Linear Diversity Analysis for *M*-ary Square Quadrature Amplitude Modulation over Nakagami Fading Channels

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We derive and analyze the exact closed-form expression for the average bit error probability (BEP) of *M*-ary square quadrature amplitude modulation (QAM) for diversity reception in frequency-nonselective Nakagami fading. A maximal ratio combining (MRC) diversity technique with independent or correlated fading cases are considered. Numerical results demonstrate error performance improvement with the use of MRC diversity reception. The presented new expressions offer a convenient way to evaluate the performance of *M*-ary square QAM with an MRC diversity combiner for various cases of practical interest.

I. Introduction

Mobile radio systems require spectrally efficient modulation schemes because the available radio spectrum is limited. With the increasing demands of various mobile communication services, transmissions at higher rates will be required in band-limited mobile radio systems. Quadrature amplitude modulation (QAM) is a very attractive technique to achieve such a high rate transmission over wireless links without increasing the bandwidth [1]. A great deal of recent attention has been devoted to the study of bit error probability (BEP) for *M*-ary square QAM [2]-[5], and an exact and general closed-form expression of the BEP for *M*-ary square QAM with arbitrary constellation size in additive white Gaussian noise (AWGN) has recently been developed in [5].

For many digital mobile communication systems, the channel is subject to fading caused by multipath propagation. The Nakagami m-distribution is a versatile statistical model, which can accurately fit experimental data for many physical propagation channels. When the fading follows the Nakagami statistics, the probability density function (pdf) of the instantaneous signal-to-noise ratio (SNR) γ of a received signal is [6]

$$f(\gamma)=\frac{\gamma^{m-1}}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^{m}\exp\left(-\frac{m}{\bar{\gamma}}\gamma\right),\ \gamma\geq 0, \qquad (1)$$

where $\bar{\gamma}$ is the average SNR, $\Gamma(\cdot)$ is the gamma function [7], and m is the fading severity parameter (m≥0.5). The m-distribution includes the half-Gaussian (m=0.5), Rayleigh (m=1), and a non-fading (m=∞) as special cases, and it can be

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made to approximate other exact or experimentally derived distributions (i.e., Rician or log-normal) by judicious choice of parameters.

Diversity receptions are effective techniques for combating the detrimental effects of channel fading and have been discussed extensively in recent years to improve the communication system performance in multi-path fading channels. Diversity techniques for improving system performance without incurring a substantial penalty in terms of implementation complexity or cost are of practical interest. For coherent communication systems, one of the most prevalent space diversity techniques is maximal-ratio combining (MRC). The MRC technique, which provides the highest average output SNR, gives the best performance [8].

Many earlier studies on MRC diversity performance derived the error rate performance for MRC diversity in Nakagami fading [9]-[11]. Aalo et al. [9] derived a symbol error probability (SEP) for MRC reception of MPSK. Annamalai et al. [10] presented symbol error probabilities for binary and *M*-ary signals with *L*-th order MRC and selection combining (SC) diversity receptions. In [11], Aalo derived bit error probabilities of phase shift keying (PSK) and frequency shift keying (FSK) for MRC reception in a correlated Nakagami fading. However, the exact closed-form BER expression of *M*-ary square QAM with MRC diversity receptions in Nakagami fading channels has not yet been derived.

Using our previous work [5], we derive and analyze the exact closed-form expression of BEP for *M*-ary square QAM signals with *L*-branch MRC diversity reception affected by a frequency non-selective slow Nakagami fading and embedded in AWGN with the results of [11]. Generally, the BEP gives a more meaningful performance measure than SEP for comparison among different modulation schemes. MRC diversity techniques with identical and independent, nonidentical and independent, and identical but correlated Nakagami fading are considered.

The rest of this paper is organized as follows: Section II describes the system model used throughout this paper. The closed-form BEP expressions of *M*-ary square QAM for MRC diversity with independent or correlated Nakagami fading cases are presented in section III. In section IV numerical results are presented. Finally, we conclude with a brief summary of our work.

II. Preliminaries

The modulated square *M*-ary QAM signal is assumed to be transmitted over a frequency non-selective slow Nakagami fading channel. The conditional probability of a bit error for an *M*-ary square QAM signal in an AWGN environment is given by [5]

$$P_b(e\,|\,\gamma) = \frac{1}{\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} P_b(k), \tag{2}$$

where $P_b(k)$ is the probability that the *k*-th bit of the in-phase components and the quadrature components are in error in terms of SNR,

$$P_b(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \left\{ \Theta(i,k,M)\cdot \operatorname{erfc}\left[\sqrt{\gamma\,\Omega(i,M)}\right] \right\}, \tag{3}$$

where $\Theta(a,b,c) = (-1)^{F(\frac{a2^{b-1}}{c})} \cdot \left[2^{b-1} - F\left(\frac{a2^{b-1}}{c} + \frac{1}{2}\right)\right]$;

$F(\cdot)$ is the floor function;

$$k \in \left\{1,2,\cdots,\log_2\sqrt{M}\right\};\ \Omega(i,M) = \frac{3(2i+1)^2\cdot\log_2 M}{2(M-1)}.$$

The average bit error probability, $\overline{P_b}$, for diversity reception in a fading channel is obtained by

$$\overline{P_b} = \int_0^\infty P_b(e\,|\,\gamma) f_D(\gamma)\, d\gamma, \tag{4}$$

where $P_b(e|\gamma)$ is the bit error probability in AWGN and $f_D(\gamma)$ is the probability density function of the SNR at the output of the diversity combiner.

By substituting (2) into (4), $\overline{P_b}$ can now be expressed as follows:

$$\overline{P_b} = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \cdot \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Theta(i,k,M)\cdot\Psi(i,M) \tag{5}$$

where

$$\Psi(i,M) = \int_0^\infty \operatorname{erfc}\left(\sqrt{\gamma\cdot\Omega(i,M)}\right) f_D(\gamma)\, d\gamma. \tag{6}$$

Equation (5) shows that $\overline{P_b}$ is solely characterized by $\Psi(i,M)$, so we will investigate $\Psi(i,M)$.

Generally, for *L*-th order MRC diversity reception, we assume that each diversity channel has independent Nakagami fading, and that the prefect channel estimation is available at the receiver. Under this assumption, the pdf of the received SNR of an *i*-th single diversity branch is [12]

$$f_D(\gamma_i) = \frac{1}{\Gamma(m_i)}\left(\frac{m_i}{\overline{\gamma_i}}\right)^{m_i} \gamma_i^{m-1} \exp\left(-\frac{m}{\overline{\gamma_i}}\gamma_i\right),\ \gamma \geq 0, \tag{7}$$

where m_i and $\overline{\gamma_i}$ are the fading index and the average SNR on the *i*-th branch, respectively.

III. Bit Error Probability of *M*-ary Square QAM

1. Independent and Identical Fading

If the diversity channels are sufficiently separated, the assumption of statistical independence between the diversity branches is valid. For *L*-branch MRC in a Nakagami fading channel, where $m_i/\overline{\gamma_i}$ is the same for all diversity branches, the pdf of the SNR at the output of the MRC diversity receiver in Nakagami-*m* fading becomes [12]

$$f_D(\gamma)=\frac{1}{\Gamma(m_T)}\left(\frac{m_T}{\overline{\gamma_T}}\right)^{m_T}\gamma^{m_T-1}\exp\left(-\frac{m_T}{\overline{\gamma_T}}\gamma\right),\ \gamma\ge 0,\qquad(8)$$

where m_T is the total fading index, $m_T=\sum_{i=1}^{L}m_i$ and $\overline{\gamma_T}$ is the total average SNR after combining, $\overline{\gamma_T}=\sum_{i=1}^{L}\overline{\gamma_i}$.

$\Psi(i,M)$ can be obtained using the following identity [13].

$$\int_0^\infty e^{-at}t^{b-1}\operatorname{erfc}\left(\sqrt{ct}\right)dt=\frac{\Gamma\left(b+\frac{1}{2}\right)}{\sqrt{\pi}b(a+c)^b}\sqrt{\frac{c}{a+c}}\,{}_2F_1\left(1,b+\frac{1}{2};b+1;\frac{a}{a+c}\right),\qquad(9)$$

where *a*, *b*, and *c* are positive real, and ${}_2F_1(\cdot)$ is the hypergeometric function and is given by [7]

$${}_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}.\qquad(10)$$

Thus $\Psi(i,M)$ is expressed as follows:

$$\Psi(i,M)=\frac{1}{\sqrt{\pi}}\frac{\Gamma\left(m_T+\frac{1}{2}\right)}{\Gamma(m_T+1)}\sqrt{\frac{\Omega(i,M)\overline{\gamma_T}}{m_T+\Omega(i,M)\overline{\gamma_T}}}\left(\frac{m_T}{m_T+\Omega(i,M)\overline{\gamma_T}}\right)^{m_T}\cdot{}_2F_1\left(1,m_T+\frac{1}{2};m_T+1;\frac{m_T}{m_T+\Omega(i,M)\overline{\gamma_T}}\right).\qquad(11)$$

Note that $\Psi(i,M)$ for MRC diversity reception under Rayleigh fading is obtained simply by substituting 1 for m_i (i.e., $m_T=L$ in (11)). In addition, when *L*=1, $m_T=m_i=m$, and $\overline{\gamma_T}=\overline{\gamma_i}=\overline{\gamma}$, $\Psi(i,M)$ in (11) is reduced to the BEP expression for a Nakagami-*m* fading channel without a diversity combiner. When *M*=4, (5) using (11) reduces to the well-known BEP of QPSK for MRC in Nakagami fading.

If $m_i=m$ and $\gamma_i=\gamma$, then $m_T=mL$ and $\overline{\gamma_T}=\overline{\gamma}L$. In this case (8) and (11) respectively become

$$f_D(\gamma)=\frac{1}{\Gamma(mL)}\left(\frac{m}{\overline{\gamma}}\right)^{mL}\gamma^{mL-1}\exp\left(-\frac{m}{\overline{\gamma}}\gamma\right),\ \gamma\ge 0,\qquad(12)$$

$$\Psi(i,M)=\frac{1}{\sqrt{\pi}}\frac{\Gamma\left(mL+\frac{1}{2}\right)}{\Gamma(mL+1)}\sqrt{\frac{\Omega(i,M)\overline{\gamma}}{m+\Omega(i,M)\overline{\gamma}}}\left(\frac{m}{m+\Omega(i,M)\overline{\gamma}}\right)^{mL}\cdot{}_2F_1\left(1,mL+\frac{1}{2};mL+1;\frac{m}{m+\Omega(i,M)\overline{\gamma}}\right).\qquad(13)$$

2. Independent and Nonidentical Fading

For *L*-branch MRC in a Nakagami fading channel, where $m_i/\overline{\gamma_i}$ is distinct across the branches and m_i is integer, the pdf of the SNR in Nakagami fading becomes [12]

$$f_D(\gamma)=\prod_{i=1}^{L}\left(\frac{m_i}{\overline{\gamma_i}}\right)^{m_i}\cdot\left[\sum_{j=1}^{L}\sum_{k=1}^{m_j}\frac{\eta_j^{(m_j-k)}}{(m_j-k)!}\frac{\gamma^{k-1}}{(k-1)!}\exp\left(-\frac{m_j}{\overline{\gamma_j}}\gamma\right)\right],\ \gamma\ge 0,\qquad(14)$$

where

$$\eta_j^{(m_j-k)}=\frac{d^{m_j-k}}{ds^{m_j-k}}\left[\prod_{l=1,l\ne j}^{L}\left(\frac{m_l}{\overline{\gamma_l}}+s\right)^{-m_l}\right]\Bigg|_{s=-m_j/\overline{\gamma_j}}.\qquad(15)$$

Following steps similar to those used above, $\Psi(i,M)$ becomes

$$\Psi(i,M)=\prod_{x=1}^{L}\left(\frac{m_x}{\overline{\gamma_x}}\right)^{m_x}\sum_{y=1}^{L}\sum_{z=1}^{m_y}\frac{\eta_y^{(m_y-z)}}{(m_y-z)!}\frac{1}{\sqrt{\pi}}\frac{\Gamma\left(z+\frac{1}{2}\right)}{\Gamma(z+1)}\sqrt{\frac{\Omega(i,M)\overline{\gamma_y}}{m_y+\Omega(i,M)\overline{\gamma_y}}}\cdot\left(\frac{\overline{\gamma_y}}{m_y+\Omega(i,M)\overline{\gamma_y}}\right)^{z}{}_2F_1\left(1,z+\frac{1}{2};z+1;\frac{m_y}{m_y+\Omega(i,M)\overline{\gamma_y}}\right),\qquad(16)$$

where

$$\eta_y^{(m_y-z)}=\frac{d^{m_y-z}}{ds^{m_y-z}}\left[\prod_{l=1,l\ne y}^{L}\left(\frac{m_l}{\overline{\gamma_l}}+s\right)^{-m_l}\right]\Bigg|_{s=-m_y/\overline{\gamma_y}}.\qquad(17)$$

3. Correlated Fading

If the diversity branches are very closely spaced, the signals on different branches are no longer independent. Here, we consider the exact BEP performance of an MRC for the detection of square QAM signals in a correlated Nakagami fading environment where the diversity branches are correlated. We assume that the fading parameters in each diversity branch are identical. The two correlation models considered are the "constant correlation model" and the "exponential correlation model."

A. Constant Correlation

For the constant correlation model, the correlation coefficient between any two branches is constant (i.e., $\rho_{ij} = \rho$, where i, j = 1, 2,…, L and $0 \le \rho \le 1$). In this case, the pdf of the SNR is given by [11]

$$f_D(\gamma) = \frac{\left(\frac{m}{\bar{\gamma}}\right)^{mL} \gamma^{mL-1} \exp\left(\frac{-m\gamma}{\bar{\gamma}(1-\rho)}\right) {}_1F_1\left(m, mL; \frac{mL\rho\gamma}{\bar{\gamma}(1-\rho)\lambda}\right)}{(1-\rho)^{m(L-1)} \lambda^m \Gamma(mL)}, \gamma \ge 0, \tag{18}$$

where ρ is a correlation coefficient and $\lambda = 1 - \rho + L\rho$. When $\rho = 0$, (18) reduces to (12) and $\Psi(i, M)$ becomes (13). For $\rho > 0$, $\Psi(i, M)$ can be obtained by using the following identity [11], [14]

$$F_2(a; b_1, b_2; c_1, c_2; x, y) = \frac{1}{\Gamma(a)} \int_0^\infty t^{a-1} e^{-t} {}_1F_1(b_1, c_1; xt) \cdot {}_1F_1(b_2, c_2; yt) dt. \tag{19}$$

Thus $\Psi(i, M)$ becomes

$$\Psi(i,M) = 1 - \frac{2}{\sqrt{\pi}} \frac{\Gamma\left(mL + \frac{1}{2}\right)}{\Gamma(mL+1)} \sqrt{\frac{\Omega(i,M)\bar{\gamma}(1-\rho)}{m + \Omega(i,M)\bar{\gamma}(1-\rho)}} \cdot \left[\left(\frac{m}{m + \Omega(i,M)\bar{\gamma}(1-\rho)}\right)^L \left(\frac{1-\rho}{\lambda}\right)\right]^m \cdot F_2\left(mL + \frac{1}{2}; 1, m; \frac{3}{2}, mL; \frac{\Omega(i,M)\bar{\gamma}(1-\rho)}{m + \Omega(i,M)\bar{\gamma}(1-\rho)}, \frac{mL\rho}{\lambda\left[m + \Omega(i,M)\bar{\gamma}(1-\rho)\right]}\right), \tag{20}$$

where $F_2(\cdot)$ is Appell's hypergeometric function and given by [14]

$$F_2(a; b_1, b_2; c_1, c_2; x, y) = \frac{\Gamma(c_1)\Gamma(c_2)}{\Gamma(a)\Gamma(b_1)\Gamma(b_2)} \cdot \sum_{i=0}^\infty \sum_{j=0}^\infty \frac{\Gamma(a+i+j)\Gamma(b_1+i)\Gamma(b_2+j)}{\Gamma(c_1+i)\Gamma(c_2+j) i! j!} \cdot x^i y^j, \quad |x| + |y| < 1. \tag{21}$$

B. Exponential Correlation

For the exponential correlation model, the correlation coefficient between any two branches decreases exponentially as the separation between them increases (i.e., $\rho_{ij} = \rho^{|i-j|}$). In this case the pdf of the SNR can be very closely approximated by [11]

$$f_D(\gamma) \approx \frac{\gamma^{\beta-1}}{\Gamma(\beta)} \left(\frac{\beta}{L\bar{\gamma}}\right)^\beta \exp\left(-\frac{\beta}{L\bar{\gamma}}\gamma\right), \gamma \ge 0, \tag{22}$$

where $\beta = \frac{mL^2}{h}$, $h = L + \frac{2\rho}{1-\rho}\left(L - \frac{1-\rho^L}{1-\rho}\right)$, $0 \le \rho \le 1$.

Following steps similar to those used above and using the following identities [11],

$$\int_0^\infty x^{s-1} e^{-\beta x} \Gamma(a,x) dx = \frac{\Gamma(s+a) {}_2F_1\left(1, s+a; s+1; \frac{\beta}{\beta+1}\right)}{s(\beta+1)^{s+a}}, \tag{23}$$

$$\Gamma\left(n + \frac{1}{2}\right) = \sqrt{\pi} 2^{-2n+1} \frac{\Gamma(2n)}{\Gamma(n)}, \quad n = 0, 1, 2, \Lambda \tag{24}$$

$\Psi(i, M)$ can be expressed as follows:

$$\Psi(i,M) = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(\beta + \frac{1}{2}\right)}{\Gamma(\beta+1)} \sqrt{\frac{h\Omega(i,M)\bar{\gamma}}{mL + h\Omega(i,M)\bar{\gamma}}} \left(\frac{mL}{mL + h\Omega(i,M)\bar{\gamma}}\right)^\beta \cdot {}_2F_1\left(1, \beta + \frac{1}{2}; \beta + 1; \frac{mL}{mL + h\Omega(i,M)\bar{\gamma}}\right). \tag{25}$$

Note that when $\rho = 0$, (25) reduces to (13).

IV. Numerical Results

By using the closed-form expressions in (5), (11), (16), and (25), we obtained the numerical results of the average BEP versus the average SNR for arbitrary square *M*-QAM with the modulation level *M*, the order of diversity *L*, and fading severity parameter *m*. Figure 1 shows the average BEP for an independent and identical fading. A close inspection of these numerical results reveals that a significant improvement in BEP performance is achieved as the order of *L* increases from 1 to 2 and the improvement in BEP performance is somewhat retained as the order of *L* increases from 2 to higher orders. For example, at a BEP of 10^{-3}, there is a 7.5 dB diversity gain using *L*=4 over *L*=2, and as much as 13 dB diversity gain using *L*=2 over *L*=1 for 4-QAM. This confirmed that the BEP performance improvement was somewhat retained when the additional diversity branch was employed to an existing higher number of diversity branches.

Figure 2 shows the average BEP for an independent and nonidentical fading, where m_i is distinct across the branches. For example, for *L*=4, $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, and $m_4 = 4$

are assumed. As Figs. 1 and 2 show, an additional 4-5 dB of SNR is required to transmit an extra bit per dimension to maintain an average BEP of 10^{-3}.

Figure 3 shows the BEP performances of 16-QAM for the exponential correlation model in Nakagami fading for various values of correlation coefficient ρ and diversity order L with m=1. Figure 3 reveals that the exponential correlation ($\rho = 0.9$) resulted in as much as 9.5, 10.5, and 11 dB loss over the independent case ($\rho = 0$) for L=2, 3, and 4, respectively, at a BEP of 10^{-3}. However, the improvement in BEP performance was achieved as the order of L increased even though the diversity branches were correlated. For example, there was a 2.5 dB diversity gain using L=4 ($\rho = 0.5$) over L =2 ($\rho = 0$). The effect of the branch correlation on the MRC diversity gain for the exponential correlation model at a BEP of 10^{-3} for 16-QAM over L=1 is shown in Table 1.

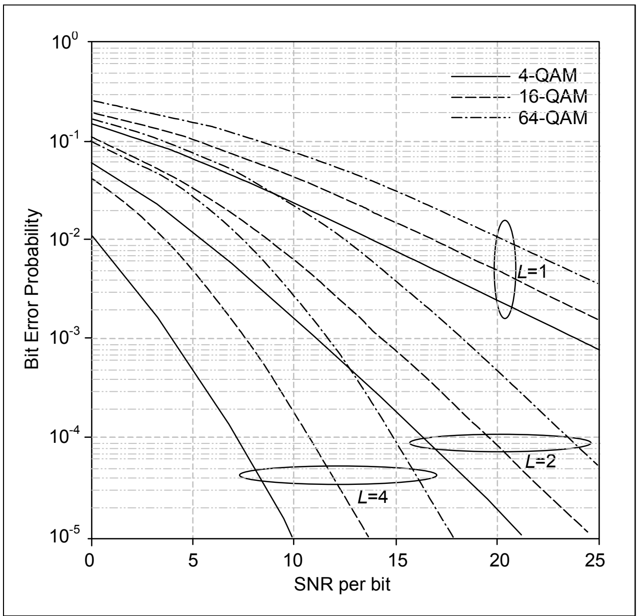

Fig. 1. Average BEP as a function of SNR for M-QAM with MRC diversity reception in an independent and identical Nakagami fading channel. (L=1,2,4, M=4, 16, 64 and m=1)

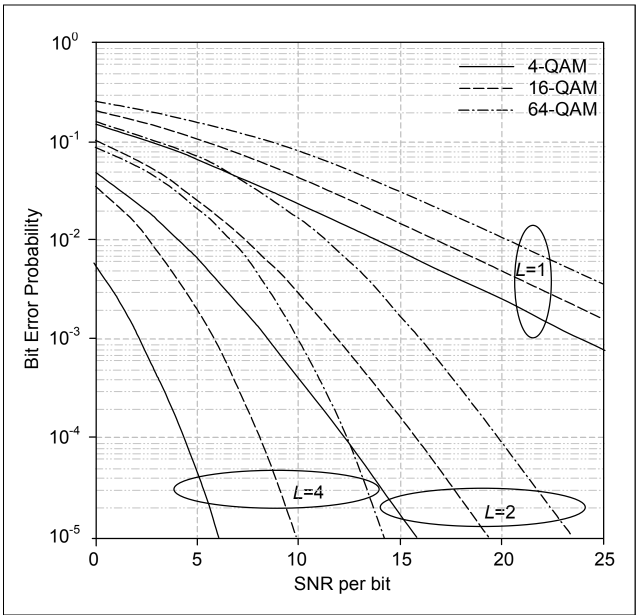

Fig. 2. Average BEP as a function of SNR for M-QAM with MRC diversity reception in an independent and nonidentical Nakagami fading channel. (M=4, 16, 64, L=1 for m=1, L=2 for m=1,2 and L=4 for m=1,2,3,4)

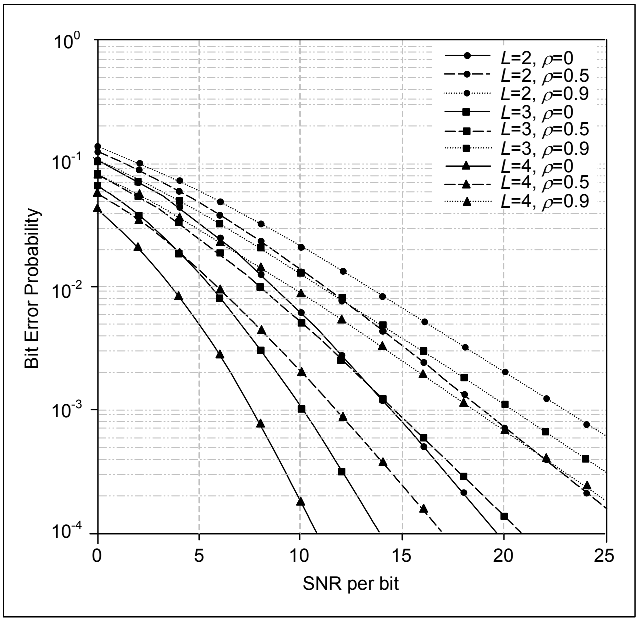

Fig. 3. Average BEP as a function of SNR for 16-QAM with MRC diversity reception in a Nakagami fading channel with exponential correlation. (L=2,3,4, ρ=0, 0.5, 0.9, m=1)

Table 1. Diversity gain at a BEP of 10^{-3} for 16-QAM with MRC diversity reception in Nakagami fading channels with exponential correlation.

ρ	m=1		m=2		m=4	
	L=2	L=4	L=2	L=4	L=2	L=4
0	12.6 dB	19.4 dB	6.9 dB	11.5 dB	4.7 dB	8.5 dB
0.5	8 dB	15.3 dB	5.2 dB	8.7 dB	3.8 dB	7.6 dB
0.9	4 dB	8.6 dB	3.5 dB	7.1 dB	3.2 dB	6.4 dB

V. Conclusions

In this paper, exact closed-form expressions for BEP of L-th order MRC diversity for M-ary square QAM signals have been derived and analyzed in identical and independent, nonidentical and independent, and identical but correlated Nakagami-*m* fading channels. The two correlation models considered were the constant correlation model and the exponential correlation model. For the particular case of m=1, the derived results easily led to the exact BEP for QAM with MRC diversity in Rayleigh fading channels. It is worth noting that for L =1, they became BEP performance in Nakagami fading without diversity. Since derived expressions are general, the exact BEP expression for various square QAM signal formats can be easily obtained by substituting parameters of interest, e.g., the signal constellation level M, fading parameter m or the order of diversity L. This offers a convenient way to evaluate the performance of arbitrary square QAM with an MRC diversity combiner for various cases of practical interest.

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