3) *A is embedded in $M(A)$;*
(4) *For all $x \in G$, $R \cong xR$.*

The following statement is a generalization of Proposition 2.11.

PROPOSITION 2.18 [3]. *Let (R, S) be a D.G. near-ring. If G is an abelian faithful D.G. (R, S)-group, then R is a ring.*

As an immediate consequence of Proposition 2.18, we have the following important corollary.

COROLLARY 2.19. *Let (R, S) be an abelian D.G. near-ring. Then R is a ring.*

LEMMA 2.20 ([2], [4]). *If R is a zero symmetric near-ring and A, B, K are R-ideals of an R-group G, then we have the following R-group:*

$$G' := [(A+K) \cap (B+K)]/[(A \cap B) + K]$$

which is abelian, and for any x, $y \in G'$, and $r \in R$, we have $(x+y)r = xr + yr$.

PROPOSITION 2.22. *Let R be a zero symmetric near-ring and G' be an R-group as in the above lemma. Then G' is a faithful $R/(0 : G')$-group. Moreover, $R' := R/(0 : G')$ becomes a ring.*

Proof. We can define the scalar multiplition as following: For $I = (o : G')$,

$$G' \times R/I \longrightarrow G'$$

defined by $(x,\ I + a) \longmapsto xa$, for all $x \in G'$ and all $I + a \in R/I$. Since $G'I = o$, this scalar multiplition is well defined, and it is easily proved that G' is faithful R/I-group. Hence the Lemma 2.10 (1) and (2) implies that $R/(o : G') = R/I$ becomes a ring. □

PROPOSITION 2.22. *Let G be a faithful monogenic R-group with generator x, where R is a zero symmetric near-ring. If I and J are right ideals of R and $I \cap J \subsetneq (0 : x)$, then R is a ring.*

Proof. From Proposition 2.12 (2) and 2.20, we have that

$$G = xR \cong R/(0:x) = [(I{+}(0:x){\cap}J{+}(0:x)]/[(I{\cap}J){+}(0:x)] = G'.$$

On the other hand, since G is faithful, by the definition, we see that

$$(0 : G') \cong (0 : G) = A(G) = 0.$$

Consequently, the Lemma 2.22 implies that R is a ring. □

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Department of Mathematics
College of Natural Sciences
Silla University, Pusan 617-736, Korea
E-mail: yucho@silla.ac.kr
yucho516@yahoo.co.kr